Units & Dimensions

L-1

Physical Quantity :-
Those quantities which can be measured is called physical quantity.

Or,

Those quantities in terms of which the law of physics will be expressed is called physical quantity.

Example :- Second law of motion.

Where, $F = ma$ — (i)

$$\left.\begin{aligned} F &= \text{Force} \\ m &= \text{Mass} \\ a &= \text{Acceleration} \end{aligned}\right\} \text{physical quantity}$$

Physical Quantity

- Basics or Fundamental Quantities
- Derived Quantities.

→ **Fundamental Quantities :-** Those quantities which are independent of the other quantities is called Basics quantities.

In Mechanics, Mass length & time are taken as fundamental quantities.

→ **Derived Quantities :-** Those physical quantities can be expressed with the help of fundamental physical quantities is called Derived physical quantities.

Eg:- Velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{L}{T} = LT^{-1}$

Acceleration = $\frac{\text{Change in Velocity}}{(\text{Time})^2} = \frac{L}{(T)^2} = LT^{-2}$

= Mass x Acceleration

= $m \times L \times (T)^{-2} = MLT^{-2}$

Unit :- To measure a physical quantity, we choose a standard, which is known as unit.

Unit

- Fundamental Unit
- Derived Unit

→ Fundamental Unit :- Unit of fundamental physical quantity is called fundamental unit.

→ Derived unit :- Unit of derived physical quantity is called Derived unit.

Unit of Derived physical quantities in C.G.S unit.

(i) Velocity = $\frac{\text{Displacement}}{\text{Time}} = \frac{cm}{s} = cms^{-1}$.

(ii) Acceleration = $\frac{\text{Change in velocity}}{\text{Time}} = \frac{cm}{s^2} = cms^{-2}$

(iii) Force = $m \times a = g\,cm^{-2}$ = dyne

(iv) Work = Force x displacement = $g \times cms^{-2} = g\,cms^{-2}$ = erg.

System Of Units :-

1) C.G.S System = In this system
→ Unit of mass = g
→ Unit of length = cm
→ Unit of time = s.

2) F.P.S System = In this system
→ Unit of mass = pound (lb)
→ Unit of length = foot (ft)
→ Unit of Time = Second (s)

(3) M.K.S System = In this System
Unit of mass = Kilogram (Kg)
Unit of length = Metre (m)
Unit of time = Second (s).

(4) S.I Unit = This System is also called as International System of units, In this system are 7 fundamental physical quantities.

P.T.O →

	Fundamental Quantity	Unit	
1.	Mass	Kg	
2.	Length	m	
3.	Time	s	
4.	Current	A	
5.	Temperature	T	
6.	Luminous Intensity	Cd	
7.	Amount of Substance	mole	

L-2

Dimension = The unit of any physical quantity in Symbol in dimenssion formula.

The power raised on the basic unit is called dimenssion.

Let unit of mass = "M".
unit of Length = "L".
unit of Time = "T".

for, example :-

(1). Velocity = $\frac{\text{displacement}}{\text{Time}} = \frac{L}{T} = [LT^{-1}]$

∴ Dimension of Velocity = 1 in length
−1 in time

(2). Acceleration = $\frac{\text{Change in Velocity}}{\text{Time}} = \frac{LT^{-1}}{T} = [L^{1}T^{-2}]$

(3) Force = Mass × Acceleration

$[F] = M \times LT^{-2}$

$\underbrace{[F] = [MLT^{-2}]}_{\text{Dimenssional Equation.}}$

(4). Area = Length × Breadth

$[A] = L \times L$

$[A] = [L^2]$

5 Volume = Length × Breadth × Height

$[V] = L \times L \times L$

$[V] = [L^3]$

6. Pressure = $\frac{\text{Force}}{\text{Area}} = \frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$

7. Surface Tension = $\frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = [MT^{-2}]$

8. Density = $\frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = [ML^{-3}]$

9. Linear Momentum = Mass × Velocity = $M \times LT^{-1} = [MLT^{-1}]$

10. Work = Force × Displacement = $MLT^{-2} \times L = [ML^2T^{-2}]$

11. Heat = ML^2T^{-2}

NOTE = DIMENSION OF ENERGY = DIMENSION OF WORK

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(12). Kinetic Energy = $\frac{1}{2}mv^2 = \frac{1}{2}M[LT^{-1}]^2$

$$[KE] = [ML^2T^{-2}]$$

(13). Potential Energy = mgh

$$[U] = M \times LT^{-2} \times L$$
$$[U] = [ML^2T^{-2}]$$

(14). Graxitional Constant (G) = $F = \frac{Gm_1m_2}{r^2}$

$$G = \frac{Fr^2}{m_1 m_2}$$

$\therefore$

$$[G] = \frac{\cancel{M}LT^{-2} \times L^2}{\cancel{M} \times M}$$
$$[G] = [M^{-1}L^3T^{-2}]$$

(15). Planck's Constant = $E = h\nu$

$$ML^2T^{-2} = hT^{-1} \qquad [\because \nu = 1/T]$$

$$[h] = \frac{ML^2T^{-2}}{T^{-1}}$$
$$[h] = [ML^2T^{-1}].$$

NOTE = ANGLE IS DIMENSIONLESS Quantity.

$$\theta = \frac{L}{r}$$

$$\theta = \frac{\cancel{L}}{\cancel{L}} = 1$$

$[\theta] = [M^0 L^0 T^0]$

[S.I. Unit of angle = Radian]

*** Those quantity which is unitless it must be diemenssionless but a diemenssionless quantity may be unitless. ***

L-3

Application of dimensional formula :-

1. Principal of homogenity = According to this principal diemenssional formula of each term in any physical equation are same.

Q. Check the equation = $S = ut + \frac{1}{2}at^2$. It is dimensionally correct or not?

Sol^n→ $S = ut + \frac{1}{2}at^2$

Diemension of $S = L$
Diemension of $ut = LT^{-1} \times L = L$
Diemension of $\frac{1}{2}at^2 = LT^{-2} \times T^2 = L$

$\therefore L = L + L$
L.H.S = R.H.S. Proved

Since diemension of each term in equation are same, Hence, it is diemensionally Correct.

Q. Check the weather the equation $S = ut - \frac{1}{3}at^2$ are dimensionally correct or not?

Solⁿ→ $S = ut - \frac{1}{3}at^2$

Diemension of $S = L$
Diemension of $ut = LT^{-1} \times T = L$
Diemension of $\frac{1}{3}at^2 = LT^{-2} \times T^2 = L$

$\therefore L = L - L$
L.H.S = R.H.S.
proved.

It is clear from the above discussion, Diemensionally correct equation may be physically correct. But diemensionally incorrect equation definetly physically incorrect.

Q. The velocity of a particle varies with time as the equation $V = at + bt^2$, where "a" & "b" are constant. Find the dimension formula of "a & b".

Solⁿ→

$V \propto t$

$\therefore V = at$

$a = \frac{V}{T}$

$a = \frac{LT^{-1}}{T}$

$a = LT^{-2}$

Similarly;

$V = bt^2$

$b = \frac{V}{t^2}$

$b = \frac{LT^{-1}}{T^2}$

$b = Lt^{-3}$. Ans.

Q Force acting on a particle is given by

$$F = a + bt^2 + \frac{ct}{d+t^2}$$, where, a, b, c & d are constant.

Find the dimension formula of a, b, c & d.

Sol^n $[F] = [a]$

$MLT^{-2} = [a]$

Again,

$[F] = bt^2$

$MLT^{-2} = [b]T^2$

$[b] = MLT^{-4}$.

Again,

$$[F] = \left[\frac{ct}{d+t^2}\right]$$

$$MLT^{-2} = \frac{[ct]}{d+T^2}$$

$$MLT^{-2} = \frac{[c]T}{dT^2}$$

$[c] = MLT^{-1}$. Ans.

Q. To check the accuracy of different physical equation relation. $T = 2\pi\sqrt{\frac{l}{g}}$ for a simple pendulum.

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~~L-3~~ L-4

Sol^n → Dimension of $T = T$

Dimension of $l = L$

Dimension of $g = LT^{-2}$

$$[T] = \sqrt{\frac{L}{LT^{-2}}}$$

$$T = \sqrt{T^2}$$

$$T = T.$$

Hence L.H.S = R.H.S.

Given formula or equation is dimensionaly Correct.

Q. To check the relation $x = x_0 + v_0 t + \frac{1}{2}at^2$

Where, x, x_0, are positions v_0 is inital velocity a is acc^n. and t is time.

Sol^n → Dimension of $x = L$

Dimension of $x_0 = L$

Dimension of $v_0 t = LT^{-1} \times T = L$.

Dimension of $at^2 = LT^{-2} \times T^2 = L$

$\therefore L = L + L + L$.

Proved

Since dimension of each term are Same, therefore given equation is dimensionally correct.

#

2.

Application - 2.

2. Conversion of a physical Quantity from one System of unit to another.

Let any physical Quantity x its numerical value is 'n' & Unit is 'u'

$$X = n.u.$$

Now, suppose,

In system I

n_1 = numerical value in system - I
u_1 = Unit in System - I
$X = n_1 u_1$ — (i)

In System - 2,

n_2 = numerical value in system - 2,
u_2 = Unit in System - 2 C.G.S System.

$$X = n_2 u_2 \quad \text{— (ii)}$$

From eq (i) & eq (ii)

$$n_1 u_1 = n_2 u_2$$

$$nu = \text{Constant}$$

$$\therefore \boxed{n \propto \frac{1}{u}}$$

Now,

In system - I

$$X = n_1 u_1$$

$$X = n_1 [M_1^a L_1^b T_1^c]$$

In system - II.

$$X = n_2 u_2$$

$$X = n_2 [M_2^a L_2^b T_2^c]$$

$$n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \times \left[\frac{L_1}{L_2}\right]^b \times \left[\frac{T_1}{T_2}\right]^c$$

Q. Pratice Questions :-

1. Convert 1N into Dyne.

Solⁿ→ We have formula,

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \times \left[\frac{L_1}{L_2}\right]^b \times \left[\frac{T_1}{T_2}\right]^c$$

Given;

$n_1 = 1$, $a = 1$, $b = 1$, $c = 2$

$$N_2 = 1\left[\frac{1kg}{1g}\right]^1 \left[\frac{1m}{1cm}\right]^1 \left[\frac{\cancel{1s}}{\cancel{1s}}\right]^2$$

$$N_2 = 1\left[\frac{1000\cancel{g}}{1\cancel{g}}\right]^1 \cdot \left[\frac{100\cancel{cm}}{1\cancel{cm}}\right] 1$$

$$N_2 = 10^5$$

$\therefore \quad N_2 = 10^5$

$$\boxed{1N = 10^5 \text{ dyne}}$$

2. Convert One joule into erg.

We have formula

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Given

$n=1, \quad a=1, b=2, c=-2.$

Now,

$$n_2 = 1\left[\frac{1kg}{1g}\right]^1 \left[\frac{1m}{1cm}\right]^2 \left[\frac{1s}{1s}\right]^{-2}$$

$$n_2 = 1 \times \left[\frac{1000\cancel{g}}{1\cancel{g}}\right]^1 \left[\frac{100\cancel{cm}}{1\cancel{cm}}\right]^2$$

$$n_2 = 1000 \times 10000 = 10^7.$$

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$n_2 = 10^7$

$\therefore \boxed{1J = 10^7 \text{ erg}}$.

Q. The universal gravitation constant have numerical value in S.I system given by $G = 6.67 \times 10^{-11}$ Nm^2 Kg^{-2}. find its value in C.G.S. System.

Sol^n → $G = 6.6 \times 10^{-11}$ Nm^2 Kg^{-2}

We have formula

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Given,

$n_1 = 6.67 \times 10^{-11}$, $a = -1$, $b = 3$, $c = -2$,

$$\therefore n_2 = 6.67 \times 10^{-11} \left[\frac{1 kg}{1 g}\right]^{-1} \times \left[\frac{1 m}{1 cm}\right]^3 \times \left[\frac{1 s}{1 s}\right]^{-2}$$

$$n_2 = 6.67 \times 10^{-11} \left[\frac{1000 g}{1 g}\right]^{-1} \times \left[\frac{100 cm}{1 cm}\right]^3.$$

$$n_2 = 6.67 \times 10^{-11} \times \frac{1}{1000} \times 1000000$$

$n_2 = 6.67 \times 10^{-8}$ dyne.

$\boxed{G = 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}}$ in C.G.S.

Application :- 3.

Established the relation between the two or more than physical quantity.

Q. Establish the time period of simple pendulum using the dimensional formula.

Sol^n→ Let the time period of simple pendulum using the dimensional formula.

→ Let time period of simple pendulum depends on mass of pendulum bob (m), length of string (l) & acceleration due to gravity (g).

$$T \propto m^a l^b g^c$$

Or

$$T = k m^a l^b g^c \quad \text{--- (i)}$$

(Where k = constant).

Put both side dimensional formula.

$$T = M^a L^b (LT^{-2})^c$$

$$M^0 L^0 T^1 = M^a L^{b+c} T^{-2c}$$

Equate both side dimension;

$\boxed{a = 0}$ — (i)

$b + c = 0$ — (ii)

$-2c = 1$

$\therefore c = -\frac{1}{2}$ — (iii)

from (ii) & (iii)

$b - \frac{1}{2} = 0$

$b = \frac{1}{2}$ Ans.

Putting value of a, b, & c in A.

$T = K M^0 l^{1/2} g^{-1/2}$

$T = k\left(\frac{l}{g}\right)^{1/2} = k\sqrt{\frac{l}{g}}$

Experimentally it is found that

Q. Coefficient of Viscosity;

Sol^n → $F = \eta \cdot A \cdot \frac{\Delta v}{\Delta x}$

$MLT^{-2} = [\eta] L^2 \times \frac{\cancel{L}T^{-1}}{\cancel{L}}$

$MLT^{-2} = [\eta] L^2 T^{-1}$

$$[\eta] = \frac{MLT^{-2}}{L^2 T^{-1}}$$

$$[\eta] = ML^{-1}T^{-1} \quad \underline{Ans.}$$

<u>Q.</u> $F = K r^a \eta^b v^c$, then value of a, b, c.

Sol^n → Put both side diamension formula.

$$MLT^{-2} = K L^a [ML^{-1}T^{-1}]^b [LT^{-1}]^c$$

$$MLT^{-2} = M^b L^{a-b+c} T^{-b-c}$$

equate.

$b = 1$ — (i)

$a - b + c = 1$ — (ii)

$-b - c = -2$ — (iii)

putting $b = 1$, in eq (ii)

$-1 - c = -2$

$-c = -2 + 1$

$\cancel{-}c = \cancel{-}1$

$c = 1$.

$\therefore\ a - b + c = 1$

$a - (-1) + 1 = 1$

$a = 1$. <u>Ans.</u>

Q. The time period of soap bubble is $T \propto P^a d^b s^c$ where P is pressure, d is density, s is surface tension. The the value of $a, b, \& c$.

Sol^n → Put both side dimension formula

$$M^0 L^0 T^1 = K [M^1 L^{-1} T^{-2}]^a [ML^{-3}]^b [M^1 T^{-2}]^c$$

$$T = M^{a+b+c} L^{-a-3b} T^{-2a-2c}$$

equate.

$a + b + c = 0$ — (i)

$-a - 3b = 0$ — (ii)

$-2a - 2c = 1$ — (iii)

from eq (i) (ii) (iii)

We get

$a = -\frac{3}{2}$.

$b = \frac{1}{2}$

$c = 1$. Ans

Q If momentum (P), area (A) & Time [T] are taken to be fundamental quanties, Then energy has

(a) $[P^1 A^{-1} T^{-1}]$
(b) $[P^2 A^1 T^1]$
(c) $[P^2 A^{-1/2} T^1]$
(d) $[P^1 A^{1/2} T^{-1}]$

Soln →

$$E = P^a A^b T^c$$

$$ML^2T^{-2} = [MLT^{-1}]^a [L^2]^b [T]^c$$

$$ML^2T^{-2} = M^a \cdot L^{a+2b} T^{-a+c}$$

equate.

$a = 1$. — (i)

$a + 2b = 2$ — (ii)

$2b = 1$ — (iii)

from eq (iii).

$b = \frac{1}{2}$ — (iv)

putting the value of b in (ii)

$a + 2b = 2$

$a + \cancel{2} \times \frac{1}{\cancel{2}} = 2$

$a = 2$

$\therefore c = -2 + 1 = -1$ Ans

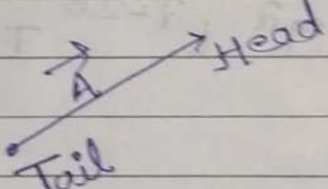
$\therefore E = P^1 A^{1/2} T^{-1}$.

-: VECTOR :-

Vector (Means an arrow):- Those physical quantity which can be represented by an arrow are Known as vector quantity and ~~there~~ those which cannot be represented by an arrow are Known as Scalar Quantity.

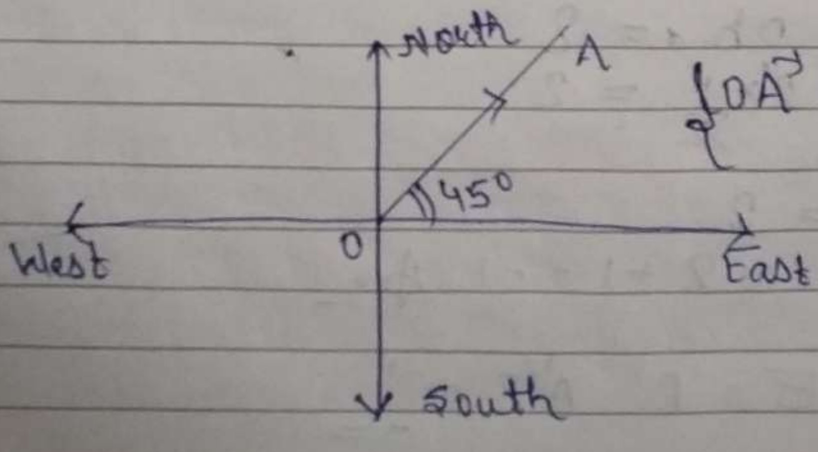
Representation of vector:-

→ A vector is represented by an arrow length of arrow representation its ~~mag~~ magnitude and Sense of arrow gives the direction of vector.

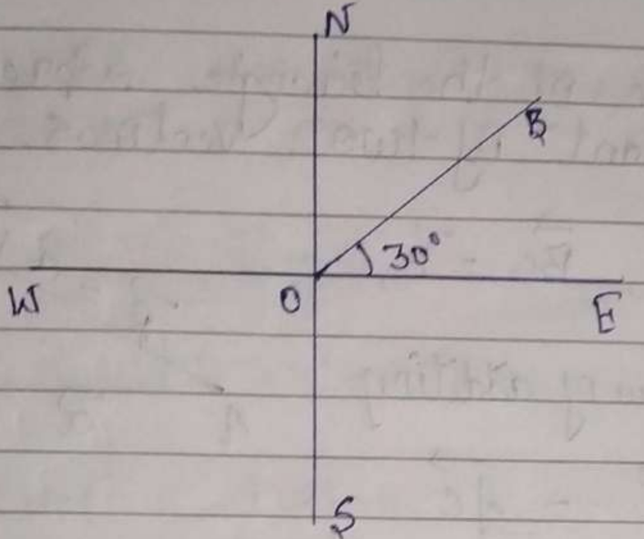


Magnitude:- Length of the vector expressed in proper unit is called magnitude, magnitude is a Scalar quantity and it will be never be negative.

Direction:- It is the angle made by the vector with ~~the help~~ the +ve direction of x- axis in anticlockwise.



{$\vec{OA}$ is in North-East Direction}



$\vec{OB}$ is at an angle of 30° north of east.

- Two or more vectors are equal only when they have same magnitude and same direction.

- If two vectors are unequal it means either, they are different in magnitude or direction or both.

- If vector is trans then it remain uneffected on the other hand it may due to rotation.

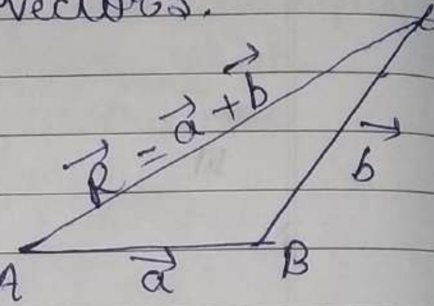
- The angle between two vectors means angle between their tails or heads and it should be smaller of two angle.

- If α is the angle between vector then it satisfy the condition $0 < \alpha < 180°$.

# of Vector :-

Triangle Law of Vector addition :- If two identecial vectors are represented by two sides of a triangle in same order than

than third side of the triangle represented sum or resultant of two vectors.



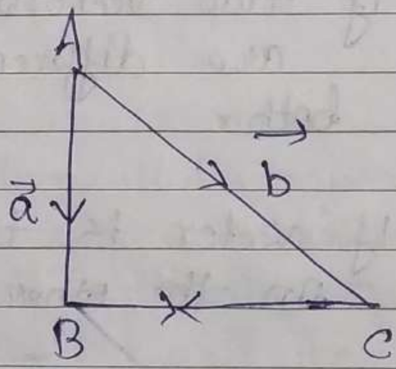
Let $\vec{AB} = \vec{a}$, $\vec{BC} = \vec{b}$,
then,
Δ A/c to Δ Law of addition

$$\vec{AB} + \vec{BC} = \vec{AC}$$

$$\text{or } \vec{a} + \vec{b} = \vec{R} \quad -(i)$$

from fig $\vec{CB} = ?$

from Δ's law



$$\vec{AC} + \vec{CB} = \vec{AB}$$

$$\vec{CB} = \vec{AB} - \vec{AC}$$

$$\therefore \vec{CB} = \vec{a} - \vec{b}.$$

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Parallelogram Law of vector addition:-

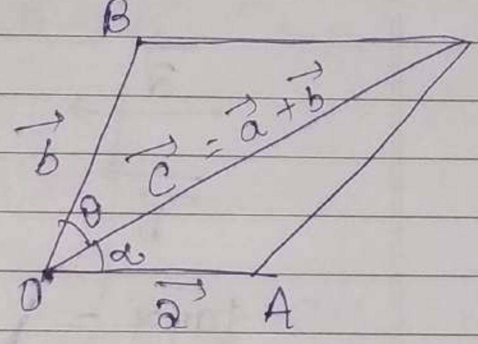
→ ~~Identical vectors~~

→ When two identical vectors are represented by two adjacent side of a parallelogram then resultant of two vectors is represented by the diagonal of the parallelogram passing through point of intersection of the two adjacent side.

$\vec{R} = \vec{a} + \vec{b}$

$\therefore R = \sqrt{a^2+b^2+2ab\cos\theta}$

$\tan\alpha = \dfrac{b\sin\theta}{a+b\cos\theta}$.



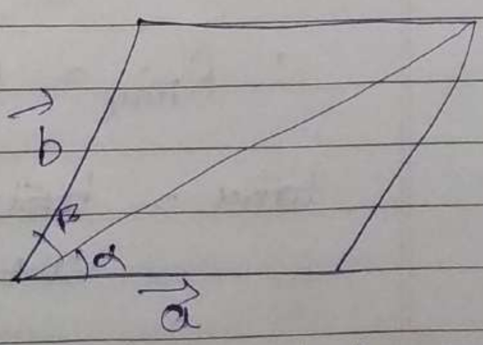
Concept :-

(i) If $a=b$, than $\alpha=\beta$
$\because \alpha+\beta = \theta \Rightarrow 2\alpha = \theta \Rightarrow \alpha = \theta/2$.
$\therefore \alpha = \beta = \theta/2$

When two vectors are equal in magnitude than their resultant bisect the angle between the two vectors.

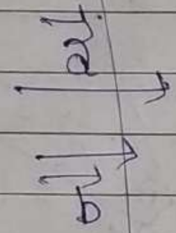
(ii) Suppose the magnitude of the two vectors are not equal.

* If $b>a$, than $\alpha>\beta$
* If $a>b$, than $\beta>\alpha$



→ If two vectors are unequal in magnitude then their resultant is inclined towards the vectors having greater magnitude.

→ For maximum value of R, $\cos\theta$ should be maximum i.e, $\cos\theta = 1$, i.e, $\theta = 0°$.



$$R_{max} = |\vec{a}| + |\vec{b}|$$

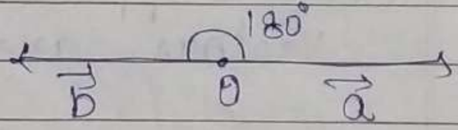
$$\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta} = \frac{b \times 0}{a+b} = 0.$$

$$\therefore \boxed{\alpha = 0}$$

When two vectors are in same direction than resultant is also in same direction and is maximum.

→ For maximum value of R, $\cos\theta$ should be minimum i.e, $\cos\theta = -1$, $\theta = 180°$.

$$R_{min} = \sqrt{a^2 + b^2 - 2ab}$$



$$\therefore R_{min} = a - b$$

$$\tan\alpha = \frac{b\sin 180°}{a + b\cos 180°} = 0, \quad \Rightarrow \boxed{\alpha = 0}$$

Hence, when two vectors are in opposite direction than their resultant is minimum & is equal to $(a-b)$ in the direction of vector having greater magnitude.

→ Vector addition obey to the commutative Law,

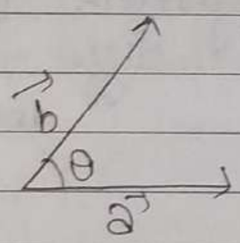
i.e $\vec{a} + b = \vec{b} + \vec{a}$.

Subtraction of Vector.

Here $\vec{a}$ & $\vec{b}$ are two vectors inclined at an angle θ

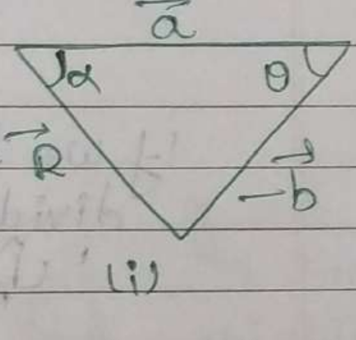
We find, (i) $\vec{a} - \vec{b} = ?$

(ii) $\vec{b} - \vec{a} = ?$



(i) $\vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = \vec{R}$

$$\cos\theta = \frac{a^2 + b^2 - R^2}{2ab}$$



$$2ab\cos\theta = a^2 + b^2 - R^2$$

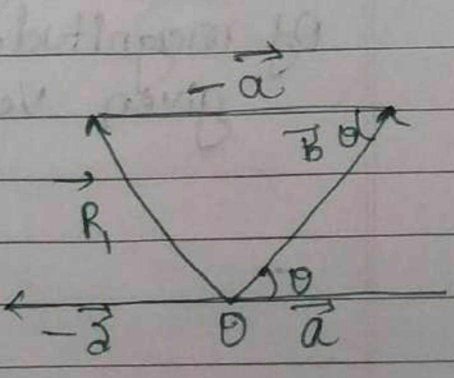
$$R^2 = a^2 + b^2 - 2ab\cos\theta$$

$$R = \sqrt{a^2 + b^2 - 2ab\cos\theta}$$

$$\tan\alpha = \frac{b\sin\theta}{a - b\cos\theta}$$ Ans.

(ii) $\vec{b} - \vec{a} = \vec{b} + (-\vec{a}) = \vec{R_1}$

$$\cos\theta = \frac{a^2 + b^2 - R^2}{2ab}$$



$\therefore R = \sqrt{a^2+b^2-2ab\cos\theta}$

$\therefore \tan\alpha = \dfrac{a\sin\theta}{b-a\cos\theta}$

Here $\vec{R} \neq \vec{R_1}$; but $|\vec{R_1}| = |\vec{R_1}|$

→ Magnitude of difference of two vectors are equal & differ direction are opposite to one another.

-: UNIT VECTOR :-

Unit Vector :- The vector whose magnitude is unity is called unit vector.

If we, have to find the unit vector of $\vec{A}$ we divide $\vec{A}$ by, by $|\vec{A}|$, it is denoted by '$\hat{u}_A$'.

$\therefore \hat{u}_A = \dfrac{\vec{A}}{|\vec{A}|}$ $\qquad \therefore \vec{A} = |\vec{A}|\,\hat{u}_A$

Hence any vector can be represented in product of magnitude & unit vector in direction of given vector.

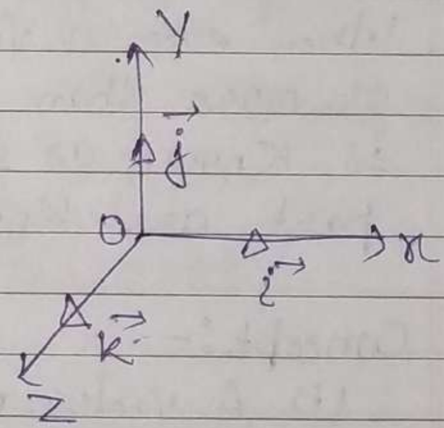
Types of Unit Vector.

- Constant Unit Vector.
- Variable Unit Vector.

(i) **Constant Unit Vector:-** The unit vector whose direction is not change during change in time is called constant unit vector.

$$|\hat{i}| = |\hat{j}| = |\hat{k}|$$

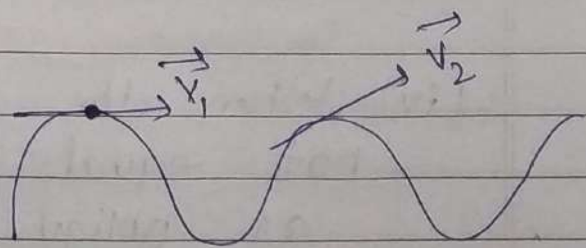
$$\vec{F} = 2N \text{ due east}$$
$$= (2N)\vec{i}$$



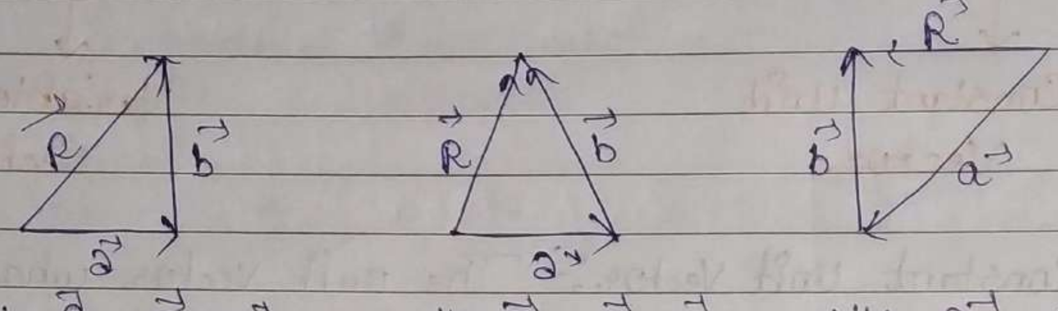
(ii) **Variable Unit Vector:-** If the direction of unit vector changes with time than it is called variable unit vector.

$$\hat{V}_1 = \frac{\vec{V}_1}{|\vec{V}_1|}$$

$$\hat{V}_2 = \frac{\vec{V}_2}{|\vec{V}_2|}$$



Resolution of Vectar :-



(i) $\vec{R} = \vec{a} + \vec{b}$ $a < R$, $b < R$

(ii) $\vec{R} = \vec{a} + \vec{b}$ $R = a = b$

(iii) $\vec{R} = \vec{a} + \vec{b}$ $R < a$

→ When ever a vector is expressed as sum of two or more than two vectors than the ~~th~~ process is Known as resolution of vector & the indivisual part are Known as components of a vectors.

→ Concept :-

(i) A vector may have infinite number of components.

(ii) Components of a vector is always vector.

(iii) The magnitude of a vector may be less than, equal to or greater than the magnitude of a vector.

(iv) When the angle between the component is not equal to 90° than these are Known as oblique components.

Resolution of Vector into Oblique Component :-

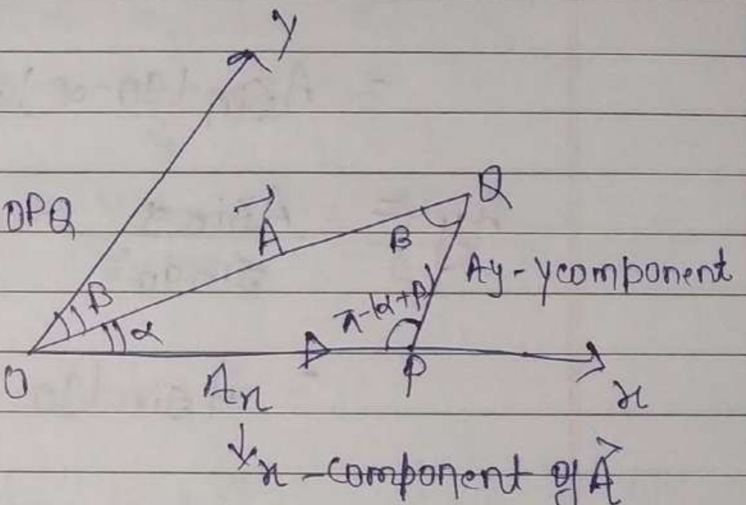
Let $\vec{A}$ makes an angle α & β with x-axis & y-axis respectively.

From, fig,

$$\vec{A} = \vec{OP} + \vec{PQ}$$

Applying, sin rule in Δs OPQ

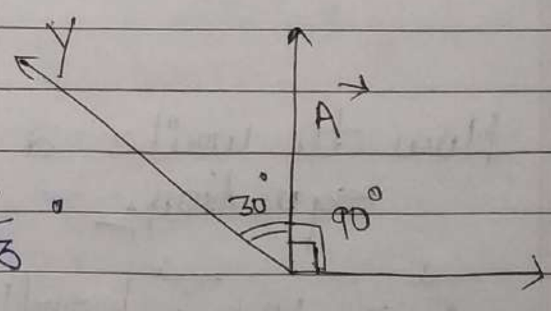
$$\frac{A}{\sin[\pi-(\alpha+\beta)]} = \frac{A_x}{\sin\beta} = \frac{A_y}{\sin\alpha}$$



$$\therefore A_x = \frac{A\sin\beta}{\sin(\alpha+\beta)}, \quad A_y = \frac{A\sin\alpha}{\sin(\alpha+\beta)}.$$

Q. In the given figure, write down the x & y component of vector.

Solⁿ → $A_x = A\dfrac{\sin 30^\circ}{\sin(90+30)}$



$$A_x = \frac{A\sin 30}{\sin 120^\circ} = \frac{A}{\sqrt{3}}.$$

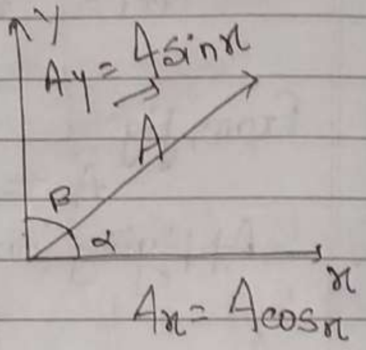
$$A_y = A\frac{\sin 90^\circ}{\sin 120} = \frac{2A}{\sqrt{3}}.$$

Here, $A_x < A$, $A_y > A$.

→ If $\alpha + \beta = \pi/2$, than

$$A_x = \frac{A \sin\beta}{\sin 90^\circ} = A\sin\beta$$

$$= A\sin(90-\alpha) = A\cos\alpha$$



$$A_y = \frac{A\sin\alpha}{\sin 90^\circ} = A\sin\alpha$$

$$= A\sin(90-\beta) = A\cos\beta$$

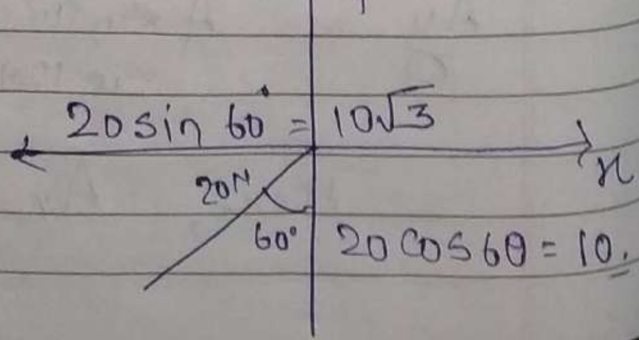
→ If a vector lies on axis than its other component will be zero.

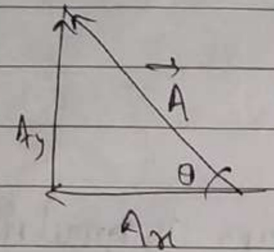
Ex. If $\vec{A}$ lies on x-axis, then

x-component of $\vec{A} = A$
y-component of $\vec{A} = 0$

How to write a vector equation of a physical equation:-

$$\vec{F} = 10(-\vec{j}) + (10\sqrt{3})(-\vec{i})$$





$\rightarrow \quad \vec{A} = A_x \vec{i} + A_y \vec{j}$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

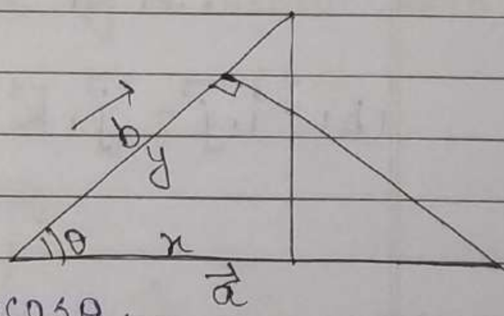
$$\cancel{\tan\theta = \frac{x \cdot comp}{y}}$$

$$\tan\theta = \frac{y\text{-comp}}{x\text{-comp}} = \frac{A_y}{A_x}$$

Dot Product of two Vector :-

→ Dot product of two vector $\vec{a}$ & $\vec{b}$ are defined as the product of magnitude of any one of the two vector and projection of other vector on first vector.

$\vec{a} \cdot \vec{b} = a \cdot$ projection of $\vec{b}$ & $\vec{a}$
$= b \cdot$ projection of $\vec{a}$ & $\vec{b}$



i.e., $\vec{a} \cdot \vec{b} = a \cdot x = b \cdot y$

$$\cos\theta = x/b = x = b\cos\theta.$$

$$\cos\theta = y/a = y = a\cos\theta.$$

$$\therefore \vec{a} \cdot \vec{b} = ab\cos\theta$$

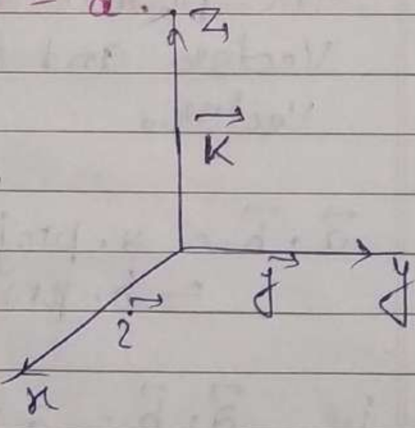
Properties:-

(i) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a} = ab\cos\theta.$

(i) The dot product of two vector obeys commulative law & result of dot product of two vectors is scalar

(ii) The dot product of two vector may be positive, negative or zero. If the two vectors are mutually ⊥r to each other their dot product becomes zero.

(ii) $\vec{a}\cdot\vec{a} = a.a.\cos\theta = a\cdot a\cdot 1 = a^2.$



(iv) (a) $\vec{i}\cdot\vec{i} = 1\times 1\cos\theta = 1$

$\therefore \hat{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1.$

(b) $\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{i} = 0.$

Let $\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$

$\vec{B} = B_x\vec{i} + B_y\vec{j} + B_z\vec{k}$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$\Rightarrow AB\cos\theta = A_xB_x + A_yB_y + A_zB_z.$$

$$\therefore \cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2+A_y^2+A_z^2}\cdot\sqrt{B_x^2+B_y^2+B_z^2}}$$

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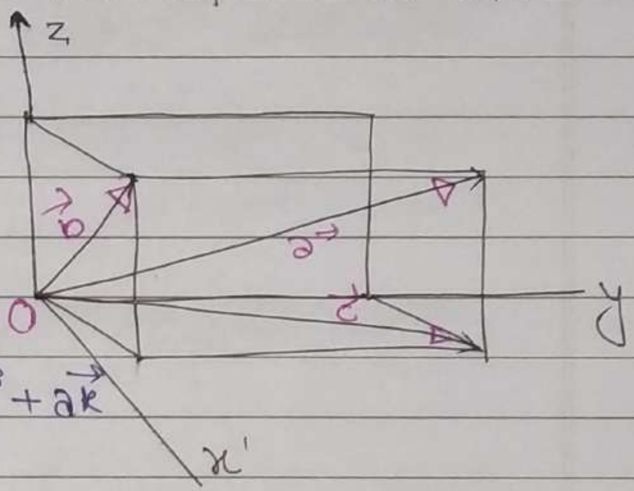
Q. Three Vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$ are represented in a cube of side a. find angle between the vectors.

(a) $\vec{a}$ & $\vec{b}$

(b) $\vec{a}$ & $\vec{c}$

(c) $\vec{b}$ & $\vec{c}$



Soln →

$\vec{b} = a\vec{i} + a\vec{k}$

$\vec{a} = \vec{b} + a\vec{j} = a\vec{i} + a\vec{j} + a\vec{k}$

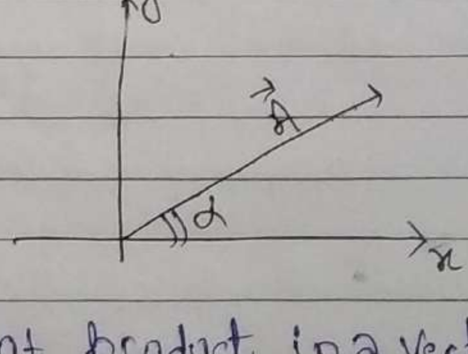
$\vec{c} = a\vec{j} + a\vec{j} + 0\vec{k}$

(i) $\cos\alpha = \dfrac{a^2 + 0 + a^2}{\sqrt{2}a\sqrt{3}a} = \dfrac{2a^2}{\sqrt{6}a^2} = 2/\sqrt{6}$

$\therefore \alpha = \cos^{-1}\left(2/\sqrt{6}\right)$

How to find the component of a vector with the help of dot product.

$\vec{A} = A\cos\alpha\,\hat{i} + A\sin\alpha\,\hat{j}$



Now,

$\vec{A}\cdot\vec{i} = A\cos\alpha = A_x$

$\vec{A}\cdot\vec{j} = A\sin\alpha = A_y$

When we take the dot product in a vector with unit vector than we get component of that Vector along that unit Vector.

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Q. $\vec{A} = 3\hat{i} + 4\hat{j} - \hat{k}$, find the magnitude of component of $\vec{A}$ along $\vec{B}$, if $\vec{B}$ is parallel to $\vec{c}$ & its magnitude ~~is~~ is 20 unit. $\vec{C} = 3\hat{i} + 4\hat{j}$.

Sol^n → $\vec{B} = \frac{20}{5}(3\hat{i} + 4\hat{j}) = 4(3\hat{i} + 4\hat{j})$
$= 12\hat{i} + 16\hat{j}$.

∴ Component of $\vec{A}$ along $\vec{B}$

$$= \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|} = \frac{(3\hat{i} + 4\hat{j})(12\hat{i} + 16\hat{j})}{\sqrt{144 + 256}}$$

$$= \frac{36 + 64}{20} = \cancel{100}/\cancel{20} = 5.$$

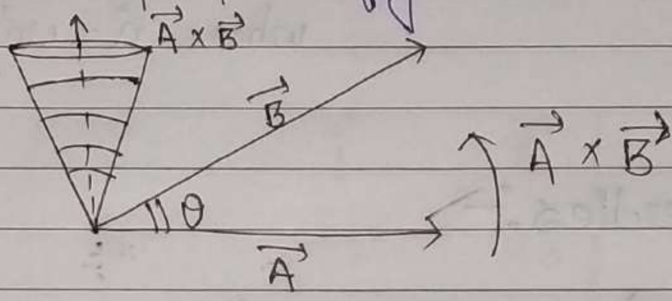
Cross Product of two Vectors.

A cross product of two vectors $\vec{A}$ & $\vec{B}$ may be defined as a new vector $\vec{C}$, whose magnitude is given by.

$|\vec{C}| = |\vec{A}||\vec{B}| \sin\theta$, where θ is the angle between $\vec{A}$ & $\vec{B}$.

→ The direction of $\vec{C}$ is defined as the perpendicular to the plane containg $\vec{A}$ & $\vec{B}$.

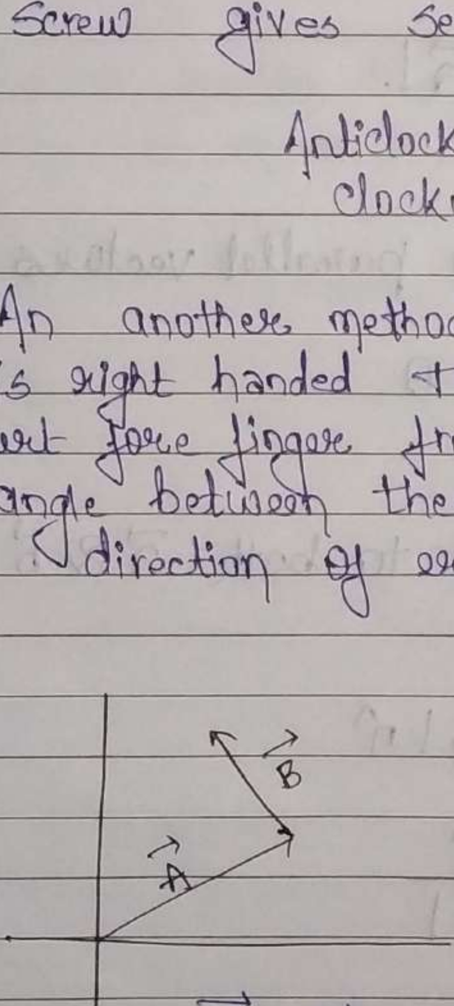
→ In order to find the sense of $\vec{C}$ (Whatever it will go up the plane or down the plane). We take a right handed screw. The axis of this screw is $\perp r$ to the plane containing $\vec{A}$ & $\vec{B}$ as shown in fig.



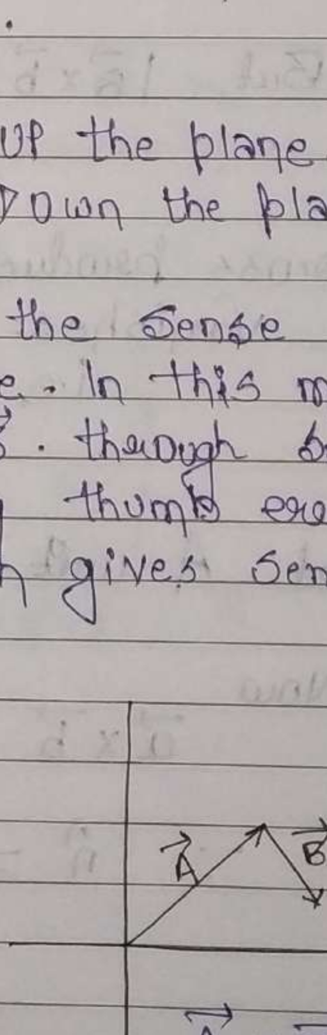
→ To find the sense of $\vec{A} \times \vec{B}$, we term the screw from $\vec{A}$ & $\vec{B}$ to smaller angle between them, than the direction of advancement tip of screw gives sense of $\vec{C}$.

Anticlockwise ⟶ Up the plane.
clockwise ⟶ Down the plane.

An another method to find the sense of $\vec{A} \times \vec{B}$ is right handed thumb rule. In this method, we curl fore finger from $\vec{A}$ to $\vec{B}$. through smaller angle between them, keeping thumb erect. than direction of erected thum gives sense of $\vec{C}$.



$\vec{A} \times \vec{B} = \odot$



$\vec{A} \times \vec{B} = \otimes$

Q. Here,

$$\vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}| \sin\theta\, \hat{n}$$

where $\hat{n}$ = unit vector $\perp^r$ to both $\vec{a}$ & $\vec{b}$.

Sol^r

Properties:-

* $$\vec{a} \times \vec{b} = ab \sin\theta\, \hat{n}$$
$$\vec{b} \times \vec{a} = ab \sin\theta\, (-\hat{n})$$

$$\therefore \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

But, $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{a}|$.

* Cross product of two parallel vectors are Vanishes.

i.e, $\vec{a} \times \vec{a} = 0$

* Let A unit vector $\perp^r$ to both $\vec{a}$ & $\vec{b}$ is $\hat{n}$

Now

$$\vec{a} \times \vec{b} = |\vec{a} \times \vec{b}|\, \hat{n}$$

$$\therefore \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

* Let $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

$\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

* Geomertical meaning of $\vec{a} \times \vec{b}$:-

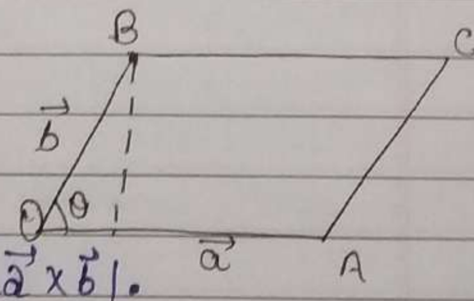
Let us consider two vectors $\vec{A}$ & $\vec{B}$ are represented by two adjacent side of parallelogram.

Such that angle between $\vec{a}$ & $\vec{b}$ is θ

Area of ||gm = Base x Height

$= a \times b \sin\theta$

$= |\vec{a} \times \vec{b}|$



∴ Hence, area of ||gm is equal to magnitude of $|\vec{a} \times \vec{b}|$.

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Force & Momentum:-

Force:- Force is a push or pull which try to change or change the state of rest or uniform motion of a body.

→ Force is a cause of translational motion it aries from the interaction of bodies either due to contact or from a distance.

→ Force may either produces deformation or acceleration. It is a vector quantity.

Mass:-

The quantity of matter contained in the body is called Mass. It is a scalar quantity & its S.I unit is kg.

Weight:-

The gravitational pull expercined by the body is known as force of gravity. If a body of mass 'm' is located at a point where acceleration due to gravity is 'g' then weight is defined as,

$$W = mg$$

Where, $g = 9.8\ m/s^2$
$= 10\ m/s^2$

Momentum :- Momentum of a is quantity of motion possesed by the body. It is equal to the product of mass and its velocity.

Momentum = Mass x Velocity

$$\vec{p} = m\vec{v}.$$

→ Momentum is a vector quantity. Its direction is along the velocity.

S.I unit of momentum = Kgm/s.

Dimension.

$$[p] = [MLT^{-1}].$$

CASE - I; If two bodies of mass m_1 & m_2 ($m_1 > m_2$) are moving with same velocity v, then,

$$\Rightarrow \frac{P_1}{P_2} = \frac{m_1 V}{m_2 V_2} = \frac{m_1}{m_2}$$

→ It means, if $m_1 > m_2$ then $P_1 > P_2$.

CASE - II; If two bodies of same mass moving with different velocity with different velocity V_1 & V_2, then,

$$\frac{P_1}{P_2} = \frac{\cancel{m}v_1}{\cancel{m}v_2} = \frac{V_1}{V_2}$$

→ It means, if $v_1 > v_2$ then $P_1 > P_2$.

CASE – III ; If two bodies moving equal momentum, then,

$$\Rightarrow P_1 = P_2$$
$$m_1 v_1 = m_2 v_2$$
$$= mv = \text{constant},$$

$$\therefore \boxed{v \propto 1/m}$$

NEWTON'S LAWS OF MOTION :-

1. Newton's 1st Law :-
This law state that, when net force acting on the body is zero, then its acceleration is zero. It means, it moves with uniform velocity along straight line.

2. Newton's 2nd Law :-
This law state that, the rate of change of momentum is directly proportional to the impressed force & take place, in the same direction in which the force acts.

This law provide relation between force & acceleration.

Measurement of Force :-

Let a body of mass 'm' moving with initial velocity (u) be acted upon by a force (f) for a time 't'. So, that its velocity becomes 'v'.

initial momentum = mu
final momentum = mv

Change in momentum = $mv - mu$.

Rate of change of momentum = $\frac{mv - mu}{t}$

A/c to Newton's 2nd law,

f = rate of ~~rate~~ ~~of~~ change of momentum

$f = \frac{mv - mu}{t}$

or, $f = \frac{m(v-u)}{t}$

$\left\{ \begin{array}{l} v = u + at \\ \Rightarrow v - u = at \\ a = \frac{v-u}{t} \end{array} \right\}$

So ;

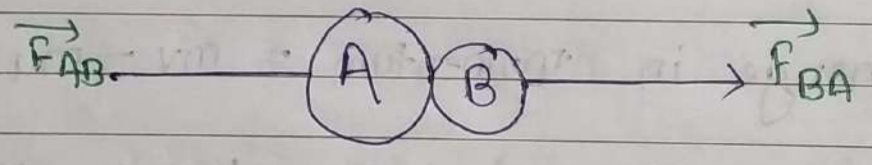
$\therefore \boxed{f = ma.}$

S.I unit of force = Newton.

3. Newton's 3rd Law :→

This law state that, to every action there is always an equal & opposite reaction.

Force in nature always occurs between pair of bodies. force on a body A by body B in equal and opposite to the force on the body B by A.

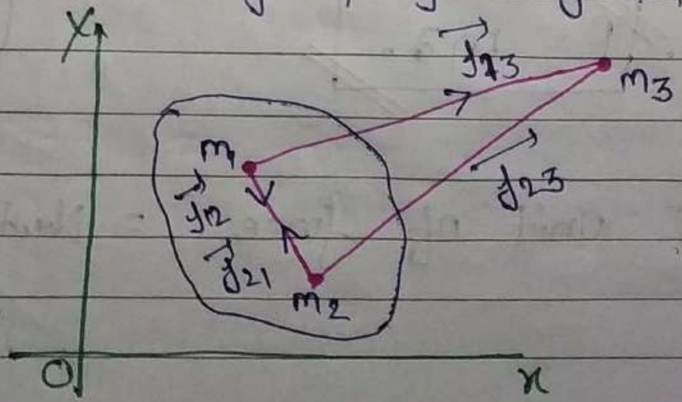


$$\vec{F}_{AB} = -\vec{F}_{BA}$$

force on A by B = - force on B by A.

CONSERVATION OF LINEAR MOMENTUM :-

- Let us consider a system containing two ~~practit~~ praticles of mass m_1 & m_2, the praticle of mass m_3 lies out side the system. whole system is observed from interial from of reference.



Rate of change of momentum of a praticle is equal to the force. A/c to Newton's Second Law.

$$\vec{F} = \frac{d\vec{P}}{dt} \quad \text{— (i)}$$

Eqⁿ for m_1

$$\vec{f}_{12} + \vec{f}_{13} = \frac{d\vec{P}_1}{dt} \quad \text{— (2)}$$

where, $\vec{P}_1$ = momentum of m_1
$\vec{f}_{12}$ = internal force on 1 due to 2
$\vec{f}_{13}$ = external force on 1 due to 3.

Eqⁿ for M_2.

$$\vec{f}_{21} + \vec{f}_{23} = \frac{d\vec{P}_2}{dt} \quad \text{— (3)}$$

Adding eqⁿ (ii) & eqⁿ (iii)

$$\Sigma(\vec{f}_{21} + \vec{f}_{12}) + \Sigma\, \vec{f}_{13} + \vec{f}_{23} = \frac{d\vec{P}_1}{dt} + \frac{d\vec{P}_2}{dt}$$

$$\Sigma \vec{F}_{int} + \Sigma \vec{f}_{ext} = \frac{d}{dt}(\vec{P}_1 + \vec{P}_2)$$

Since, $\Sigma \vec{f}_{int} = 0$

$$\Rightarrow \Sigma \vec{f}_{ext.} = \frac{d\vec{P}}{dt} \quad \text{— (4)}$$

Here $\vec{P} = \Sigma P_i$ = Total momentum of the system
Hence, total momentum of system depends up on external force only. internal force do not play any role.

if $\Sigma \vec{F_{ext}} = 0$, then,

$$\frac{d\vec{P}}{dt} = 0$$

$$\Rightarrow \vec{P} = \text{Constant.}$$

Hence, when vector sum of external forces on the system in internal frame of reference frame is equal to zero than total momentum of the system remains considered in both magnitude as well as in direction. this is known as principle of conservation of linear momentum

Concept:-

$$\Sigma \vec{f_{ext}} = \frac{d\vec{P}}{dt}$$

$$\Rightarrow \Sigma f_x = \frac{dP_x}{dt} \quad \&$$

$$\Sigma f_y = \frac{dP_y}{dt}.$$

There may be a system in which net external force is not equal to zero but force along a particular direction suppose along x- axis is zero. than total momentum of the system along that particular direction will remain conserved:-

$$\text{if } \Sigma f_x = 0$$

$$\Rightarrow \frac{dP_x}{dt} = 0$$

$$\Rightarrow \boxed{P_x = \text{constant}}$$

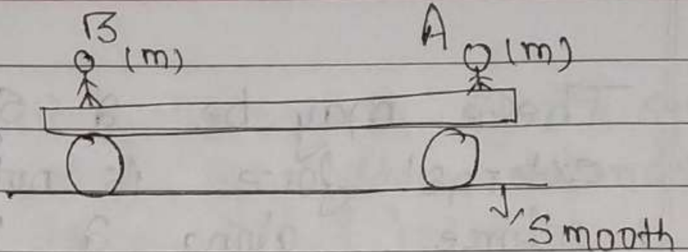
→ Rembere that, if external force on a system is equal to zero than total momentum will remain conserved however momentum of indivisual particle of system may change.

→ If system is observed from non-inertial fram of reference

$$\Sigma \vec{F}_{seudo} + \Sigma \vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

In this case conservation of Linear momentum will be applicable provided vector. Sum of Pseudo forces & all external forces will be zero.

Q.



initially the whole System in rest. both the man jumps one by one with velocity a cart trolley. What will be final velocity of trolley.

Sol^n $\vec{V}_{mT} = \vec{V}_m - \vec{V}_T$

$\vec{V}_m = \vec{V}_{mt} + \vec{V}_t$

$= 4 + 0 = 4.$

Taking mass + trolley as a system. Since there is no horizontal force on this System therefore momentum along horizontal direction will remain constant.

Let $\vec{V}_1$ = velocity of trolley when man A jump

$P_i = P_f$

$= 0 = mu + [-(M+m)v_1]$

$v_1 = \frac{mu}{M+m}$

Now, when man jump velocity of man B wrt ground when he jumps of

$\vec{V}_m = \vec{V}_m + \vec{V}_T$

$$-u - v_1$$

$$= -u - \frac{mu}{M+m} = -\frac{(M+2m)u}{M+m}.$$

$$P_i = P_f$$

$$\Rightarrow -(M+m)v_1 = -\frac{mu(M+2m)}{M+m} + Mv_2$$

$$\Rightarrow -mu + \frac{mu(M+2m)}{M+m} + mv_2$$

$$\left[\frac{M+2m}{M+m} - 1\right] mu = Mv_2$$

$$\frac{m^2u}{M+m} = Mv_2$$

$$\therefore v_2 = \frac{m^2u}{M(M+m)}. \quad \underline{\text{Ans.}}$$

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CIRCULAR MOTION.

→ When a particle ~~mavo~~ moves about a fixed point in such a way that its distance from the fixed point remains always same, then, the path followed by the particle will be a circle. Fixed point is called centre and fixed distance is called radius of circle.

Such type of motion is called circular motion or angular motion.

* **Time Period (T):-** It is time taken by the body to compalete one rotation. During this time the body traces an angle 2π at the centre.

* **Frequency (n):-** The number of rotation completed by a body in one second is called its frequency.

$\because$ In T Second no. of rotation $= 1$

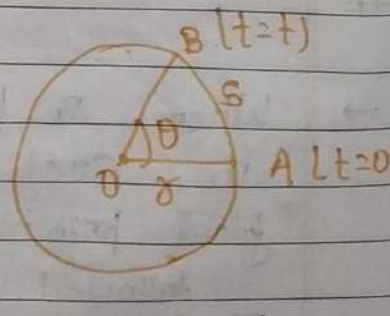
$\therefore$ 1 second no. of rotation $= \frac{1}{T}$

$\therefore$ Frequency $= \boxed{n = \frac{1}{T}}$

ANGULAR DISPLACEMET (θ):-

The angle described by the particle during time 't' from its initial position is called angular displacement.

Angular displacement $= \frac{\text{Arc } AB}{\text{Radius}}$



Or,

$$\boxed{\theta = \frac{s}{r}} \quad \text{— (i)}$$

ANGULAR VELOCITY (ω):-

The rate of change of angular displacement is known as angular velocity.

(or).

It is defined as angle described by the particle per second.

Angular velocity = $\boxed{\omega = \theta / t}$

→ If T is the time period, then $\theta = 2\pi$.

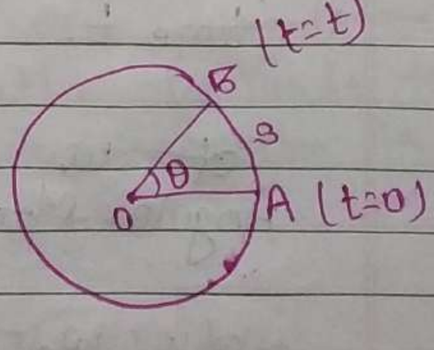
$$\therefore \omega = \frac{2\pi}{T} \qquad \left(\because n = \frac{1}{T}\right)$$

$$\therefore \boxed{\omega = 2\pi n} \text{ (rad/s)}.$$

Relation between Linear velocity & Angular velocity(ω):-

Suppose a particle starting from a A goes to position B in time 't' as shown in fig.

te. Linear Velocity / Speed.



$$V = \frac{AB}{T} = \frac{S}{T}$$

$$\because \theta = \frac{S}{r} \quad \therefore S = \theta . r$$

$$\therefore V = \frac{\theta . r}{T} = \left(\frac{\theta}{t}\right) r = \omega r$$

$$\therefore \boxed{V = \omega r}$$

Linear Velocity = Radius × Angular Velocity.

ANGULAR ACCELERATION (α) :-

The rate of change of angular velocity is called angular acceleration.

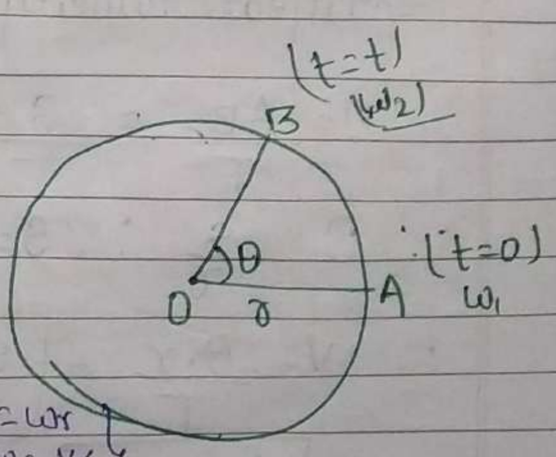
Angular Acceleration :-

$$\alpha = \frac{\text{Change in angular Velocity}}{\text{Time Taken}}$$

$$\therefore \boxed{\alpha = \frac{\omega_2 - \omega_1}{t}}$$

Relation Between Linear Acceleration & Angular Acceleration :-

Consider a particle moves in a circle of radius r. ω_1 be the angular velocity at A. And ω_2 be the angular velocity at B after time t.



~~$\alpha = \omega_1$~~

Angular ~~Mor~~ Acceleration

$$\alpha = \frac{\omega_2 - \omega_1}{t} = r$$

$$\alpha = \frac{\frac{v_2}{r} - \frac{v_1}{r}}{t} \quad \left\{ \begin{array}{l} v = \omega r \\ \therefore \omega = v/r \end{array} \right\}$$

Or, $\alpha = \frac{1}{r} \cdot \left(\frac{v_2 - v_1}{t}\right) = \frac{1}{r} \cdot a$

$\therefore \boxed{\alpha = \frac{a}{r}}$ or $\boxed{a = r\alpha}$

$\therefore$ $\boxed{\text{Linear Acceleration} = \text{Radius} \times \text{Angular Acceleration}}$

Circular Motion
↓
Uniform Circular Motion | Non-Uniform Circular Motion.

(i) Uniform Circular Motion:-

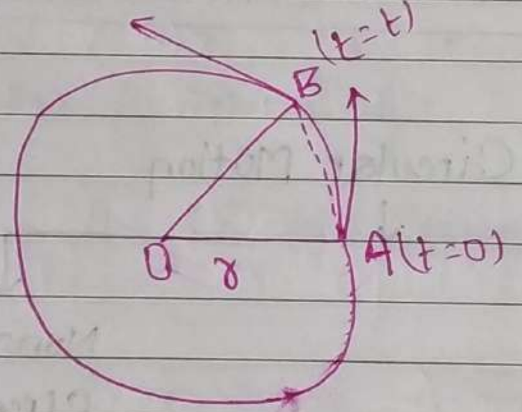
When a particle moves in a circular path with constant speed then the motion is known as uniform circular motion.

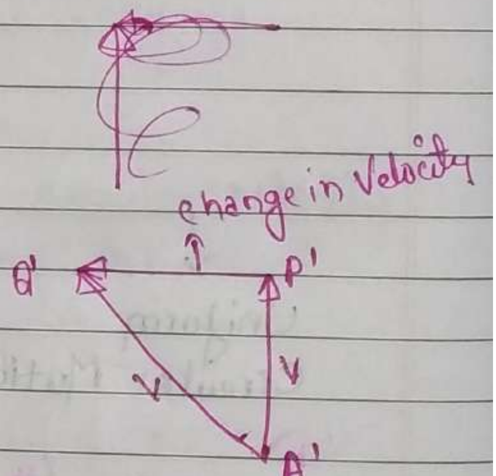
In uniform circular motion speed is constant but velocity changes due to change in direction of motion. Hence, there is an acceleration which is exactly towards the centre of the circle. This acceleration is known as centripital acceleration.

* Expression for centripital Acceleration:-

Suppose a particle of mass 'm' moving

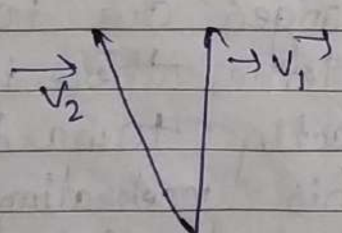
on a circular path of radius r & Centre O with constant Speed V. as Shown in fig. When the particle is at point A at t=0, its velocity v is along the tangent AP. After time 't' is its velocity v is along the tangent BQ.





The particle when moves from A to B has cover a distance AB in time t.

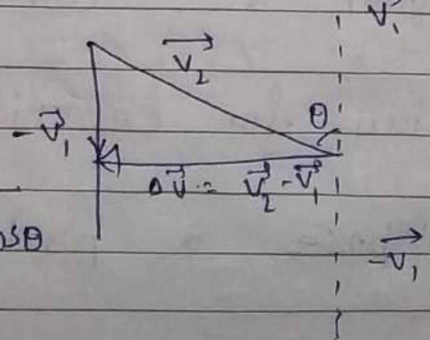
AB = velocity x time.



We, want change in velocity

$$\Delta\vec{V} = V_2 - V_1 = V_2 + (-V_1)$$

$$\Delta\vec{V} = \vec{V_2} - \vec{V_1}$$



$|\vec{v_1}| = v$

$|\vec{v_2}| = v$

$$|\Delta\vec{v}| = \sqrt{v_2^2 + v_1^2 - 2v_1v_2\cos\theta}$$

Or, $\Delta v = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$

$$\Delta v = \sqrt{2v^2 - 2v^2\cos\theta}$$

$$\Delta v = \sqrt{2v^2 - 2\sin^2\theta/2}$$

$$= 2v\sin\frac{\theta}{2}$$

For small angle $\sin\theta/2 \simeq \theta/2$

$$\therefore \Delta v = 2v.\frac{\theta}{2} = v.\theta.$$

$\therefore$ Acceleration

$$a = \frac{\Delta v}{t} = v.\frac{\theta}{t} = v.\omega$$

$\because$ $v = \omega r$, $\therefore \omega = v/r$

$$\therefore a = v.\frac{v}{r} = v^2/r$$

$\therefore$ centripital acceleration

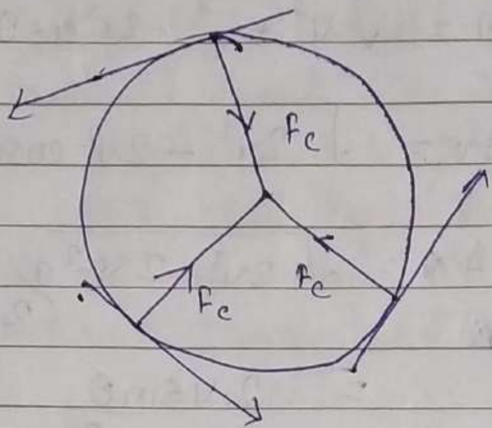
$$\boxed{a_c = \frac{v^2}{r}}$$

This acceleration is directed toward the centre of circle.

Centripital Force :-

f_c = mass × Centripital Acceleration

$$\boxed{F_c = \frac{m.v^2}{r}}$$



$\because$ $v = \omega r$

$\therefore F_c = m \cdot \frac{\omega^2 r^{\cancel{2}}}{\cancel{r}}$

$$\boxed{F_c = m\omega^2 r}$$

Also, $\omega = 2\pi n$

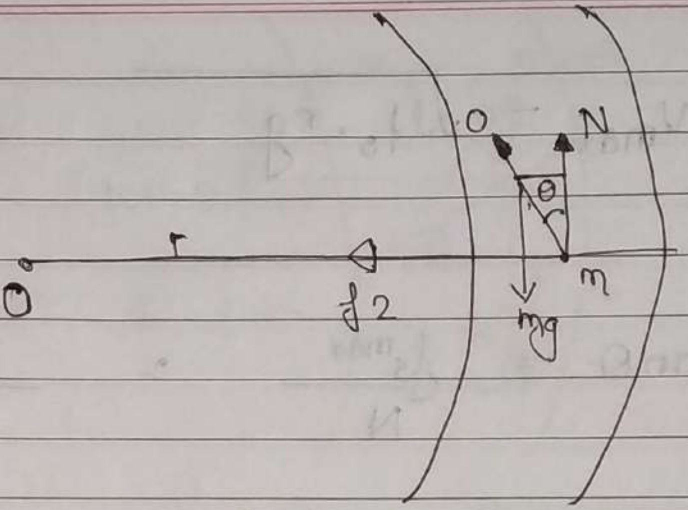
$\therefore F_c = m \cdot (2\pi n^2)\, r$

$$\therefore \boxed{F_c = 4\pi^2 n^2 m r}$$

Bending of Cyclist :

(18th May)

When a cyclist going round on the horizontal circular road, it bend himself a little from the the vertical position in order to avoid slipping. In this case ~~to avoid~~ ~~so~~ the tire of cycle have tendency.



to slids away from the centre and the necessary centripital force acts towards the centre. This force is static friction.

Equation for vertical equilibrim.

$$N = mg \quad \text{—(i)}$$

Equation for circular motion

$$f_s = \frac{mv^2}{r} \quad \text{—(ii)}$$

$$\therefore f_s^{max} = \mu \cdot N$$
$$= \mu_s \cdot mg$$

Also,

$$\therefore f_s \leq f_s^{max}$$

$$\frac{\cancel{m}v^2}{r} \leq \mu_s \cdot \cancel{m}g$$

$$v^2 \leq \mu_s \cdot rg$$

$$v \leq \sqrt{\mu_s \cdot rg}$$

$$v_{max} = \sqrt{\mu_s \cdot rg}$$

Also,

$$\tan\theta = \frac{f_s^{max}}{N} = \frac{\cancel{m}v^2}{r \times \cancel{m}g}$$

$$\tan\theta = \frac{v^2}{rg}$$

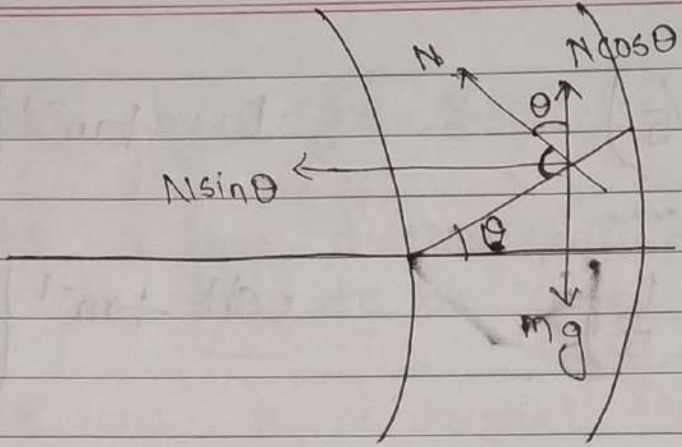
$$\boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$

Angle of bending.

* Banking Of Roads:-

To avoid the skiding of a car and for safe driving the road is banked in such a way that outer part of the road is raised a little above towards the centre of the curve. This is known as banking of road or track.

The angle through which the outer edge of the circular track is raised above the inner edge is called the angle of banking.



In this case, horizontal component of normal force provides necessary centripital force for circular motion & vertical component balanced the weight of car.

$$\therefore \quad N\sin\theta = \frac{mv^2}{r} \quad \text{— (i)}$$

$$N\cos\theta = mg \quad \text{— (ii)}$$

Now, eq^n – (i) ÷ eq^n (ii)

$$\frac{\cancel{N}\sin\theta}{\cancel{N}\cos\theta} = \frac{\cancel{m}v^2}{r\times \cancel{m}g}$$

$$\tan\theta = \frac{v^2}{g}$$

$$\therefore \boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$

Q. A circular curve of a longway is designed for traffic moving at 72 km/h. If the radius of the curve path is 100 m, the correct angle of banking of road should be;

(a) $\tan^{-1}(2/5)$ (b) $\tan^{-1}\left(\frac{3}{5}\right)$

(c) $\tan^{-1}\left(\frac{1}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{4}\right)$

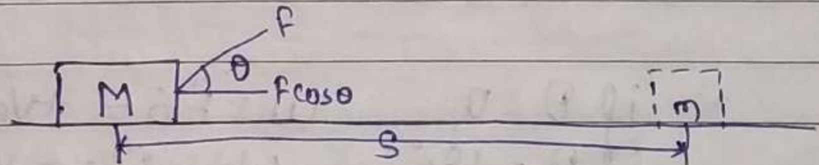
Sol^n →

Wrok done by a force :-

→ if due to a force displacement is produced than work is said to be done by the force.

(A) Wrok done by a constant force :-

The magnitude of the work done by constant ~~work~~ force is defined as product of component of force along displacement and magnitude of displacement.



Component of F along displacent = $F\cos\theta$

Work done (W) = $(F\cos\theta) \times S$

$W = FS\cos\theta.$

= magnitude of force x component of disp. along F.

Hence wrok done by constant force is also defined as product of magnitude of force and component of displacement along force.

$$W = \vec{F} \cdot \vec{S}.$$

Since, work done is dot product of force & displacement it is scalar quantity.

Concept:-

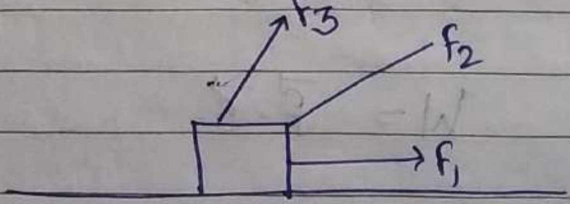
i) If a body is under reaction of several forces then total work done on the body is equal to sum of work done by induvisual forces.

(ii) $\Rightarrow W = F S \cos\theta \quad . \quad 0 < \theta \leq 180^\circ$

if $\theta = 0$, $W = FS = +ve$
if $\theta = 180^\circ$, $W = -FS = -ve$
if $\theta = 90^\circ$, $\Rightarrow W = \underline{0}$.

(iii) If in general mis conception that workdone by static friction is always zero which is not true. Work done by static friction may be any thing +ve, -ve or zero, depending upon situtation:-

(4) Wrok done by resulting force on a body is equal to sum of work done by indivisual force.



Let due to the force $f_1, f_2, \ldots f_n$ a body suffer a displacement $\vec{s}$.

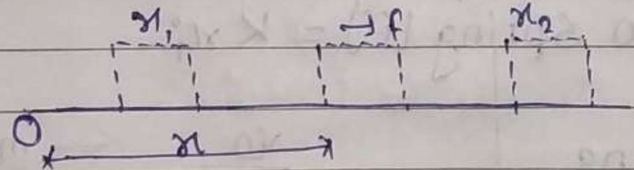
Total work done

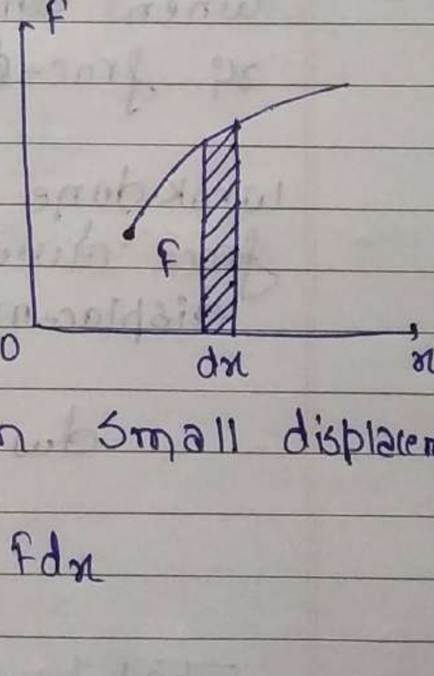
$$W = \vec{f_1}\cdot\vec{s} + \vec{f_2}\cdot\vec{s} + \ldots + \vec{f_n}\cdot\vec{s}$$

$$= (\vec{f_1} + \vec{f_2} + \ldots \vec{f_n})\cdot\vec{s}$$

$$= \vec{f}\cdot\vec{s}.$$

~: Wrok done by variable force :~



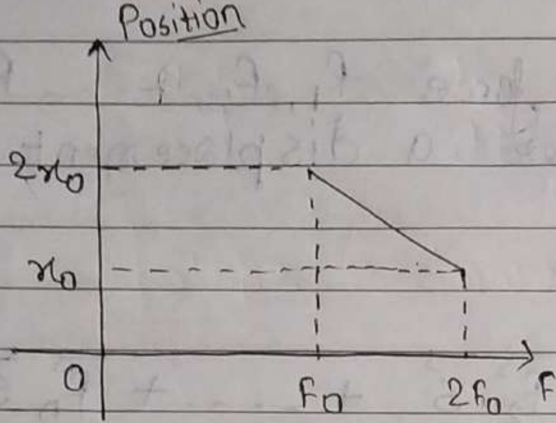


During small displacement dx, force f may be assume to be constant, work done in small displacement

$$dw = f\,dx\cos\theta = f\,dx$$

Area of strip $= f\cdot dx$

$\therefore$ Total work done $W =$ Sum of area of all strip $= \int f\,dx =$ area of f-x curve



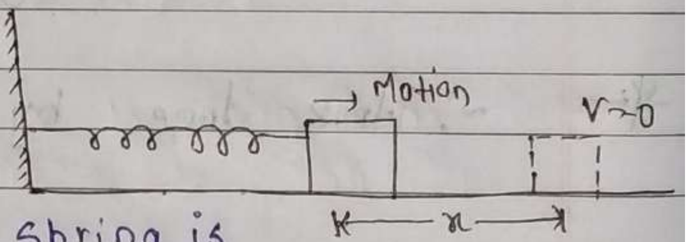
* Calculate work done by the force:

$$W = F_0 \times x_0 + \frac{1}{2} \times F_0 x_0$$

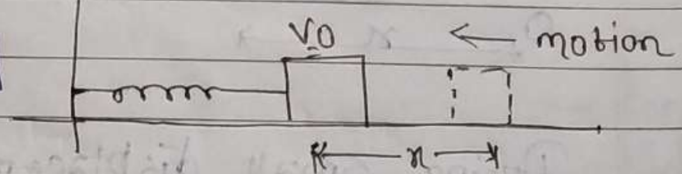
$$= \frac{3}{2} F_0 x_0$$

Wrok done by Spring Force :-

x = Total elongation produced by spring.



When elongation in a spring is, x_i froce due to spring $(F) = Kx_i$



Wrok done by spring froce during small displacement dx;

$$dw = F_s \cos\theta$$
$$= Kx_i\, dx_i \cos 180^\circ$$
$$= -Kx_i\, dx_i$$

Total work done by spring force

$$W_{spring} = -K\int_0^x x_1\, dx_1 = \frac{-1}{2} Kx^2$$

→ If initially spring is at its natural length work done by spring force on the body during elongation or compression is $-\frac{1}{2}Kx^2$.

POWER

→ Power is defined as rate of work done.

Average power is defined as ratio of total work done and time taken to do the work.

If ΔW be the amount of work done in time interval Δt then

$$P_{av} = \frac{W_{total}}{Time} = \frac{\Delta W}{\Delta T}$$

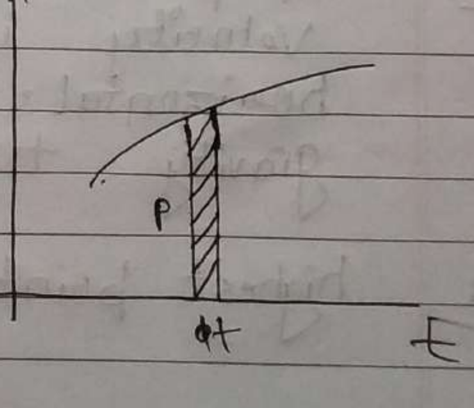
if $\Delta t \to 0$, then

$$\boxed{P = \frac{dw}{dt}}$$

※ Concept :-

Total area $= \int P dt$

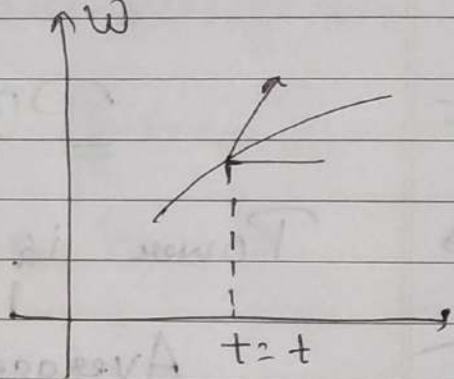
$= \int dw = W_T$



Area of P—t Curve gives total work done.

$$\text{Slope} = \frac{dw}{dt}$$

$$\boxed{\text{Slope} = \text{Power}}$$



→ $\text{Power}(P) = \frac{dw}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt}$

$$= \vec{F} \cdot \vec{V} = FV\cos\theta$$

→ instantaneous Power is defined as dot-product of force & velocity.

$$P = \vec{F} \cdot \vec{V} = fV\cos\theta$$
$$0 \leq \theta \leq 180^\circ$$

✳ Concept :-

$$P_{av} = \frac{W_T}{\Delta t} = \frac{\int P dt}{\int dt}$$

$$\boxed{\langle P \rangle = \frac{\int P dt}{\int dt}}$$

Q. A particle of mass m is projected with velocity is at an angle θ with horizontal. Calculate Power delivered by gravity to the praticle at:-

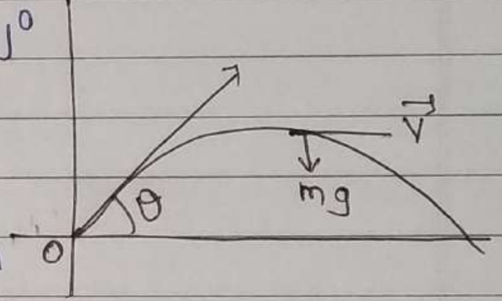
(i) highest point (ii) Final point (iii) at any time

Sol^n (i) At the highest Point

$$P = \vec{F}\cdot\vec{V} = 0$$

(ii) $P = \vec{F}\cdot\vec{V} = mgu\cos(90-\theta)^0$

$= mgu\sin\theta$

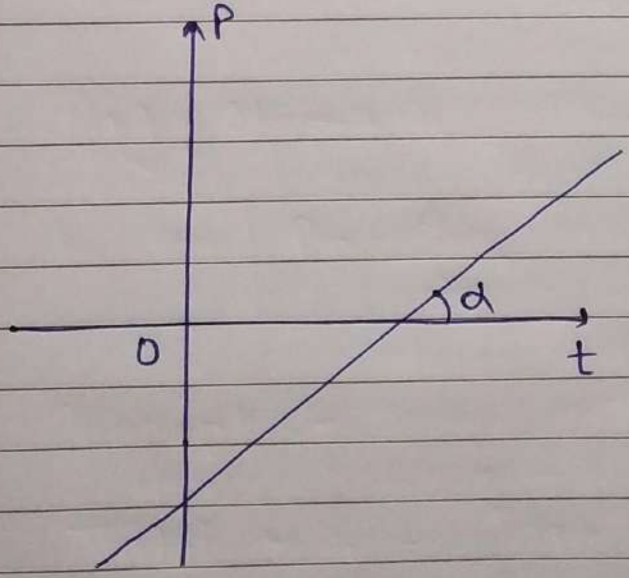


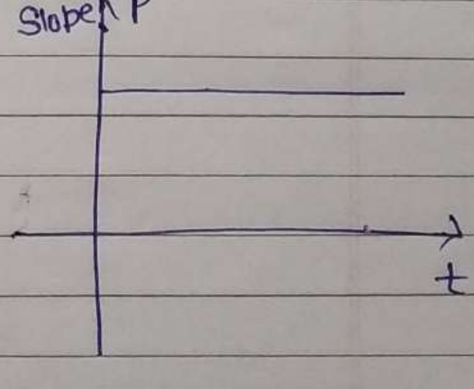
(iii) $\vec{V} = u\cos\hat{i} + (u\sin\theta - gt)\hat{j}$

$$P = \vec{F}\cdot\vec{V} = mg(-\hat{j})\cdot\{u\cos\hat{i} + (u\sin\theta - gt)\hat{j}\}$$

$$-mg(u\sin\theta - gt)$$

$$= -mgu\sin\theta + mg^2t.$$





Rotational Mechanics:-

Rigid body:- If radhe of a body cannot be changed by application of large force however large, than the body is said to be rigid.

Or, The distance between two consecutive particles of a body cannot be changed by application of very large force. the body a said to be rigid.

Pure translational Motion:-

→ When a body travells such that each & every particle of body suffer same displacement than the motion is known as translatory motion.

For, translatory Motion:

$$\vec{S_1} = \vec{S_2} = \vec{S_3} = \ldots = \vec{S_n}$$

$$\vec{v_1} = \vec{v_2} = \vec{v_3} = \ldots = \vec{v_n} \quad (\vec{v}, \text{ say})$$

$$\vec{a_1} = \vec{a_2} = \vec{a_3} = \ldots = \vec{a_n} \quad (\vec{a}, \text{ say})$$

$$\vec{v}_{am} = \frac{m_1\vec{v_0} + m_2\vec{v_0} + \ldots + m_n\vec{v_0}}{m_1 + m_2 + \ldots + m_n} = \vec{v_0}$$

$$\vec{a}_{cm} = \vec{a}_0.$$

Pure Rotational Motion:-

→ When a body moves such that all the particle of a body moves on a circular path whose centre is lies on a straight line than the motion is known as pure rotational motion and the straight line is called as axis of rotation.

The direction of positive axies of roation is taken by right hand-screw rule.

During pure rotational motion different particle of a body have a different distances in a given time interval however all the particles of the body suffer same angular displacement i.e.,

→ $S_1 \neq S_2 \neq S_3 \neq ------ \neq S_n$

$\vec{v}_1 \neq \vec{v}_2 \neq \vec{v}_3 \neq ----- \neq v_n$

$a_1 \neq a_2 \neq a_3 ------ \neq a_n$

$\omega_1 = \omega_2 = \omega_3 = ---- = \omega_n$

$\theta_1 = \theta_2 = \theta_3 = ---- \theta_n$

$\alpha_1 = \alpha_2 = \alpha_3 = ------ = \alpha_n$

classmate

Date
Page

θ, ω, & α are parameter of rotational motion.

Angular Displacement :- Angular displacement of a particle in a given time interval is define as the amount of angle turned by the particle during that interval.

Angular displacement is always taken along positive axis of rotation.

$$\vec{\theta_1} + \vec{\theta_2} \neq \vec{\theta_2} + \vec{\theta_1}$$

Therefore, large angular displacement is not a vector quantity however.

$$d\vec{\theta_1} + d\vec{\theta_2} = d\vec{\theta_2} + d\vec{\theta_1}$$

$\therefore$ Small angular displacement is a vector quantity.

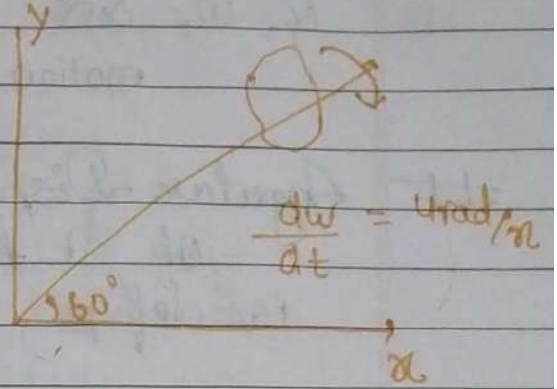
Angular Velocity :- To give complete knowledge regrading $\vec{\omega}$, we must aspect the following:

i) Location of axis
ii) Sense of rotation
iii) Rate of rotation
iv) Orientation of axis

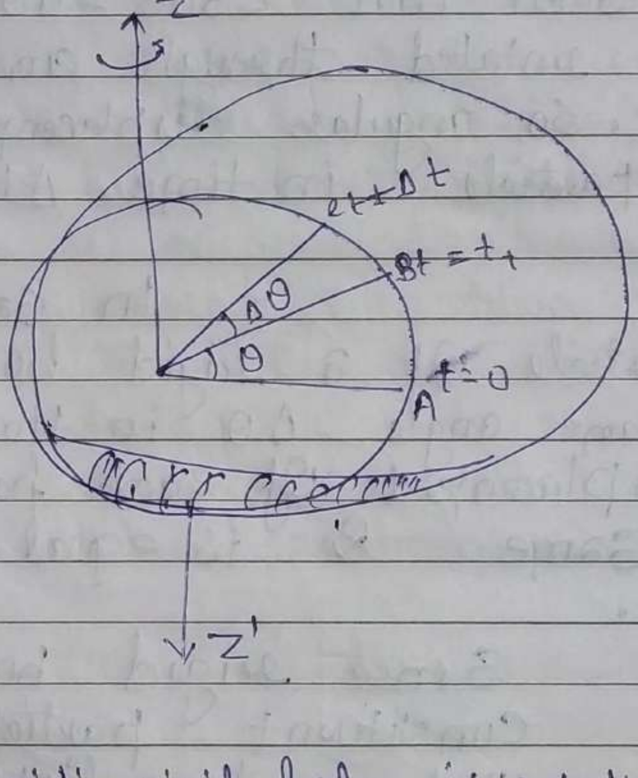
$$\vec{\omega} = 4\cos 60\,\hat{i} + 4\sin 60\,\hat{j}$$

$$= 4 \times \frac{1}{2}\hat{i} + 4 \times \frac{\sqrt{3}}{2}\hat{j}$$

$$\vec{\omega} = 2\hat{i} + 2\sqrt{3}\hat{j}$$



Rotational Kinematics:-



Suppose a ~~ridig~~ rigid body is rotating about 'z, z' axis as shown in above figure. Every particle of the rigid body will rotates in a circle whose radius will be equal to its ⊥r distance from axis of rotation.

Suppose a particle is at 'A' at $t = 0$, goes to the position B at $t = t_1$

θ, ω, & α are parameter of rotational motion.

Angular Displacement :- Angular displacement of a particle in a given time interval is define as

Subtending angle θ at centre after further time Δt goes to the position such that $\angle BOC = \Delta\theta$. So the particle has rotated through an angle $\Delta\theta$ in time Δt. So, angular displacement of this particle in time Δt is $\Delta\theta$.

In same way every particle of a rigid body rotates with same angle $\Delta\theta$ in time Δt, so, angular displacement of each particle will be same & is equal to $\Delta\theta$ in time Δt.

Since rigid body is composed of constituent particle, so, we can say that angular displacement of whole rigid body in time Δt is $\Delta\theta$

Angular velocity of rigid body

$$\vec{\omega} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity $\boxed{\omega = \frac{d\theta}{dt}}$

Angular Acceleration

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Relation among Rotational Variables:-

(A) $\alpha = \frac{d\omega}{dt}$

$d\omega = \alpha dt$

$= \int_{\omega_0}^{\omega} d\omega = \alpha \int_0^t dt$

$= \boxed{\omega = \omega_0 + \alpha t}$

(B) $\omega = \omega_0 + \alpha t$

$\frac{d\theta}{dt} = \omega_0 + \alpha t$

$\Rightarrow d\theta = (\omega_0 + \alpha t) dt$

$\Rightarrow \theta = \omega_0 + \frac{1}{2}\alpha t^2$

(C). $\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} \times \frac{d\theta}{dt}$

$\omega d\omega = \alpha d\theta$

$$\boxed{\omega^2 = \omega_0^2 + 2\alpha \Delta\theta}$$

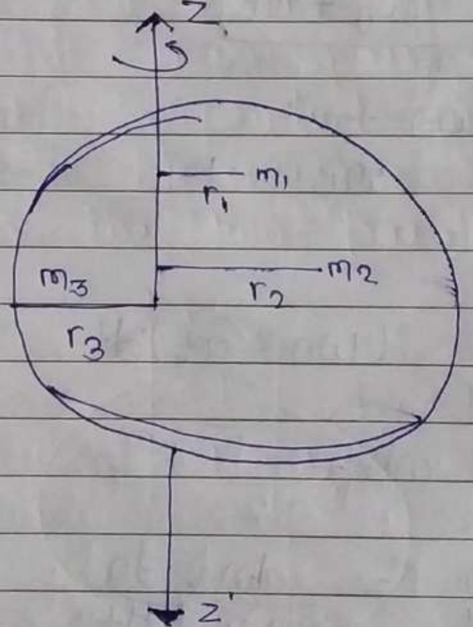
Ratational interia or Moment of inertia :-

Moment of interia is the properties of a body by vertue of which it resist any change in its position of rest or in uniform angular velocity about given axis of rotation.

⇒ Moment of Inertia upon two factors :-

(i) Masses of consituent particle of the body.

(ii) Ir distance from axis of rotation.



Let $m_1 = m_2 = m_3 = m_n = m$

$$I = mr_1^2 + mr_2^2 + mr_3^2 + \dots + mr_n^2$$

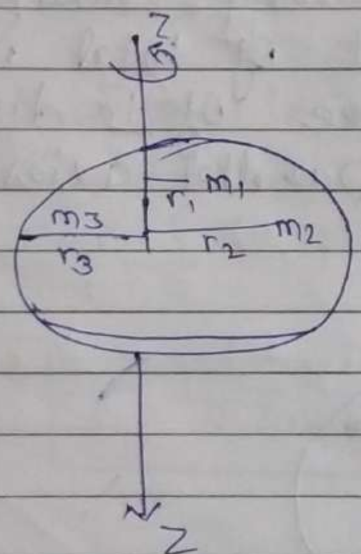
$$= m \times n \left(\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n} \right)$$

$$\boxed{I = \Sigma m_i r_i^2}$$

Moment of interia of a system of particle may be defined as sum of product of masses of the constiutent particle by square of ⊥r distance from axis of rotation.

Radius of Gyration: Radius of gyration is that ⊥r distance from the axis of rotation whose square is multiplied by total mass of the body gives the moment of interia of a body.

$I = mK^2$; where K = Radius of gyration



Let $m_1 = m_2 = m_3 = m_n = m$.

$$I = mr_1^2 + mr_2^2 + mr_3^2 + \dots + mr_n^2$$

$$= m \times n \left(\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n} \right)$$

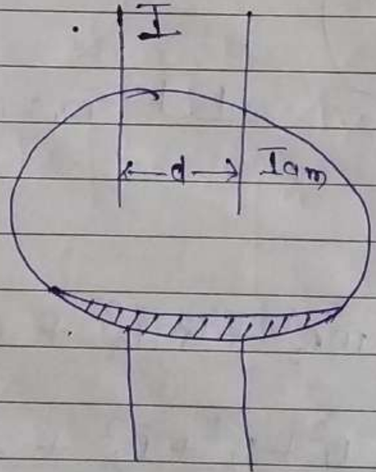
$$= MK^2 = M \left(\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n} \right)$$

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$$

Radius of gyration is nothing but root mean square of ⊥r distance from constituent particle from axis of rotation.

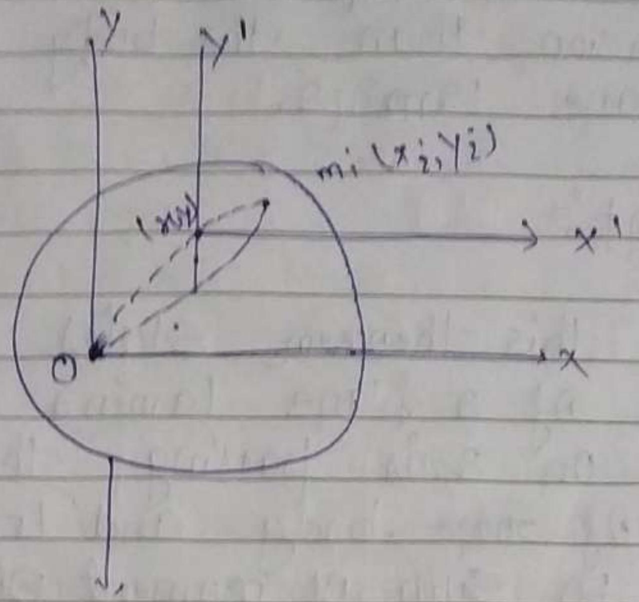
Parallel axis theorem:

This theorem states that moment of interia of a given body about given axis of rotation is equal to the sum of moment of interia of the body about a parallel axies passing through centre of mass & product of total mass of a body by square of ⊥r distance between the two parallel axies.



$$I = I_{cm} + Md^2$$

Proof :-



Axis of rotation are ⊥r. to plane are is passed through (0,0) other is passing through (x_0, y_0)

$$I = \sum m_i \{(x_i - x_0)^2 + (y_i - y_0)^2\}$$

$$= \sum m_i \{x_i^2 + x_0^2 - 2x_0 x_i + y_i^2 + y_0^2 - 2y_0 y_i\}$$

$$= \sum m_i (x_i + y_i)^2 - 2x_0 x_i \sum m_i x_i - 2y_0 \sum m_i y_i + (x_0^2 + y_i^2) \sum m_i$$

$$\boxed{I = I_{cm} + Md^2}$$

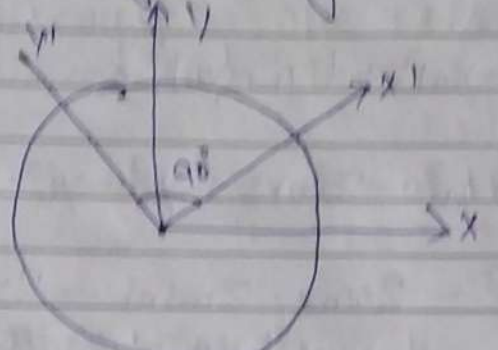
Prependicular axies theorem :-

Prependicular axis theorem valid for plane lamina. If one dimension of a body of a body is very much.

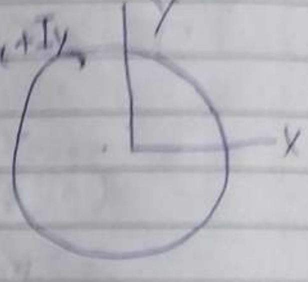
negligible in comparision to other two dimension than the body is said to be plane lamina.

Statement:-

This theorem states that moment of interia of a plane lamina or a body about an axis passing through the plane of same body and ⊥r to the plane is equal to sum of moment of interia about any two mutually ⊥r axis in the plane of the body provided the three axis passing through same point. (i)

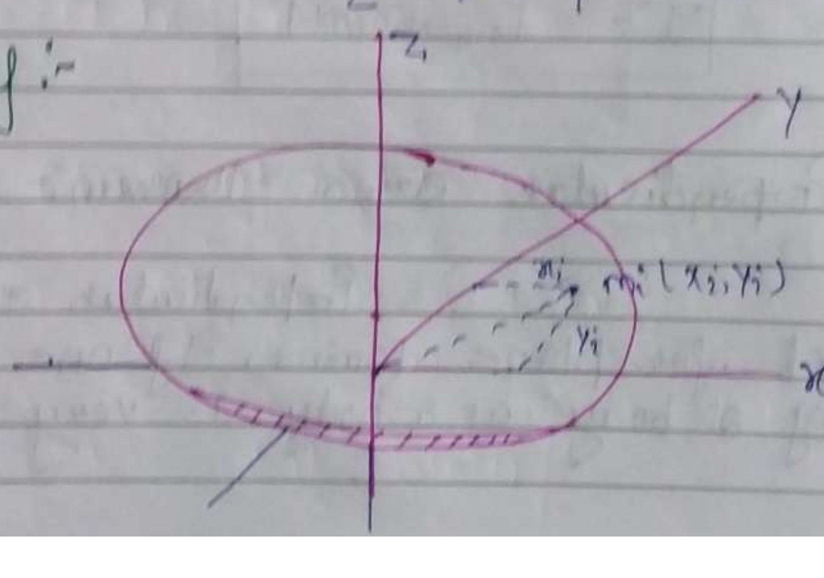


$I_z = I_x + I_y$



$$I_z = I_x + I_y$$

* Proof :-

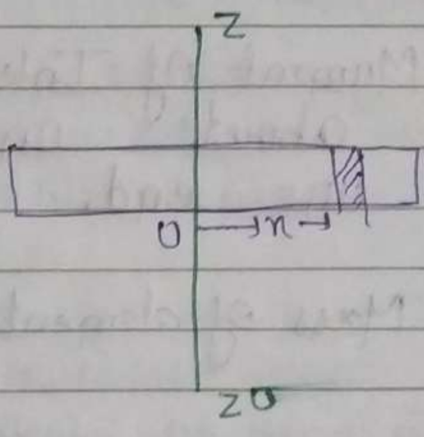


$$I_x = \sum m_i y_i^2$$
$$I_y = \sum m_i x_i^2$$

$$I_x + I_y = \sum m_i \cdot (x_i^2 + y_i^2)$$
$$= I_z$$

$$\therefore \boxed{I_z = I_x + I_y}$$

(i) Moment of interia of a uniform road about an axies ⊥r to the road & passing throught th Cms.



Mass of element

$$dm = \left(\frac{M}{L} dx\right)$$

MoI of the element

$$dI = \left(\frac{M}{L} dx\right) x^2$$

∴ MoI of the rod

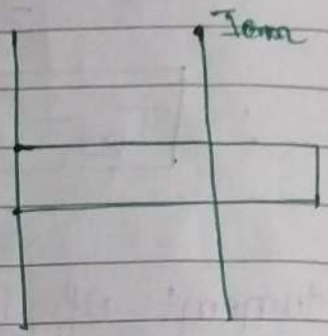
$$I = \int_{-L/2}^{L/2} \frac{M}{L} dx\, x^2$$

$$\boxed{I = \frac{M}{12}}$$

(2) Moment of Interia of uniform rod about an axis passing througth one end and ⊥r to rod.

$$I = I_{cm} + Md^2$$

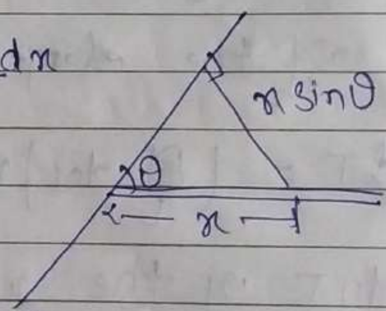
$$= \frac{ML^2}{12} + \frac{ML^2}{4}$$



$$\therefore \boxed{I = \frac{ML^2}{3}}$$

(iii) Moment of Interia of the uniform rod about on axis passing througth one end.

Mass of element $= \frac{m}{l} dx$



MoI of the element

$$dI = \left(\frac{M}{L} dx\right) (x \sin\theta)^2$$

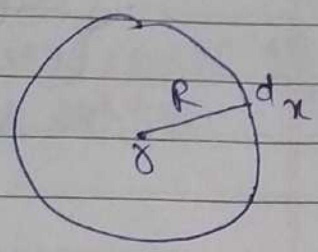
$\therefore$ MoI of rod

$$I = \frac{M}{L} \sin^2\theta \int_0^L x^2 dx$$

$$\boxed{I = \frac{ML^2}{3} \sin^2\theta}$$

(iv) Moment of Interia ring about an axis passing through c.m and ⊥r to plane of ring.

$$I_z = \int dm\, R^2$$



$$\{I_z = MR^2\}$$

Case I : Moment of Interia of ring about diameter

$$I_z = I_x + I_y$$

$$Mr^2 = 2I_x$$

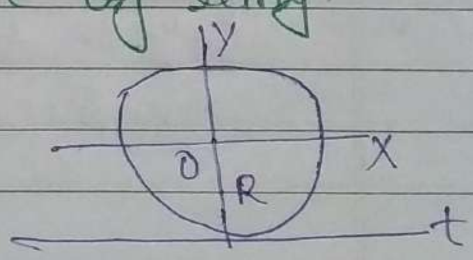
$$I_x = \frac{MR^2}{2}.$$

Case III - Moment of Interia of ring about tangent & ⊥r to the plane.

$$I_{tn} = I_z + MR^2$$

$$= MR^2 + MR^2 = 2MR^2.$$

Case II : Moment of Interia of ring about tangent in the plane of ring.



$$I_t = I_x + MR^2$$

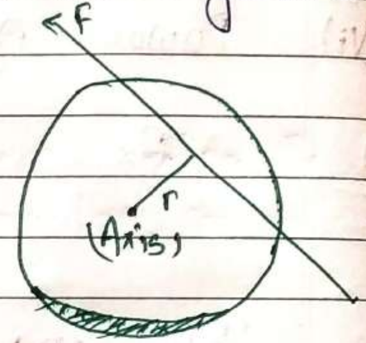
$$I_t = \frac{MR^2}{2} + 2MR^2$$

$$\therefore \left\{ I_t = \frac{3}{2} MR^2 \right\}$$

MOMENT OF FORCE / Torque :

When a force is applied to a body which does not passes through the axis of rotation, Than the magnitude of angular acceleration provided in a body depends on two factors:

i) Magnitude of Force
ii) ⊥r distance from axes of rotation



On the basis of these two factors we defined a new physical quantity is called torque.

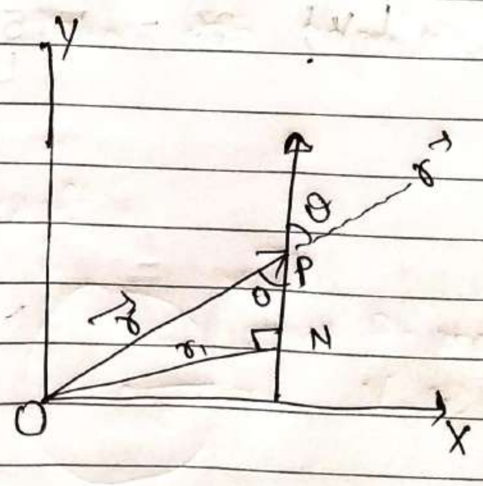
Torque due to force about a point:
Suppose a particle in x-y plane whose position vector $\vec{r}$. $\vec{F}$ is the force acts on the particle as shown in fig:

From fig:

$\sin\theta = \frac{ON}{OP} = \frac{r_1}{r}$

$r_{\perp} = r\sin\theta$

⊥r distance from the axis of rotation.



Torque due to force is defined as vector product of position vector & applied force on the body.

$$\vec{\tau} = \vec{r} \times \vec{F} \leftarrow (i)$$

Also

$$\tau = F \times (r \sin\theta)$$

Or, $\boxed{\tau = F \times r_1}$

Hence, Torque may be defined as product of magnitude of applied force and $\perp^r$ distances from the axis of rotation.

Now, $\vec{\tau} = \vec{r} \times \vec{F}$

if $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$.

then,

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

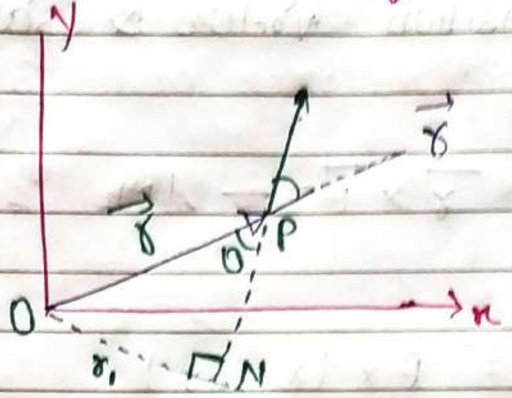
$$\vec{\tau} = i(y f_x - zF_y) - j(xF_z - zF_x) + k(xF_y - yF_x)$$

Angular Momentum of a particle:

$\sin\theta = ON/OP$

$ON = OP \sin\theta$

$r_1 = r \sin\theta$.



The vector product of position vector & linear momentum is called angular momentum

$$\vec{L} = \vec{r} \times \vec{P} \quad \text{--- (i)}$$

Also.

$$|\vec{L}| = rp \sin\theta = p \times (r\sin\theta)$$

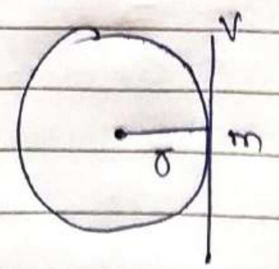
$$\therefore L = p \times r_1$$

Hence, angular momentum is also known as moment of momentum.

→ Concept:-

When a particle moves in a circle of radius of with speed v then its angular momentum.

$$L = p \times r = mvr = m\omega r^2$$



$P = mv$

$v = \omega r$.

Angular momentum of rigid body in pure rotational motion :-

$$\vec{L} = \vec{L_1} + \vec{L_2} + \vec{L_3} + \dots + \vec{L_n}$$
$$= m_1 r_1^2 \vec{\omega} + m_2 r_2^2 \vec{\omega} + \dots m_2 r_n^2 \vec{\omega}.$$

$$\vec{L} = \vec{\omega} \, (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2)$$

$$\vec{L} = \vec{\omega} \sum_{i=0}^{n} m_i r_i^2 = \vec{\omega} \cdot I$$

Where : $I = \sum m_i r_i^2$

$$\therefore \boxed{\vec{L} = I \vec{\omega}}$$

* Kinetic Energy in pure rotational motion :-

$$K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \dots + \frac{1}{2} m_n v_n^2$$

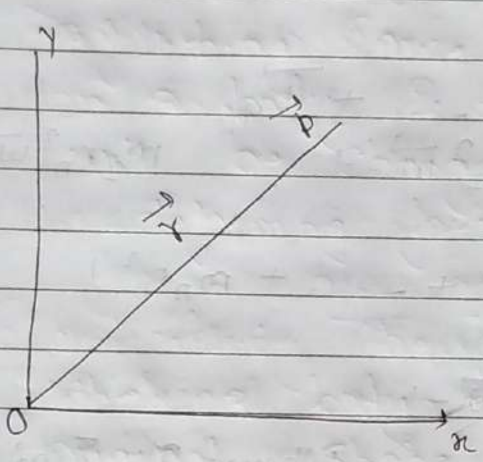
$$= \frac{1}{2} m_1 r_1^2 \omega^2 + \frac{1}{2} m_2 r_2^2 \omega^2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$= \frac{1}{2} \omega^2 (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2)$$

$$= \frac{1}{2} \omega^2 \sum_{i=0}^{n} m_i r_i^2 = \frac{1}{2} \omega^2 . I$$

$$\therefore \boxed{K = \frac{1}{2} I \omega^2}$$

* Relation between torque & angular momentum :-



Angular momentum for a particle.

$$\vec{L} = \vec{r} \times \vec{p} \quad \text{--- (i)}$$

Differetiating both side of equations :-

i) With respect to time :

$$\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

$$\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F} + \vec{v} \times m\vec{v}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} + m(\vec{v} \times \vec{v}^{\,0})$$

$$\therefore \boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

for a rigid body,
Rate of change of momentum of a rigid body is equal to net torque acting on it

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

Conservation Of Angular Momentum :-

Let, $\vec{L} = \sum \vec{L_i}$ = Total angular momentum of a rigid body.

$\vec{\tau}_{ext}$ = Net external torque on the rigid body.

Then, we know that

$$\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

if. $\vec{\tau}_{ext} = 0 \Rightarrow \vec{L}$ = constant.

Hence, if total torque due to external forces acting on a system is equal to zero then total angular momentum of the system will remain conserved. This is known as principle of conservation of angular momentum.

Concept :-

$$\because \vec{L} = I\vec{\omega}$$

$$\frac{d\vec{L}}{dt} = I \cdot \frac{d\vec{\omega}}{dt}$$

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Properties Of Matter :-

* **Deforming force :-** When a force is applied on a body & Shape & size of the body will be changed, then the force is called deforming force.

* **Elasticity :-** The property of a body virtue of which a body trends to regain its original shape & size after the removal of deforming force is called elasticity.

The body which regain its original shape & size after removal of deforming force is called elastic body or perfectly elastic body.

* **Plasticity :-** The property of matter by virtue of which a body does not regain its ~~angle~~ original shape & size after removal of deforming force is called plasticity & the body is known as plastic.

The elastic & plastic nature of material are only idealized concepts as it is not realized in pratice. No, body is perfectly plastic, All the body found in nature lie between these two extreme limits.

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Stress :-

When a body is subjected to external deforming force, the change in its shape & size due to relative displacement of its molecules. As a result an internal restoring force developed inside the body which try to regain its original shape & size.

The restoring force per unit area set up inside the body is called Stress.

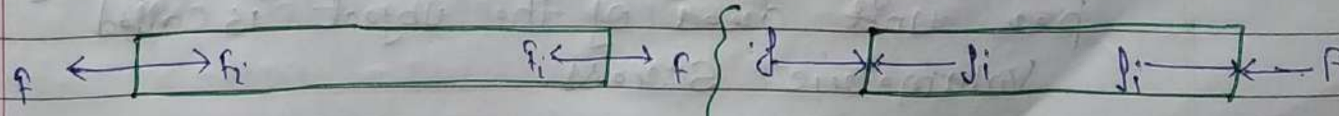
$$\text{Stress} = \frac{\text{Restoring force}}{\text{Area}} = \frac{f_i}{A}$$

S.I unit of stress = N/M^2

Types Of Stress :-

1) Longitudinal stress

→ Tensile stress
→ Compressive stress.



Tensile stress $= \frac{f_i}{A}$

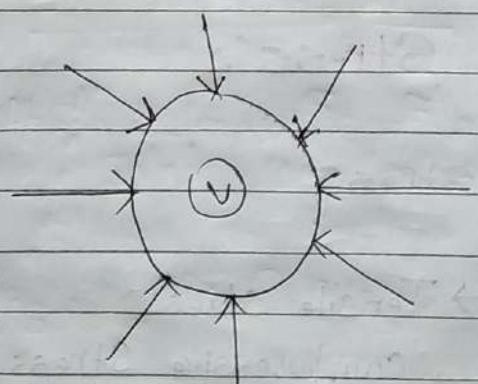
Compressive Stress $= \frac{f_i}{A}$

→ When a force acts normal to the Surface then ~~reso~~ restoring force per unit area is called normal stress or longitudinal stress.

* Tensil Stress :- When two equal & opposite forces are applied on the rod, then restoring force normal to the cross-sectional area of rod per unit area is called tensil, in this case length may increase.

ii) Bulk Stress or Volume Stress :-

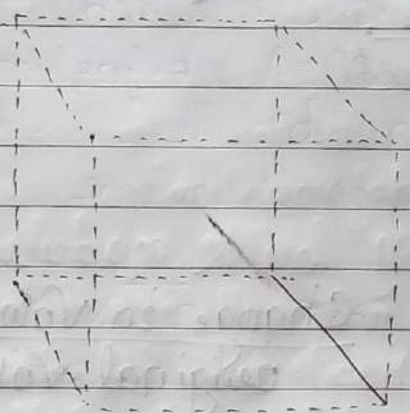
When an object is immersed in a fluid the fluid exerts force on the surface of the objects as shown in figure.

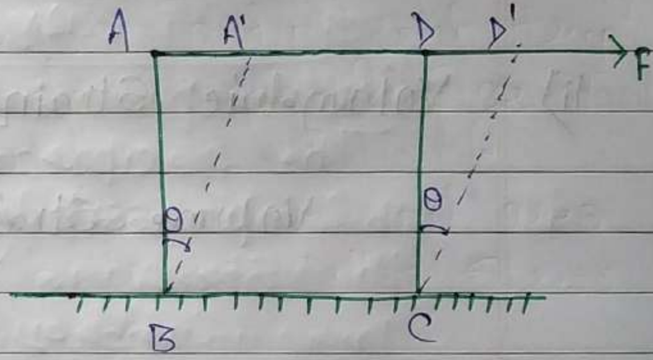


The force acting ⊥ to the surface of object per unit area of the object is called Volumetric stress.

$$\text{Bulk Stress} = F/A$$

iii) Tangential Stress Or Shearing Stress:-





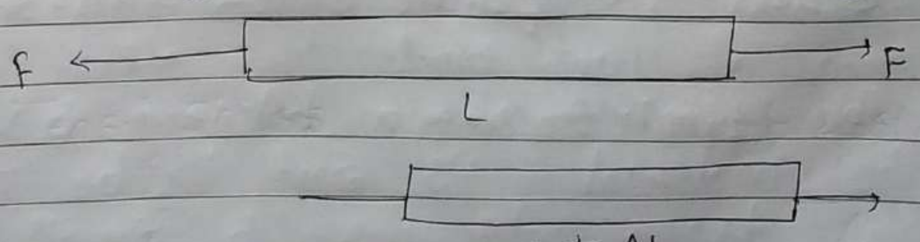
Sharing Stress = F/A
= Tangential force per unit area.

✳ **Strain:-** When a deforming force acts on a body then its shape & size will be changed & the body is said to be deformed or strained.

$$\text{Strain} = \frac{\text{Change in length (dimension)}}{\text{Original dimensions.}}$$

Types Of Strain:-

i) Longitudinal Strain:- It is defined as the ratio of change in length to the original length.



longitudinal Strain = $\dfrac{\text{Change in length}}{\text{Original length}}$

$$= \frac{\Delta L}{L}.$$

ii) Volumetric Strain:-

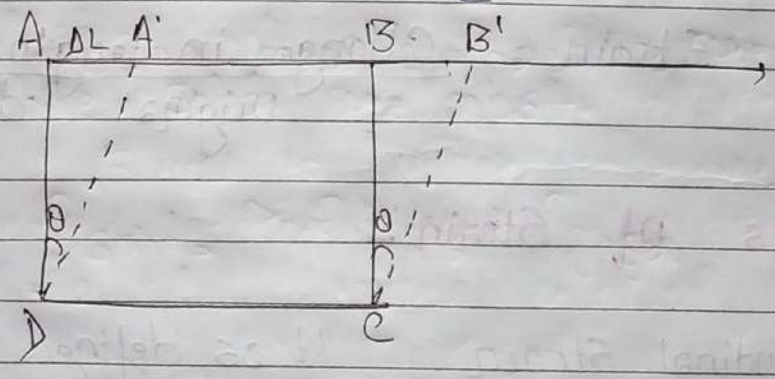
Volume Strain = $\dfrac{\text{Change in Volume}}{\text{Original Volume}}$

$$= \frac{-\Delta V}{V}.$$

iii) Shearing Strain (θ):-

θ = angle relates by Shearing strain

$$\theta \simeq \tan\theta = \frac{\Delta L}{L}$$



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Hooke's LAW:- Hooke's law states that within the elastic limit, Stress is directly proportional to strain.

i.e, Stress $\propto$ Strain

Or, Stress $= E \times$ Strain.

where, E, - proportionality constant known as modulus of elasticity.

$$\therefore \boxed{E = \frac{\text{Stress}}{\text{Strain}}}$$

$$\text{S.I unit of } E = \frac{N/m^2}{1} = N/m^2$$

$$\text{dimension} = [E] = [ML^{-1}T^{-2}]$$

Types of Modulus of Elasticity:-

i) Young's Modulus:- (Y), It is defined as the ratio of longitudinal stress to the longitudinal Strain.

$$Y = \frac{\text{longitudinal Stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L}$$

ii) Bulk Modulus (B):- It is defined as the ratio of volumetric stress to Volume Strain,

$$B = \frac{\text{Volumetric Stress}}{\text{Volumetric Strain}} = \frac{F/A}{\Delta V/V}$$

$$B = \frac{\Delta p}{-\Delta V/V}, \quad \Delta p = \text{Change in pressure}$$

$$\boxed{B = -V \cdot \frac{\Delta p}{\Delta V}}$$

The reciporcal of bulk modulus is called compreessibity.

$$\text{Compressibility} = \frac{1}{B} = \frac{-1}{V} \cdot \frac{\Delta V}{\Delta p}$$

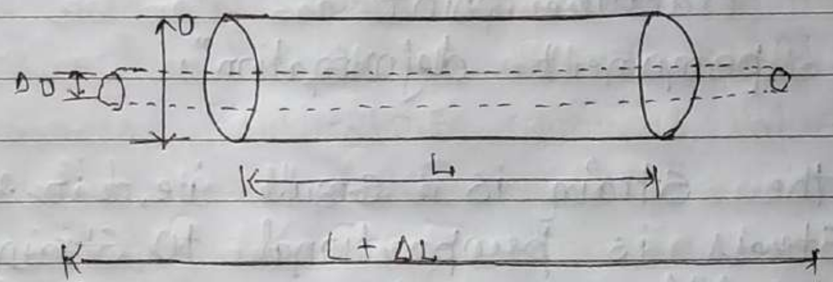
iii) Modulus of Rigidity :- (η), It is defined as the ratio of tangential stress to the shearing strain.

$$\boxed{\eta = \frac{F/A}{\theta}}$$

→ Gases being mass compressible are least elastic while solid are most.

$$E_{solids} > E_{liquids} > E_{gases}$$

Paisson's Laws :- The ratio of lateral strain to the longitudinal strain is called paisson's ratio It is denoted by 'σ'.



Lateral strain $= \dfrac{-\Delta D}{D}$

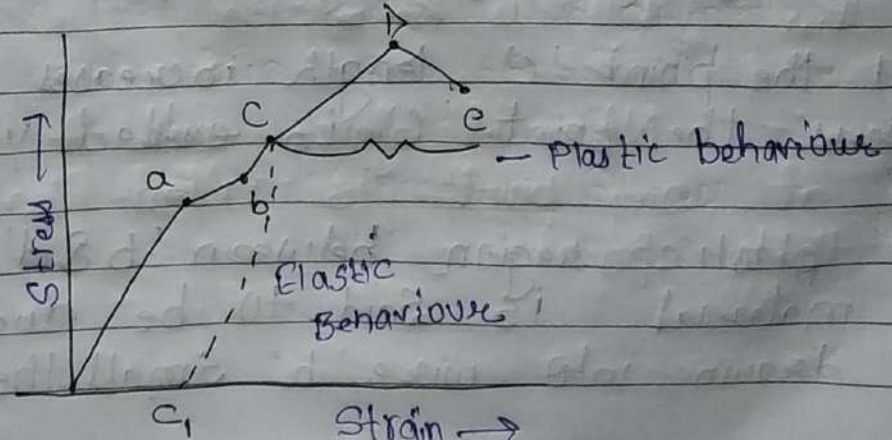
$\sigma = \dfrac{\text{Lateral strain}}{\text{longitudinal strain}} = \dfrac{-\Delta D/D}{\Delta L/L}$.

$\therefore \sigma = \dfrac{-b}{D} \cdot \dfrac{\Delta D}{\Delta L}$

σ = Unitless and dimensionless its lies between 0 & $+1/2$

$\therefore$ i.e, $\boxed{0 < \sigma < 1/2}$

* Stress — Strain Curve :-



"a = proportinality limit.
b = Elastic limit
c = Yield point
d =
e = fracture point
o' = pemanent deformation".

i) When the strain is a small i.e, (in region OA) the stress is proportional to strain i.e, Hook's law invalid. So point 'a' is called proportinality limit.

ii) If strain to increase a little bit (i.e, in region ab). Hook's law is not valied but wire region its orginal length after removal of deforming force. So, point 'b' is called elastic limit.

iii) If the wire is strectched beyond the elastic limit (b), the strain increase more (c).

iv) When stress is further increase very large strain produced & come to be the state of fracture & finally break down.

v) Beyond the point 'c', length increases without stress the point 'c' is called yield point.

vi) If the plastic region between 'b & d'. is 'a' large the material is said to be ductile & can be drawn into wire 'b' small the material is Said to be brittle as it is break soon after the elastic limit is crossed.

Q. A wire of length 1m is stretched by a force 10N. The area of cross-section of the wire is 2×10^{-6} m^2 & Y is 2×10^{11} N/m^2. Calculate the (i) Stress (ii) Strain (iii) The increase in length of the wire.

Sol^n · Given :- $L = 1m$, $F = 10N$.

$A = 2\times10^{-6} m^2$, $Y = 2\times10^{11} N/m^2$

i) $Stress = \frac{F}{A} = \frac{10^5}{2\times10^{-6}} N = 5\times10^{6} N m^2$

ii) $Y = \frac{stress}{strain}$

$Strain = \frac{Stress}{Y}$

$= \frac{5\times10^{6} \times N \times \cancel{m^2}}{2\times10^{11}\times N \times \cancel{m^2}} = 2.5\times10^{-5}$

iii) $Strain = \frac{\Delta L}{L}$

$\Delta L = L \times Strain$

$= 1\times 2.5\times10^{-5} m$

$= 2.5\times10^{-5} m$. Ans.

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Fluid Mechanics:-

* Fluid :- Those substances which can be flow is called fluid.
Liquid & gas can be flow, so, these are the fluids.

* Mass density (ρ):- Density of material is defined as mass per unit volume of the material.

$$\therefore \text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

→ S.I unit of ρ = Kg/m^3.

* Relative density (R.D):- The ratio of density of substance to the density of water at 4°c is called relative density.

$$R.D = \frac{\text{Density of Substance}}{\text{Density of water at } 4^\circ C}$$

$\therefore$ Density of Substance = R.D × density of water at 4°C.

$$\rho_w = 1.0 \times 10^3 \, kg/m^3.$$

* Pressure:- Normal force exerted on unit area at any point is called pressure at that point.

$$\boxed{P = \frac{F}{A}}$$

S.I unit $= N/m^2$
$= 1$ Pascal
$= 1\ (Pa)$

$1\ Pa = 1\ N/m^2$
$1\ bar = 10^5\ N/m^2$
1 torr = 1 mm of Hg height.

* Atomic Pressure (atm):-

The gaseous envelope surrounding the earth is called the ~~atomic pressure~~ atomsphere. The atomsphere can be supposed to the divided into number of layers parallel to the surface of earth, each layer support the weight of all other layer above it & thus subjected a pressure due to their weight.

This pressure is called atomic pressure.

$$P_0 = 1.01 \times 10^5\ N/m^2.$$

* Variation of Pressure with depth:-

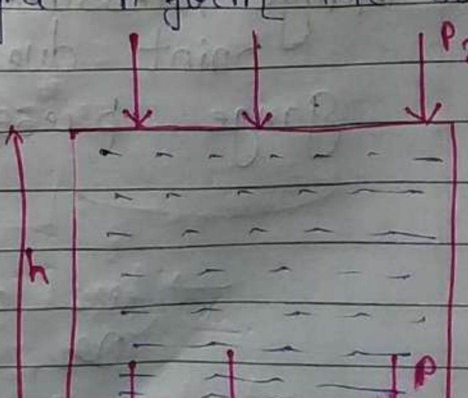
Let us consider a beaker containing liquid of density 'P' upto the height 'h' form the bottom

$\therefore$ density $= \frac{Mass}{Volume}$

Mass = density x Volume

Mass = $P.(A.h)$

where A = area of cross Section of beaker.



So, $m = \rho \cdot A \cdot H$

force on the surface of bottom of beaker = Gravity force.

$= mg = \rho \cdot A \cdot h \cdot g$.

So, pressure on the bottom on beaker,

$$P = F/A = \frac{\rho \cdot \cancel{A} \cdot h \cdot g}{\cancel{A}}$$

$$\therefore \boxed{P = \rho \cdot h \cdot g}$$

→ This pressure is due to liquid only if we consider atomicspheric pressure 'P_0' also, then total pressure at the bottom,

$$\Rightarrow \boxed{P_{Total} = P_0 + \rho g h}$$

* Absolute Pressure:- The total pressure at a point including the contribution of the liquid as well as that of the atmosphere is called absolute pressure.

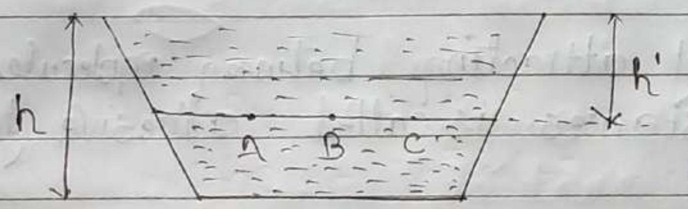
* Gauge Pressure:- The total pressure at a point due to the liquid only is called gauge pressure.

$$P_{absolute} = P_{atm} + P_{gauge}$$

$$P_{Total} = P_0 + \rho g h$$

Or, $P_{Total} - P_0 = \rho g h$ = Gauge pressure.

Concept:-

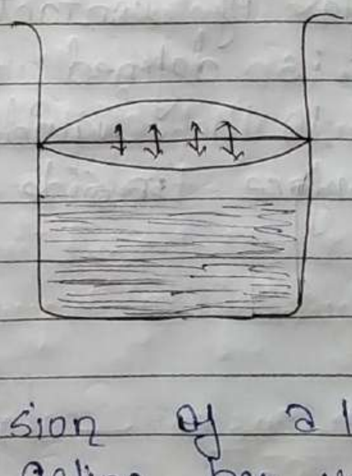


$P_A = P_B = P_c = \rho g h$

The pressure is ~~sta~~ same everywhere on a horizontal plane at depth 'h'.

Surface Tension:-

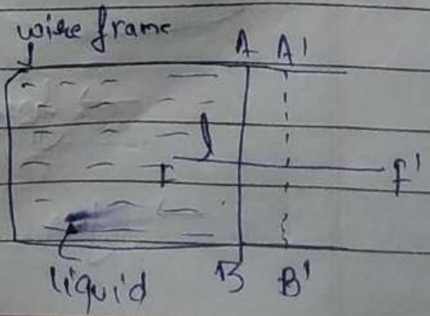
The property of liquid due to which its free surface ~~tire~~s tries to have minimum surface area & behaves like a stretched elastic memberane is called surface tension.



Surface tension of a liquid is measured by the force acting per unit length on either side of an imaginary line drawn on the free surface of liquid.

Surface tension. $\boxed{T = \frac{F}{L}}$, S.I unit of T: N/M

* Surface Energy:- Let us consider ~~a~~ a wire frame & movable framing a loop.



Force of Adhesion :-

The force of attraction between molecules of different substances is called adhesive force

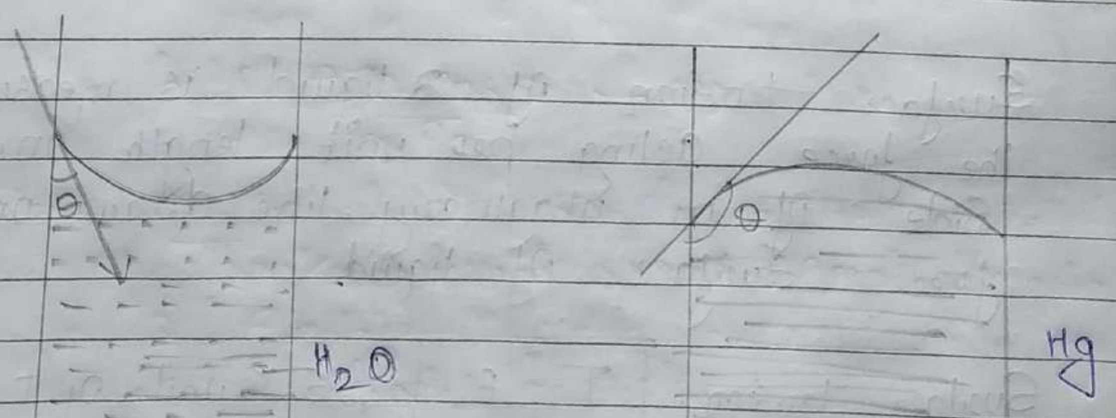
e.g.

i) Adhesive force help us to write on the paper with link.

ii) Water wets the glass plate due to adhesive force.

Angle of Contact :-

Angle of contact between a liquid & a solid is defined as the angle between the tangent to the liquid surface & the solid surface inside the liquid.



→ for concave meniscus, $\theta < 90°$

→ for convex meniscus, $\theta > 90°$

Significance of Angle of Contact:-

→ If angle of contact (θ) is acute (i.e, $\theta < 90^\circ$) then,

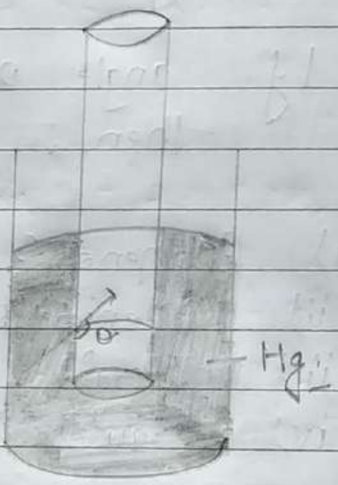
i) Cohensive force is smaller than adhesive.
ii) Meniscus of liquid is concave.
iii) Liquid rises in capillary tube.
iv) Liquid wets the well of the solid.

If angle of contact (θ) is obtuse (i.e, $\theta > 90^\circ$) then,

i) Conhensive force is greater than the adhesive.
ii) Meniscus of liquid convex.
iii) Liquid falls down in capillary tube.
iv) Liquid does not wet, the wall of solid.

Capillarity or Capillary Action:-

A glass tube of very fine bore throughout the length of the tube is called capillarity or tube. If the capillary tube is dipped in water the water wets the inner side of the tube & rises in it. If same capillary tube is dipped in the mercury then the tube is mercury in the tube is depressed.



→ The phenomenon of the rise or fall of liquid in a capillary tube is called capillarity or capillarity action.

Let, r = radius of capillarity tube.
T = Surface tension of liquid
h = height of liquid in a capillary tube.
ρ = density of liquid.

$$h = \frac{2T \cdot \cos\theta}{r \cdot \rho g} \quad \text{— (i)}$$

The expression is called acent formula.

$$\boxed{h \propto \frac{1}{r}}$$

$$\therefore h_1 r_1 = h_2 r_2$$

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* Force of Cohesion :-

The force of attraction between the molecules of same substances is called force of cohesion.

e.g. (i) Two drops of liquid coalesce (i.e, combine due to cohesive force.

ii) it is very difficult to break a drop of mercury into small droplets due to large cohesive force.

Viscosity

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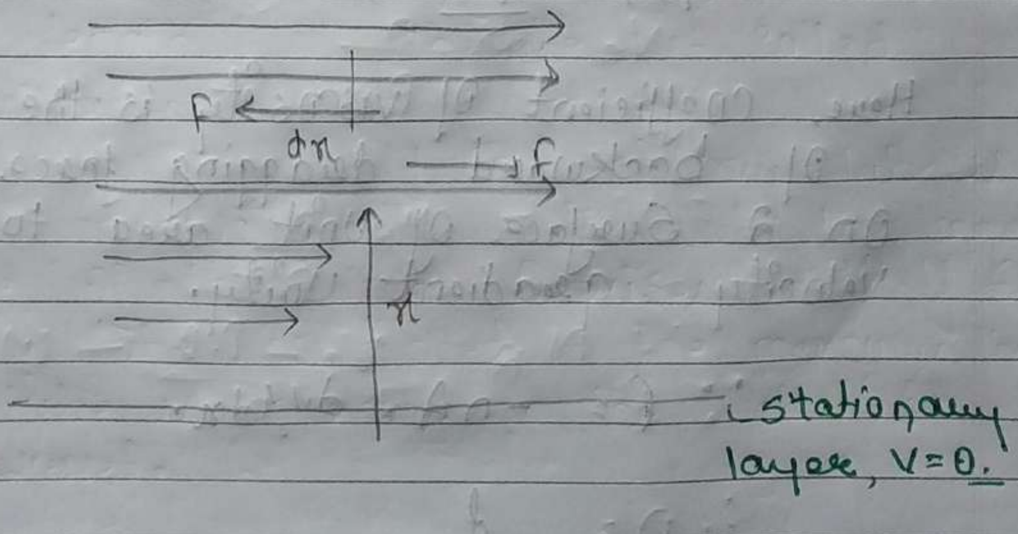
It is the property of a liquid or gas by virtue of which an opposing force (internal friction) comes into play between different layers of a liquid, whenever there is a relative motion between these layers of the liquid is called viscosity.



Viscous force 'f' depend upon:-

i) nature of the fluid.
area of the surface (A)
Velocity gradient (dv/dx)

A/c to Newton's

$$F \alpha A \cdot \frac{dv}{dx}$$

$$F = -\eta A \cdot \frac{dv}{dx} \quad --- (i)$$

Where η = coefficient of viscocity, it depends on nature of fluid.

if $A = 1m^2$, $\frac{dv}{dx} = 1sec^{-1}$, then

$$|F| = \underline{\eta.}$$

Here, coefficient of viscosity is the magnitude of backward dragging force acting on a surface of unit area to maintain velocity gradient unity.

$$\therefore F = -\eta A \cdot dv/dx.$$

$$\therefore \eta = \frac{f}{A \cdot \left(\frac{dv}{dx}\right)}$$

S.I unit of $\eta = \dfrac{N}{m^2 \times \frac{\cancel{m}}{s \times \cancel{m}}} = \dfrac{N.S}{m^2}$ = Poiseville (PL).

C.G.S unit of $\eta = \dfrac{dyne.se}{cm^2}$ = Poise.

$\boxed{1 \text{ poiseville} = 10 \text{ poise}}$

Dimension of $\eta = \dfrac{MLT^{-2}}{L^2 \times T^{-1}} = [ML^{-1}T^{-1}]$.

* Effects of temperature on Viscosity of liquid:-

$$\eta \propto \frac{1}{\sqrt{T}}$$

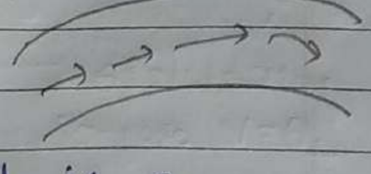
for gas. $\boxed{\eta \propto \sqrt{T}}$

Viscosity $\propto$ adultration.

* Flow of liquid:-

i) Stream line or Steady or laminar flow:-

* flow of liquid is solid to be stream line when at a particular point velocity of any particle of the liquid is Same.



It means in case of stream-line flow velocity at particular point is independent of time.

(ii) Turbulent or Unsteady or non-laminar flow:-

when the velocity of particle of liquid at a particular point changes with time then motion is known as turbulent motion.

Critical Velocity & Reynold's number:-

It is minimum velocity up to which flow of liquid remain steady.

Critical velocity depends upon:-

i) Viscosity of the fluid (η).
Density of the fluid (ρ).
Diameter of the tube,
it is found that,

$$V_c \propto \frac{\eta}{\rho D}$$

$$\therefore V_c = N_R \cdot \frac{\eta}{\rho \cdot D}$$

$$\therefore N_R = \frac{V_c \cdot \rho \cdot D}{\eta} \quad \text{— (i)}$$

Here, N_R = Reynold's Number.

↳ if $N_R < 2000$, then flow is steady
↳ if $N_R > 3000$, then flow is turbulent
↳ if $2000 < N_R < 300$, then flow is unstable.