UNIT-3 :- MEASURE OF LOCATION AND DISPERSION

(1) Mean :-

(a) Direct method :-

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

(b) Assumed mean method :-

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

(c) Step deviation method :-

$$\bar{x} = \frac{\Sigma f_i u_i}{\Sigma f_i} \times h + a$$

where, $u_i = \frac{x_i - A}{h}$

(2) Median :-

case I :- n = even number :

$$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{ or } \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

case II :- when n = odd number :-

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term (observation)}$$

$$\text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$$

(3) **Mode** :- Maximum no. of observations.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Exercises :-

1. 2, 3, 4, 5, 0, 1, 3, 3, 4, 3.

Sol^n :- Arrange in ascending order.

0, 1, 2, 3, 3, 3, 3, 4, 4, 5.

$$\text{Mean} = \frac{0+1+2+3+3+3+3+4+4+5}{10} = \frac{28}{10} = 2.8$$

$$\text{Median} = \left(\frac{n}{2}\right)^{th} \text{ or } \left(\frac{n}{2}+1\right)^{th} \text{ term}$$

$$= \left(\frac{10}{2}\right)^{th} \text{ or } \left(\frac{10}{2}+1\right)^{th} \text{ term}$$

$$= 5^{th} \text{ or } 6^{th} \text{ term}$$

$$= \frac{6}{2} = 3$$

Mode = 3, because its frequency is appear more than other frequency i.e 4 times.

2. Given data is -

42, 39, 48, 52, 46, 62, 54, 40, 96, 52, 98, 40, 42, 52, 60.

Sol^n :- Here, $n = 15$

arrange in ascending order.

39, 40, 40, 41, 42, 46, 48, 52, 52, 52, 54, 60, 62, 96, 98

$$\text{Mean} = \frac{39+40+40+41+42+46+48+52+52+52+54+60+62+96+98}{15}$$

$$= \frac{822}{15} = 54.8$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term} = \left(\frac{15+1}{2}\right)^{th} \text{ term} = 8^{th} \text{ term} = 52$$

Mode = 52.

Exercise :- 13.1

1.

No. of plants	No. of houses (f_i)	x_i	$f_i x_i$
0-2	1	1	1
2-4	2	3	6
4-6	1	5	5
6-8	5	7	35
8-10	6	9	54
10-12	2	11	22
12-14	3	13	39
	$\Sigma f_i = 20$		$\Sigma f_i x_i = 162$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{162}{20} = 8.1$$

Mean no. of plants per house is 8.1.

By Assumed mean method :

2.

Daily wages	No. of workers(f_i)	(x_i)	$d_i = x_i - a$ (a = 550)	$f_i d_i$
500 - 520	12	510	-40	-480
520 - 540	14	530	-20	-280
540 - 560	8	a → 550	0	0
560 - 580	6	570	20	120
580 - 600	10	590	40	400
	$\Sigma f_i = 50$			$\Sigma f_i d_i = -240$

$$\text{Mean} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 550 + \left(\frac{-240}{50}\right)$$

$$= 550 + (-4.8)$$

$$= 545.2$$

Mean daily wages of workers is Rs. 545.2

3.

Daily pocket	f_i	x_i	$f_i x_i$
11 - 13	7	12	84
13 - 15	6	14	84
15 - 17	9	16	144
17 - 19	13	18	234
19 - 21	f	20	20f
21 - 23	5	22	110
23 - 25	4	24	96
	$\Sigma f_i = 44 + f$		$\Sigma f_i x_i = 752 + 20f$

Soln. - given, Median = 18.

Median $= l + \left(\frac{n/2 - cf}{f}\right) \times h$

$18 = \cancel{21 + \frac{44f\,5 - 20}{5}} \quad 21 + \left(\frac{44+f-20}{5}\right)$

Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$18 = \frac{772f}{44+f}$

$792 + 18f = 772f$

Mean $= \frac{\sum f_i x_i}{\sum f_i}$

$18 = \frac{752 + 20f}{44+f}$

$792 + 18f = 752 + 20f$

$792 - 752 = 20f - 18f$

$40 = 2f$

$f = \frac{\cancel{40}}{\cancel{2}}$

$\boxed{f = 20}$ Ans

4.

No. of heartbeats per min.	No. of women (f_i)	x_i	$f_i x_i$
65-68	2	66.5	133
68-71	4	69.5	278
71-74	3	72.5	217.5
74-77	8	75.5	604
77-80	7	78.5	549.5
80-83	4	81.5	326
83-86	2	84.5	169
	$\Sigma f_i = 30$		$\Sigma f_i x_i = 2277$

Soln. - Mean $= \frac{2277}{30} = 75.9$ Ans

(1) find the mean and variance for the data :-
6, 7, 10, 12, 13, 4, 8, 12

sol^n:- $\bar{x} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$.

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9
Σx_i		$\boxed{(\Sigma x_i - \bar{x})^2 = 74}$

• variance $(\sigma^2) = \frac{\Sigma(x_i - \bar{x})^2}{8}$

$= \frac{74}{8}$

$= 9.25$

• Standard deviation (S.D)

$= \sqrt{\sigma^2}$

$= \sqrt{9.25} = 3.04$

• Co-efficient of variance (C.V) $= \frac{S.D}{\bar{x}} \times 100$

$= \frac{3.04}{9} \times 100$

$= \frac{304}{9}$

$= 33.7$

ii) Mean and variance for the first n natural number

sum of first n natural numbers $= \frac{n(n+1)}{2}$

Solⁿ:- Mean $(\bar{x}) = \frac{n(n+1)}{2n} = \frac{n+1}{2}$

$$\text{variance}(\sigma^2) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{n}\sum\left(x_i - \frac{n+1}{2}\right)^2$$

$$= \frac{1}{n}\left\{\sum_{i=1}^{n} x_i^2 - 2\sum_{i=1}^{n} x_i\left(\frac{n+1}{2}\right) + \sum_{i=1}^{n}\left(\frac{n+1}{2}\right)^2\right\}$$

$$= \frac{1}{n}\left\{\frac{n(n+1)(2n+1)}{6} - 2\times\frac{n(n+1)}{2}\times\frac{n+1}{2} + \frac{(n+1)^2}{4}\times n\right\}$$

$$= (n+1)(2n+1) - \frac{(n+1)^2}{2} + \left(\frac{n+1}{4}\right)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \qquad \left\{\because 1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}\right\}$$

$$= \frac{(n+1)}{2}\left\{\frac{2n+1}{3} - \frac{n+1}{2}\right\}$$

$$= \frac{n+1}{2}\left\{\frac{2(2n+1) - 3(n+1)}{6}\right\}$$

$$= \frac{n+1}{2}\left\{\frac{4n+2-3n-3}{6}\right\} = \frac{n+1}{2}\times\frac{n-1}{6}$$

$$= \frac{n^2-1}{12} \quad \text{Ans}$$

(i) find the mean deviation about the mean for the data.
4, 7, 8, 9, 10, 12, 13, 17

Solⁿ:- Mean $(\bar{x}) = \frac{4+7+8+9+10+12+13+17}{8} = \frac{80}{8} = 10$

$|x_i - \bar{x}| = 6, 3, 2, 1, 0, 2, 3, 7 = \Sigma(x_i - \bar{x}) = 24$

$M.D(\bar{x}) = \sum_{i=1}^{n} \frac{|x_i - \bar{x}|}{n} = \frac{24}{8} = 3$ Ans

(ii) Mean deviation about median for the data.
13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17

Solⁿ:- $n = 12$
ascending order:-
10, 11, 11, 12, 13, 13, 14, 16, 16, 17, 17, 18

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th}\text{ term} + \left(\frac{n}{2}+1\right)^{th}\text{ term}}{2}$$

$$= \left(\frac{6^{th} + 7^{th}}{2}\right)\text{ term}$$

$$= \frac{13+14}{2} = 13.5$$

$|x_i - M| = 3.5, 2.5, 2.5, 1.5, 0.5, 0.5, 2.5, 2.5, 3.5, 3.5, 4.5$

$M.D(M) = \frac{\Sigma(x_i - M)}{n} = \frac{28}{12} = \frac{7}{3} = 2.33$ Ans

(iii) find the mean ~~median~~ deviation about the mean for the data:-

x_i	f_i	$x_i f_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	7	35	9	63
10	4	40	4	16
15	6	90	1	6
20	3	60	6	18
25	5	125	11	55
	$\Sigma f_i = 25$	$\Sigma f_i x_i = 350$		$\Sigma f_i \lvert x_i - \bar{x} \rvert = 158$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{350}{25} = 14$$

$$M.D(\bar{x}) = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i} = \frac{158}{25} = 6.32$$

(iv) Mean deviation about the mean.

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	$\Sigma f_i = 80$	$\Sigma f_i x_i = 4000$		$\Sigma f_i \lvert x_i - \bar{x} \rvert = 1280$

$$\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{400\cancel{0}}{8\cancel{0}} = 50$$

$$M.D(\bar{x}) = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i} = \frac{128\cancel{0}}{8\cancel{0}} = 16$$

(v) Mean deviation about median

x_i	f_i	c.f	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	$\Sigma f_i = 26$			$\Sigma f_i \lvert x_i - M \rvert = 84$

$N = 26 = \frac{N}{2} = \frac{26}{2} = 13$

$$\text{Median } (M) = \frac{(13^{th}) + (14^{th}) \text{ observation}}{2}$$

$$= \frac{7+7}{2} = 7$$

$$M.D(M) = \frac{\Sigma f_i \lvert x_i - M \rvert}{\Sigma f_i}$$

$$= \frac{84}{26}$$

$$= 3.25$$

5.1

(8) Mean deviation about median.

x_i	15	21	27	30	35
$f_i(n)$	3	5	6	7	8

x_i	$f_i(n)$	c.f	$\|x_i - 30\|$	$f_i\|x_i - 30\|$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	$\Sigma f_i = 29$			$\Sigma f_i\|x_i-30\| = 148$

$= \frac{N}{2} = \frac{29}{2} = 14.5$

The c.f just greater than 14.5 is 21 and corresponding value of x_i is 30.

Mean-deviation about mean $= \frac{\Sigma f_i |x_i - 30|}{\Sigma f_i} = \frac{148}{29} = 5.1$

(4) Mean deviation about mean $\Rightarrow$ 157.92

income per day	No. of persons (f_i)	x_i	$f_i x_i$
0 - 100	4	50	200
100 - 200	8	150	1200
200 - 300	9	250	2250
300 - 400	10	350	3500
400 - 500	7	450	3150
500 - 600	5	550	2750
600 - 700	4	650	2600
700 - 800	3	750	2250
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 17900$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{17900}{50} = 358$$

$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
50 - 358 = 308	1232
150 - 358 = 208	1664
108	972
8	80
92	644
192	960
292	1168
392	1176

$\Sigma f_i \lvert x_i - \bar{x} \rvert = 7896$

$$M.D = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i} = \frac{7896}{50} = 157.92.$$

(1.28)

(10) Mean deviation about mean

Height in cm	No. of boys (f_i)	x_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247
	$\Sigma f_i = 100$		$\Sigma f_i x_i = 12530$		$\Sigma f_i \lvert x_i - \bar{x} \rvert = 1128.3$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{12530}{100} = 125.3$$

$$\text{Mean-Deviation} = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{\Sigma f_i}$$

$$= \frac{1128.3}{100}$$

$$= 11.28$$

$\boxed{\Sigma f_i = n}$

(10.39)

11. Mean deviation about median

Marks	No. of girls (f)	c.f.	x_i (Mid-values)	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
0-10	6	6	5	22.86	137.16
10-20	8	14	15	12.86	102.88
20-30	14	28	25	2.86	40.04
30-40	16	44	35	7.14	114.24
40-50	4	48	45	17.14	68.56
50-60	2	50	55	27.14	54.28
	$\Sigma f_i (n) = 50$				$\Sigma f_i \lvert x_i - M \rvert = 517.16$

$\frac{n}{2} = \frac{50}{2} = 25$

$l = 20, \; h = 10, \; f = 14, \; c.f = 14.$

(M) Median $= l + \frac{\left(\frac{n}{2} - c.f\right)}{f} \times h$

$= 20 + \left(\frac{25 - 14}{14}\right) \times 10 = 20 + \frac{11}{\cancel{14}_7} \times \cancel{10}^5$

$= 20 + \frac{55}{7}$ 7.85

$= 27.85$

Mean-deviation about Median $= \frac{\Sigma f_i \lvert x_i - M \rvert}{\Sigma f_i} = \frac{517.16}{50} = 10.35$

$\boxed{\Sigma f_i = n}$

(7.35)

(12) Mean deviation about median

Age	(n) Number (f_i)	Mid-values (x_i)	c.f	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
16-20	5	18	5	20	100
21-25	6	23	11	15	90
26-30	12	28	23	10	120
31-35	14	33	37	5	70
36-40	26	38	63	0	0
41-45	12	43	75	5	60
46-50	16	48	91	10	160
51-55	9	53	100	15	135
	(n) $\Sigma f_i = 100$				$\Sigma f_i \lvert x_i - M \rvert = 735$

$= \frac{N}{2} = \frac{100}{2} = 50$

Median class is 35.5-40.5

$l = 35.5$, $f = 26$, $c.f = 37$, $h = 5$

$$\text{Median } (M) = l + \left(\frac{n/2 - cf}{f}\right) \times h$$

$$= 35.5 + \frac{50 - 37}{26} \times 5$$

$$= 35.5 + \frac{13}{26} \times 5 = 35.5 + \frac{5}{2}$$

$$= 35.5 + 2.5 = 38.$$

Mean deviation about median: $\frac{\Sigma f_i \lvert x_i - M \rvert}{(n)\Sigma f_i} = \frac{735}{100} = 7.35$

(13) find the mean and variance for the first 10 multiple of 3. :-

sol^n:- first 10 multiples of 3 are:-
3, 6, 9, 12, 15, 18, 21, 24, 27, 30

$$\text{Mean}(\bar{x}) = \frac{3+6+9+12+15+18+21+24+27+30}{10} = \frac{165}{10} = 16.5$$

x_i	$(\bar{x} - x_i)$	$(\bar{x} - x_i)^2$
3	-13.5	182.25
6	-10.5	110.25
9	-7.5	56.25
12	-4.5	20.25
15	-1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25

$$\Sigma(x_i - \bar{x})^2 = 742.5$$

$$\text{variance}(\sigma^2) = \frac{\Sigma(x_i - \bar{x})^2}{n} = \frac{742.5}{10} = 74.25$$

(14) find the mean and variance for the data.

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
6	2	12	-13	169	338
10	4	40	-9	81	324
14	7	98	-5	25	175
18	12	216	-1	1	12
24	8	192	5	25	200
28	4	112	9	81	324
30	3	90	11	121	363
	$\Sigma f_i = 40$	$\Sigma f_i x_i = 760$			$\Sigma f_i(x_i - \bar{x})^2 = 1736$

$$\text{Mean}(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{760}{40} = 19$$

$$\text{variance}(\sigma^2) = \frac{\Sigma f_i(x_i - \bar{x})^2}{\Sigma f_i} = \frac{1736}{40} = 43.4$$

15. find the mean and variance for the data:-

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
92	3	276	-8	64	192
93	2	186	-7	49	98
97	3	291	-3	9	27
98	2	196	-2	4	8
102	6	612	2	4	24
104	3	312	4	16	48
109	3	327	9	81	243
	$\Sigma f_i = 22$	$\Sigma f_i x_i = 2200$			$\Sigma f_i(x_i - \bar{x})^2 = 640$

$$\text{Mean}(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2200}{22} = 100$$

$$\text{variance}(\sigma^2) = \frac{\Sigma f_i(x_i - \bar{x})^2}{\Sigma f_i} = \frac{640}{22} = 29.09$$