VISCOSITY

The property of a moving fluid (liquid or gas) to oppose the relative motion between its layers is called viscosity of the fluid.

viscous force :
when a liquid (or fluid) flows, a backward dragging force is developed between its layers that retards the motion of liquid. This drag acts tangentially on the layers of the liquid is called viscous force

(Or)

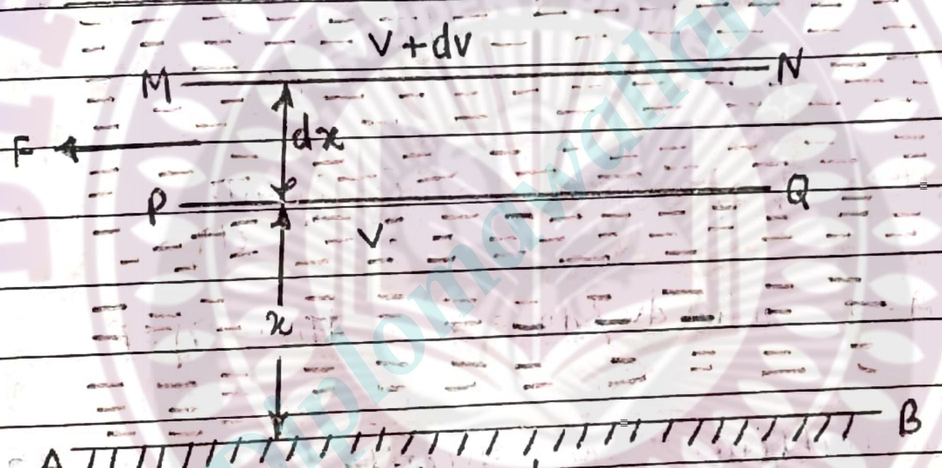
The force which oppose the relative motion of adjacent layers of the fluid is called viscous force of the liquid.

Gradient - Gradient of a physical quantity means "Rate of change of the physical quantity with respect to distance".

Newton's Law of Viscosity and Cofficient of Viscosity -

Let us consider the steady flow of a liquid over a horizontal solid surface AB. Consider two parallel layer PQ and MN, each of area A, at a distance x and $x+dx$ from the solid surface AB and moving with velocities V and $V+dV$ respectively. The increase in velocity divided by the distance over which this change occurs (i.e dv/dx) is called the velocity gradient and is measured perpendicular to the direction of velocity.

Liquid Surface



Solid surface

According to Newton, the viscous force F between the layers depends upon the following factors -

(i) The viscous force F is directly proportional to area A of the fluid layers in contact.

i.e, $F \propto A$ —— (1)

(ii) The viscous force F is directly proportional to the lateral velocity gradient $\frac{dv}{dx}$ betn the layers

$$F \propto \frac{dv}{dx} \quad \text{------} \quad (2)$$

Combining these two equation, we have

$$F \propto A \frac{dv}{dx}$$

$$F = -\eta A \frac{dv}{dx}$$

Definition of η. $F = \eta A \frac{dv}{dx}$ .....magnitude

If $A = 1$ and $dv/dx = 1$ then $\eta = F$

Hence coefficient of viscosity of a liquid is defined as the tangential force required to maintain a unit velocity gradient betn two parallel layers each of unit area.

unit of η. $\eta = \frac{F}{A \times \text{velocity gradient}}$

$$\text{SI unit of } \eta = \frac{1\,N}{1\,m^2 \times s^{-1}} = 1\,Ns\,m^{-2}$$

we find that the SI unit of η is Nsm^{-2}. It is also called decapoise or pascal second (Pa s).

Hence the coefficient of viscosity of a liquid is 1 decapoise (or 1 Nsm^{-2}). If a tangential force of 1 N is required to maintain a velocity gradient of 1 ms^{-1}/m between two parallel layers each of area 1 m^2

The corresponding C.G.S unit is dyne-s/cm^2 called poise and is in common use.

Clearly, 1 decapoise = 10 poise.

Dimension formula of η

$$\eta = \frac{F}{A \times \text{velocity gradient}}$$

$$[\eta] = \frac{[MLT^{-2}]}{[L^2][LT^{-1}]/[L]} = [ML^{-1}T^{-1}]$$

Discussion :

① The viscosity of an ideal liquid is zero.

② The coefficient of viscosity of a liquid decrease with the increase in temp. and vice-versa. However, the coefficient of viscocity of gases increases with the increase in temp.

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FLUID FLOW

The study of fluids in motion is called fluid dynamics.

① Nonviscous fluid : There is no internal friction between the adjacent layers of the fluid.

② Incompressible fluid : Density of the fluid is constant.

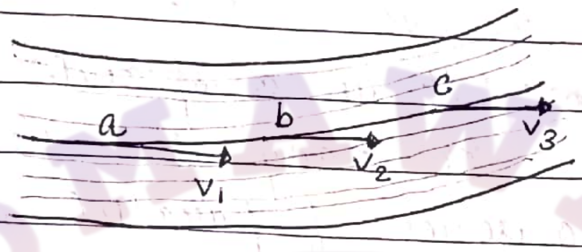
③ steady fluid : Velocity, density and pressure at each point in the fluid do not change with time.

Types of Liquid flow

The liquid flow is of two main types viz
① streamline flow or steady flow
② Turbulent flow.

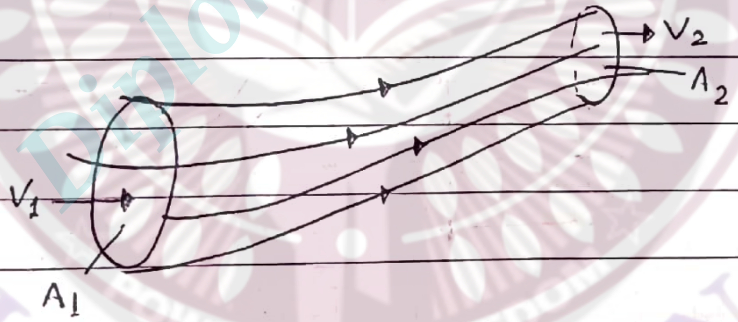
① streamline flow or steady flow
The flow of a liquid is said to be streamline flow or steady flow if all the liquid particles that pass any given point follow the same path at the same speed (i.e they

have the same velocity.)



The line abc is called streamline. At each point, the direction of velocity is tangent to the streamline.

A streamline is a curve whose tangent at any point is along the direction of the velocity of the liquid particle at that point.



streamline cannot cross. If they were to do so, particles reaching the intersection would not have a unique velocity at that point in space. A bundle of streamlines ~~represent~~ is called a tube of flow. Since the streamlines represent the paths of particles, we see that no liquid can flow into or out of the sides of a tube of flow. In a steady

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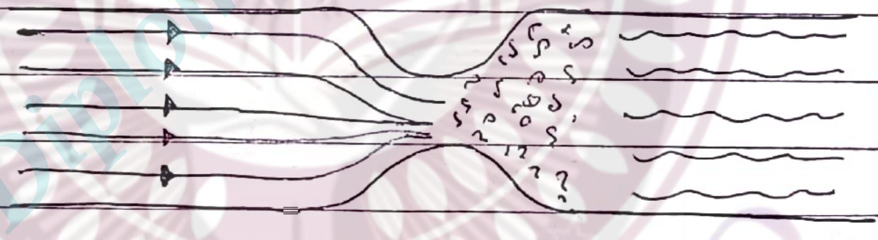
flow, the velocity, density and pressure at each point in the liquid do not change with time.

Laminar flow: It is a special case of streamline or steady flow in which the liquid flows as a series of parallel layer (laminae) and no one layer crosses another layer. Thus smooth streamline flow is known as laminar flow.

(2) Turbulent flow
The flow of a liquid is said to be turbulent flow or disorderly flow if the speed and direction of the liquid particles passing any point change with time.



REYNOLDS NUMBER

when a liquid flows in a pipe with a small velocity, the flow is smooth streamline or laminar flow. As the velocity is gradually increased, a stage is reached when the liquid flow becomes turbulent. whether the liquid flow is laminar or turbulent is decided by Reynolds number (N_R). The expression for Reynolds number is given by;

$$\boxed{\text{Renynolds number, } N_R = \frac{\rho v D}{\eta}}$$

where, ρ = Density of the liquid
v = Average velocity of flow
D = Diameter of the tube or pipe
η = Coefficient of viscosity of the liquid.

Experiments show that
- if N_R is less than 2000, the flow is laminar (or steady).
- If N_R is greater than 3000, the flow is turbulent.
- If N_R is betn 2000 and 3000, the flow is unstable. It may change from laminar to turbulent and vice versa.

Discussion: The following points may be noted-

(1) N_R is a pure number (all of its units cancel) that determines the nature of flow of the liquid through a pipe.

(2) The value of N_R is independent of the system of units used for the measurement of various quantities in Eq-

$$N_R = \frac{\rho v D}{\eta}$$

CRITICAL VELOCITY

That velocity of liquid flow upto which its flow is steady and above which its flow becomes turbulent is called critical velocity.

Experiments show that for cylindrical pipes critical velocity is given by :

$$v_c = \frac{1100\,\eta}{r\rho}$$

where, η = Coefficient of viscosity of the liquid
r = Radius of pipe
ρ = Density of the liquid.

STOKES' LAW

When a solid body moves through a fluid at rest, the fluid exerts a viscous force on the solid body to oppose the motion of the body.

The magnitude of the viscous force depends on
1. The shape and size of the solid body.
2. the speed of the body
3. The coefficient of viscocity of the fluid.

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Derivation of Viscous force for free fall of spherical body through viscous medium:

stok's observed that the viscous force F depends upon
① The radius r of the free falling body.
② The coefficient of viscosity η of the fluid and
③ The terminal velocity v_T of the free falling body in the fluid.

So, we assume a relation

$$F \propto r^a v_T^b \eta^c$$

$$F = K r^a v_T^b \eta^c \quad \text{——— ①}$$

where, K is some constant and its value can only be known by experiment, a, b and c are the exponents of r, v_T and η respectively which can be obtained by dimensional ~~ang~~ analysis.

Relation (1) may now written as

$$[MLT^{-2}] = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c$$

or

$$[MLT^{-2}] = [M^c L^{a+b-c} T^{-b-c}] \quad \text{——— ②}$$

Now by the principle of homogeneity of the dimension, equating exponents of M, L and T on both the sides of relation (2), we have -

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(i) $c = 1$

(ii) $a + b - c = 1$

(iii) $-b - c = -2$

After solving these equations we have $a = 1$, $b = 1$ and $c = 1$.

∴ Relation (2) becomes $F = k r v_T \eta$

Stok's slowed by exp. that when the medium be continous up to infinity then, $k = 6\pi$ ∴ $F = 6\pi r v_T \eta$

Importance of stockes' law –

1. It is used in Millikan's experiment for the measurment of charge on an electron.
2. It accounts for the formation of clouds.
3. It explains why large rain drops hurt much more than small ones when they fall on you. It is not just that they are heavier; they are actually falling faster.
4. It is used to find the size of small particles.
5. It is useful in the consideration of geological processes in which the rate of sedimentation is important.

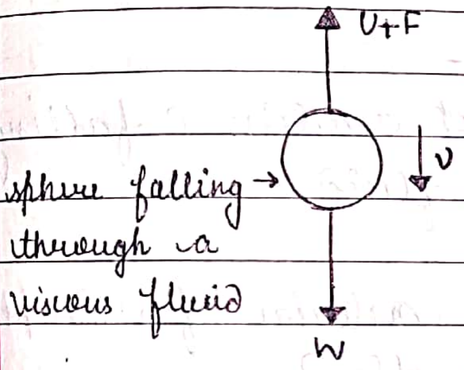
TERMINAL VELOCITY

Consider a small sphere falling freely from rest through a large column of a viscous fluid. The forces acting on the sphere are:

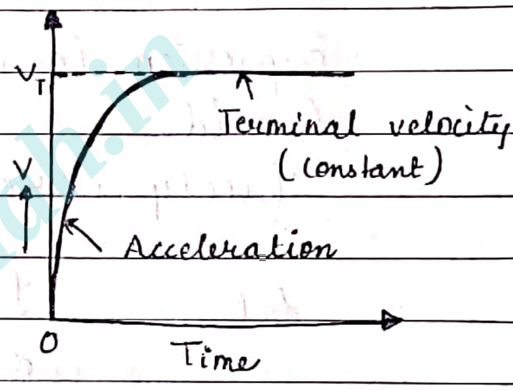
① weight W of the sphere acting vertically downwards.

② upthrust U equal to the weight of the fluid displaced.

③ viscous drag F acting vertically upward i.e, in a direction opposite to motion of the sphere.



(i)



(ii)

Initially, the downward force W is greater than the upward force, U+F, and the sphere accelerates downwards. As the velocity of the sphere increases, the magnitude of velocity dependent viscous force F (stokes' law) also increases. Eventually, a stage is reached when U+F = W. Since the resultant force on the sphere is zero, it now moves downwards with a constant maximum velocity, called terminal velocity V_T.

The maximum constant velocity acquired by a body while falling freely through a viscous medium is called terminal velocity of the body.

Fig (ii) shows that variation of the velocity of the velocity v of the freely falling sphere till the terminal velocity V_T is reached.

Expression for terminal velocity -
Consider a small sphere of radius r falling freely through a viscous fluid

Let,
ρ = Density of the material of the sphere
σ = Density of the fluid
η = coefficient of viscosity of the fluid

weight of sphere, $W = mg = \left(\frac{4}{3}\pi r^3 \rho\right) g$

$$\cancel{g} = \frac{4}{3}\pi r^3 \rho g$$

upthrust, $U = \frac{4}{3}\pi r^3 \sigma g$

At terminal velocity $V_T = F = 6\pi \eta r V_T$

We know that at the terminal velocity, $W = U + F$

or, $\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v_T$

or. $\frac{4}{3}\pi r^3 (\rho-\sigma) g = 6\pi\eta r v_T$

or, $\boxed{\text{Terminal velocity, } v_T = \frac{2}{9} \times \frac{r^3(\rho-\sigma)g}{\eta}}$

This eqⁿ gives the expression for the terminal velocity of a spherical body falling freely through a fluid.

Discussion - The following points are worth nothing -

① The terminal velocity (v_T) of a spherical body falling freely through a viscous fluid is directly proportional to the square of its radius. This means that for a given medium, the terminal velocity of a large sphere is greater than that of a small sphere of the same material. For this reason, bigger raindrops fall with greater velocity as compared with smaller ones.

② The terminal velocity (v_T) of a spherical body is directly proportional to the difference in the densities of the body and the fluid (i.e, $\rho-\sigma$). If the density of the fluid is greater than that of body (i.e, $\sigma > \rho$), then terminal velocity is negative.

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This means that the body instead of falling, moves upward. This is why bubbles of air rise up in water.

(3) The terminal velocity (V_T) of a spherical body is inversely proportional to the coefficient of viscocity of the fluid. This means that more viscous the fluid, the smaller the terminal velocity that the spherical body will ~~aq~~ acquire.

(4) If ρ, σ and η are known, measuring V_T can provide information regarding the size of the spherical particle. R.A. Millikan employed this technique in his historic measurement of charge on an electron.

Importance of Viscosity -

(1) The viscosity of blood depends upon the concentration of red blood corpuscles. Therefore, viscosity of blood can be used to detect blood corpuscle deficiency.

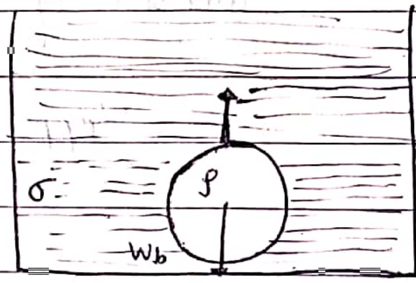
(2) Viscosity of air or liquid is used in providing damping torque in measuring instruments.

(3) Oil used as a lubricant should have proper value of viscosity.

(4) The viscosity of oil helps in applying bracks.

(5) Blood circulation through arteries depends upon the viscosity of blood.

Buoyant Force or Upthrust



The buoyant force on a body wholly or partially submerged in a fluid is equal to the weight of the fluid displaced by the body.

i.e, Buoyant force, F_B = weight of fluid displaced by the body immerged wholly or partially in the fluid.

We consider a spherical body.
Let, its radius = r
Density of the material of the body = ρ
We consider a fluid of density = σ

Let, $\rho > \sigma$

weight of the body, $W_b = \frac{4}{3}\pi r^3 \rho \cdot g \downarrow$ acting vertically downwards

Let, body is immerged in the fluid
volume of fluid displaced by body $V_L = \frac{4}{3}\pi r^3$ (fully immerged)

Its weight, $W_L = \frac{4}{3}\pi r^3 \sigma g \uparrow$ acting vertically upward, called upthrust or buoyant.

weight of the body in fluid, called apparent weight

$$W_{app} = W_b \downarrow - W_L \uparrow$$
$$= \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$
$$W_{app} = \frac{4}{3}\pi r^3 g(\rho - \sigma). \downarrow$$

- Effect of Temperature on viscosity:

(a) Liquid : The viscosity of liquids in general falls rapidly with rise of temperature.

(b) Gases : In the case of gases, viscosity increases with increases in temperature.

- Effect of pressure on viscosity:

(a) Liquid : For water, viscosity decreases with increases in pressure. Except this all liquid's viscosity increases in pressure.

(b) Gases : Viscosity of a gas is quite independent of pressure at ordinary temperature.