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UNITS AND MEASUREMENTS

Introduction, Need of measurement and unit in engineering and science, Unit of physical quantity and requirements of standard unit, System of units, Fundamental and derived quantities and their units, Least count and range of vernier caliper, Micrometer screw gauge and spherometer, Accuracy and precision of instruments, Error and estimation of errors, Significant figures, Rules and identification of significant figures.

1.1 INTRODUCTION

In physics we are required to measure the physical quantities. Measurement consists of the comparison of an unknown quantity with a known fixed quantity. Development of physics is mostly due to very accurate and precise instruments which can measure the size of physical quantity to high degree of accuracy.

1.1.1 Need of Measurement and Unit in Engineering and Science

The main purpose of measurement in engineering and science is to determine whether a job has been manufactured to the requirements of specification. Measurement is compulsory part of development technology. Accuracy of measurement depends on

(1) Method of measurement.

(2) Measuring instrument.

Measurement consists of the comparison of given quantity with standard.

e.g. Length of table is 3 metre.

i.e. Any measurement consists of two parts.

The first is the number which indicates the magnitude of quantity and second indicates the standard. In the above example, 3 is the magnitude and metre is the standard (unit) of that quantity. It gives exact sense that the length of the table is 3 times the standard.

All the quantities measured must be clearly defined so that there should not be any confusion or doubt in between two parties. e.g. customer asks for 3 metre cloth to a shopkeeper. Here metre is the standard for the length and which is fixed and well accepted by universe (society). Hence, there is no confusion between shopkeeper and customer.

Specific examples :

Measurement is very important in science and engineering in the following ways.

Need in science :

1. **To predict scientific laws :** The correlation between the two or more quantities can be found out by measuring the changes occurred in their magnitudes under different conditions.

For example, consider gas of fixed mass (m) and with constant absolute temperature (T). While predicting relation between its pressure and volume, one has to check the changes occurred in these quantities by accurate measurement. "With the measurement of pressure at different volumes of gas, it is found that as pressure of gas increases, its volume decreases under fixed mass and temperature." This becomes law (Boyle's law).

2. **To verify the law :** If some one wants to verify the law or scientific statements, the concerned quantities should be measured accurately under given conditions.

For example, to verify Archimede's principle, one should calculate volume of solid by using mathematical formula and compare this value with the volume of liquid displaced by the same solid.

Need In Engineering :

1. **Correct sense of the quantities :** For example, measurement of mass, length, diameter etc. must be clear for development of engineering product.

2. **In quality assurance of product :** Quality depends on various factors, like strength, finishing, precision, correctness, user friendliness, etc. These factors should be measured prior to launching the product in market. In every industry, this care is taken by the department of metrology and quality control.

1.1.2 Unit of a Physical Quantity and Requirements of Standard Unit

Definition of unit : The standard used for measurement of a physical quantity is called unit of that quantity.

In the above example, 3 metre is the length of the table. Here metre is the standard (unit) used for the measurement of the length.

Requirements of standard unit :

The unit selected should have following characteristics :

1. It should be universally accepted (i.e. accepted by all).
2. It should be definite and well defined.
3. It should be invariable (fixed) with time and place.
4. It should be easily reproducible.
5. It should be easily comparable with other similar units.
6. Its size should be such that the quantities measured with it should not be too large or too small.
7. It should be readily available.

1.1.3 System of Units

The system of units which are in use are (i) C.G.S., (ii) M.K.S., (iii) F.P.S. and (iv) S.I.

Here length, mass and time are taken as basic physical quantities. Using this basic quantities, many physical quantities can be derived.

(i) **C.G.S. system :** In this system, the units of length, mass and time are Centimetre, Gram and Second.

(ii) **M.K.S. system :** In this system, the units of length, mass and time are Metre, Kilogram and Second.

(iii) **F.P.S. system :** In this system, the units of length, mass and time are Foot, Pound and Second.

(iv) **S.I. system :** S.I. is the abbreviation of the French name System Internationale (International system of units).

To have a common platform between different countries, a common system is proposed and which is accepted by whole world.

This system consists of seven fundamental (basic) units with the help of which, unit of any quantity in engineering and science can be derived. Table 1.1 gives these quantities and their units along with their symbols.

Table 1.1

Fundamental physical quantity	Unit	Symbol
1. Length	metre	m
2. Mass	kilogram	kg
3. Time	second	s
4. Electric current	ampere	A
5. Temperature	kelvin	$^{\circ}\text{K}$
6. Luminous intensity	candela	cd
7. Amount of substance	mole	mol

The following additional units are used in S.I. system as supplementary units.

Table 1.2 : Supplementary units

Supplementary physical quantity	Unit	Symbol
1. Plane angle	radian	rad
2. Solid angle	steradian	sr

1.1.4 Fundamental and Derived Quantities and their Units

Physical quantities are divided into two groups :

- (i) Fundamental quantities and
- (ii) Derived quantities.

Fundamental Quantities :

The physical quantities which does not depend on any other quantity are called fundamental physical quantities.

The unit of fundamental quantity is called fundamental unit. It does not depend on any other unit.

These are the basic (fundamental) quantities (units) with the help of which unit of any quantity in engineering and science can be derived. The following are fundamental physical quantities along with their units.

Table 1.3

Fundamental physical quantity	Fundamental unit
1. Length	metre
2. Mass	kilogram
3. Time	second
4. Electric current	ampere
5. Temperature	kelvin
6. Luminous intensity	candela
7. Amount of substance	mole

Derived Quantities :

The physical quantities which are derived using two or more fundamental quantities are called derived quantities.

The units of derived quantities are called derived units. These units depend on two or more fundamental units.

As we have seen there are seven fundamental quantities. The remaining all quantities are derived quantities.

e.g.
$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{\text{Mass}}{l \times b \times h}$$

It is derived using two fundamental quantities i.e. mass and length. Following are few derived quantities along with their units.

Table 1.4

Derived physical quantity	Derived unit
Area	square metre
Volume	cubic metre
Velocity	metre/sec
Acceleration	metre/sec ²
Force	newton
Pressure	newton/metre ²

As an engineer we should be able to convert the units of same quantity in different systems. Recommended prefixes for information of multiples and sub-multiples of units are as shown below :

Table 1.5

Value Indicated	Prefix	Symbol
10 ¹²	tera	T
10 ⁹	giga	G
10 ⁶	mega	M
10 ³	kilo	k
10 ²	hecto	h
10 ¹	deca	da
10 ⁻¹	deci	d
10 ⁻²	centi	c
10 ⁻³	milli	m
10 ⁻⁶	micro	μ
10 ⁻⁹	nano	n
10 ⁻¹²	pico	p
10 ⁻¹⁵	femto	f
10 ⁻¹⁸	atto	a

1 m → 10⁻³ m

1.2 LEAST COUNT AND RANGE OF THE INSTRUMENT

Definition of least count : Least count of the instrument is the smallest possible reading which can be measured accurately using that instrument.

i.e. It is the smallest possible reading that we can measure accurately using given instrument (device).

Range of the instrument : It is the minimum to maximum reading that can be measured accurately using that instrument.

i.e. It is the minimum to maximum reading, that we can measure using the given instrument (device).

1.2.1 Least Count of Vernier Caliper

Vernier caliper is a device (instrument) using which, outer and inner dimensions of the small object can be measured accurately. This vernier caliper consists of two scales : (1) Main scale and (2) Vernier scale.

Least count of vernier caliper (formula) :

$$\text{Least count} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on vernier scale}}$$

Vernier caliper type 1 :

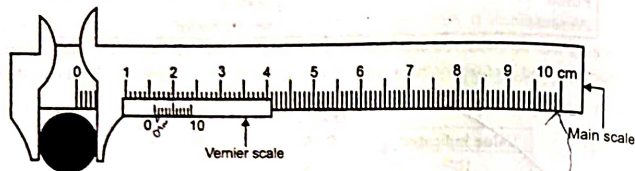


Fig. 1.1

In Fig. 1.1, the calibrated scale on main scale is in centimeter. There are 10 divisions in 1 cm. i.e. 1 cm is divided into 10 equal parts.

∴ Smallest division on main scale is 0.1 cm.

$$\begin{aligned} \text{Least count} &= \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}} \\ &= \frac{0.1 \text{ cm}}{10} = 0.01 \text{ cm} \end{aligned}$$

L.C. = 0.01 cm

and

Range = 0 to 10 cm

Vernier caliper type 2 :

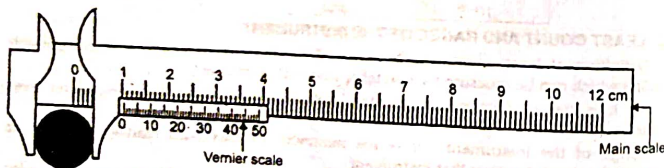


Fig. 1.2

In the above vernier caliper, the scale on main scale is in centimeter. Again in 1 cm, there are 10 divisions, hence smallest division on main scale is 0.1 cm. But there are total 50 divisions on vernier scale.

$$\begin{aligned} \therefore \text{Least count} &= \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on vernier scale}} \\ &= \frac{0.1 \text{ cm}}{50} = 0.002 \text{ cm} \end{aligned}$$

L.C. = 0.002 cm

and

Range = 0 to 12 cm

1.2.2 Least Count of Micrometer Screw Gauge

Micrometer screw gauge is a device (instrument) using which, outer dimensions of smaller object can be measured more accurately. Micrometer screw gauge consists of two scales : (1) Main scale on axial line and (2) Circular scale.

Micrometer screw gauge type 1 :

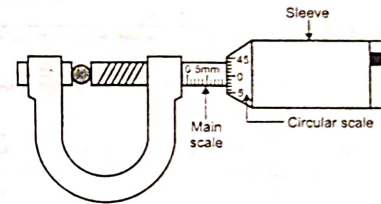


Fig. 1.3

Pitch : It is the distance between two successive threads.

How to find out (measure) pitch : Give one complete rotation to sleeve and record corresponding displacement in the main scale reading. The (displacement) shift in the main scale reading gives pitch.

Formula for least count of micrometer screw gauge :

$$\begin{aligned} \text{Least count} &= \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}} \\ &= \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}} \end{aligned}$$

In the above (diagram) micrometer screw gauge,

$$\text{L.C.} = \frac{1 \text{ mm}}{50} = \frac{0.1 \text{ cm}}{50}$$

L.C. = 0.002 cm

Range : In the above diagram, range cannot be expressed because the maximum reading portion is hidden.

Micrometer screw gauge type 2 :

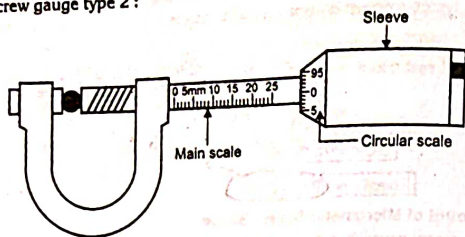


Fig. 1.4

In the above (diagram) micrometer screw gauge,

$$\begin{aligned} \text{Least count} &= \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}} \\ &= \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}} \\ &= \frac{1 \text{ mm}}{100} \\ &= \frac{0.1 \text{ cm}}{100} \end{aligned}$$

2.5 mm = 0.25 cm

$$\therefore \text{L.C.} = 0.001 \text{ cm}$$

and

$$\text{Range} = 0 \text{ to } 25 \text{ mm i.e. } 0 \text{ to } 2.5 \text{ cm}$$

1.2.3 Least Count of Spherometer

Spherometer is a device (instrument) using which increase (elongation) or decrease in length of wire can be measured (e.g. Young's modulus of elasticity Y experiment), radius of curvature or height of curved part can be measured.

It consists of two parts : (1) Main scale and (2) Circular scale.

Spherometer type 1 :

Main scale is calibrated in centimeter. In between 1 cm there are 10 divisions. i.e. 1 cm is divided into 10 equal parts. Therefore, smallest division on main scale is 0.1 cm. There are total 100 divisions on circular scale.

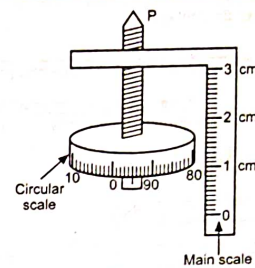


Fig. 1.5

Least count of spherometer (formula) :

$$\begin{aligned} \text{Least count} &= \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}} \\ \text{L.C.} &= \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}} = \frac{0.1}{100} \end{aligned}$$

$$\therefore \text{L.C.} = 0.001 \text{ cm}$$

and

$$\text{Range} = 0 \text{ to } 3 \text{ cm}$$

Spherometer type 2 :

Again main scale is in centimeter. There are 10 divisions in 1 cm. i.e. 1 cm is divided into 10 equal parts. Therefore, smallest division on main scale is 0.1 cm. There are total 100 divisions on circular scale.

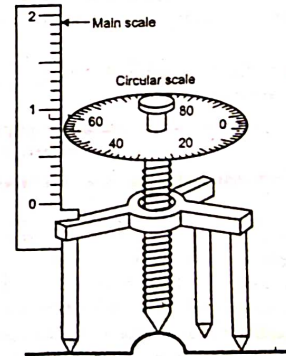


Fig. 1.6

Least count of spherometer (formula) :

$$\text{Least count} = \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}}$$

$$\text{L.C.} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}}$$

$$= \frac{0.1 \text{ cm}}{100}$$

$$\text{L.C.} = 0.001 \text{ cm}$$

and

$$\text{Range} = 0 \text{ to } 2 \text{ cm}$$

1.3 ACCURACY, PRECISION, ERROR AND ESTIMATION OF ERRORS

1.3.1 Accuracy and Precision of Instruments

Accuracy : In any measurement, the possibility of error is bound to arise. No measurement is exact. Hence, accuracy of measurement is most important aspect.

Definition : Accuracy is the agreement of the result of a measurement with the true value of the measured quantity.

The accuracy of the instrument is its ability to give correct results.

The accuracy of measurement depends upon quality of instrument and selection of proper instrument e.g. diameter of ball bearing can be measured more accurately by micrometer than that of vernier.

Accuracy of measurement depends on human limitations i.e. sense of hearing, sense of touch, sense of sight and systematicness.

Accuracy also depends on conditions of surroundings like temperature, pressure, humidity etc.

Definition of precision : Precision is defined as the repeatability of a measuring process.

In physics, for any measurement, we take number of readings and find out the average (mean) i.e. set of measurements. In any set of measurements, the individual measurements are scattered about the mean and precision tells us about the performance of measurement. Less repeatability is a sign of less accuracy and more repeatability is a sign of more accuracy. If number of readings are concentrated near average reading then precision is said to be more and if readings are scattered about mean reading then precision is said to be less.

Following set of readings illustrate distinction between accuracy and precision.

e.g. Exact diameter of ball bearing is 2.5 mm (true value).

Table 1.6

Set 1	Set 2	Set 3
2.4 mm	2.3 mm	2.49 mm
2.4 mm	2.4 mm	2.49 mm
2.45 mm	2.48 mm	2.5 mm
2.45 mm	2.56 mm	2.51 mm
2.5 mm	2.66 mm	2.51 mm
Mean = 2.44 mm	Mean = 2.48 mm	Mean = 2.5 mm
It is more precise but not accurate.	It is not precise (since scattered) but accurate.	It is more precise and also accurate (since matching with true value).

1.3.2 Error and Estimation of Errors

Error : An error is a fault, which may occur even in the most careful observation. Error arises due to human limitations and instrumental limitations. Errors cannot be completely eliminated but can be reduced. Error is deviation of measurement from standard value.

There are three types of errors :

- Instrumental or constant error :** This error is caused due to faulty instrument.
- Systematic error :** This error is caused due to defective settings or adjustment or unsystematicness of the experimenter.
- Random error :** This error is caused due to changes in experimental conditions, human limitations.

Estimation of Errors :

As we know errors cannot be eliminated but can be reduced for which estimation of error is required, which helps us to understand accuracy and precision of the measurement process. It also helps to calibrate the instrument.

- Absolute error :** The difference between the corrected reading and the mean (average) reading is called 'Absolute error.' (Dec. 2010)

Suppose, the diameter of a rod is measured by Micrometer Screw Gauge at ten different places. Corrected reading is obtained by adding zero error in each observation. Then the average or mean value of these corrected readings is found out.

The difference between mean reading and corrected reading is calculated. It is absolute error. i.e. $\delta_{A_n} = |A_n - A_m|$, where A_n = corrected reading, A_m = average corrected reading.

- Average absolute error :** The mean of absolute errors is known as 'Average absolute error'. i.e. $\delta_{\text{avg}} = \frac{\delta_{A_1} + \delta_{A_2} + \dots + \delta_{A_n}}{n}$

3. **Relative error** : It is the ratio of average absolute error to corrected average value.

$$R = \frac{\delta_{avg}}{A_m} \quad (\text{Dec. 2010})$$

4. **Percentage error** = $\frac{\text{Average absolute error} \times 100}{\text{Average reading}}$ (Dec. 2010)

i.e. = relative error $\times 100 = \frac{\delta_{avg}}{A_m} \times 100$

Percentage error is defined as a way of expressing a relative error as a fraction of 100.

Sequence in estimating the errors :

- To calculate corrected reading.
Add zero error if zero error is negative and subtract zero error if it is positive from given reading.

$$\text{Corrected reading} = \text{Given reading} \pm \text{Zero error}$$

- To calculate corrected average (A_m),

$$A_m = \frac{A_1 + A_2 + A_3 + \dots + A_n}{n}$$

- To calculate absolute error (δ_{A_i}),

$$\delta_{A_1} = |A_1 - A_m|$$

$$\delta_{A_2} = |A_2 - A_m|$$

:

$$\delta_{A_n} = |A_n - A_m|$$

- To calculate average absolute error (δ_{avg}):

$$\delta_{avg} = \frac{\delta_{A_1} + \delta_{A_2} + \delta_{A_3} + \dots + \delta_{A_n}}{n}$$

- To calculate relative error,

$$R = \frac{\delta_{avg}}{A_m}$$

- Percentage error, % error = $R \times 100$

1.3.3 Significant Figures, Rules and Identification of Significant Figures

Using calculators, we compute multiplication, divisions and we get answers in many digits after decimal point. i.e. too long answers (e.g. 0.2430687692). But we have to think that how many digits are really meaningful and where to restrict this figure.

Also if answer is too long, then there are more chances of misentering these digits.

Hence, the final result must always be restricted and rounded to a number of digits consistent with accuracy. In simple language, consider only those digits which are meaningful i.e. significant figures.

A significant figure is defined as a figure in any place (in number) which is reasonably trustworthy (meaningful). (Dec. 2010)

Significant digits : Digits that are meaningful in assigning a true value or realistic value to a result.

Rules to determine significant figure :

- The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are significant figures.
- Zeros is significant** if it is used to indicate a specific magnitude e.g. 106 km/hr. If zero comes in between two numbers then it is significant.

Zero is not significant if it is used to fix the decimal point or if it is used to fill the places of discarded digits.

Some examples,

- 0.00476 :** Here significant figures are 4, 7 and 6. Zero is not significant (meaningful) since it is used to fix decimal point.
- 1024 :** Here, all digits including zero i.e. 1, 0, 2, 4 are significant (meaningful).
- Bangalore is at a distance of 1200 km from Mumbai.**

In this case 1 and 2 are significant, zero may or may not be significant since distance may or may not be exact 1200 km.

Case 1 : If actual distance is exact 1200 km (or 1199.5 km) then all the four digits are significant (meaningful), hence significant figure is 4.

To convey this properly, we can write this as 1.200×10^3 km.

Case 2 : If exact distance is 1196 km and if it is rounded to 1200 km, then 1, 2 and leftmost zero is significant. Hence, significant figure is 3.

To convey this properly, we can write this as 1.20×10^3 km.

Case 3 : If exact distance is 1173 km and if it is rounded to 1200 km, then both the zeros are not significant (meaningful). Only 1, 2 are significant.

Hence, significant figure is 2. To convey this we can write 1.2×10^3 km.

If we express speed of sound as 320 m/s (at 22°C), it means it is closer to 320 than to 319 or 321. And if we express speed of sound as 320.0 m/s it means that it is closer to 320.0 than to 319.9 or 320.1. Thus, 320.0 is 10 times precise than 320.

Some examples :

Value	Number of significant figure
1. 1.080	4
2. 0.0018	2
3. 0.00180	3
4. 0.001800	4
5. 4.38×10^{11}	3

Handwritten notes:
 1.080 = 4 sig fig
 0.0018 = 2 sig fig
 0.00180 = 3 sig fig
 0.001800 = 4 sig fig
 4.38 x 10^11 = 3 sig fig
 France 28-11-2019

Rounding of approximate numbers :

It is the discarding of insignificant digits in a number, discarding of insignificant digits on the right of the decimal point.

To round a number to 'n' significant figures, discard all digits to the right of n^{th} place.

e.g. $\pi = 3.141592654 \xrightarrow[\text{3 significant figures}]{\text{to round it to}} 3.14.$

Rules or conventions for rounding of approximate numbers :

1. If the first digit of discarded number is less than 5, leave the n^{th} digit unchanged.

e.g. $1.12345678 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 1.123$

(\therefore first discarded digit is 4 which is less than 5 keep (n^{th}) i.e. 4th digit (here 3 unchanged.)

2. If the first digit of discarded number is greater than 5 then increase the n^{th} digit by 1.

e.g. $6.124756 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 6.125$

(\therefore first discarded digit is 7 which is greater than 5, hence increase (n^{th}) 4th digit by 1 i.e. $4 + 1 = 5$).

3. If the first digit of discarded number is exact 5, then leave n^{th} digit unchanged if it is an even and add 1 to it if it is odd.

e.g. $3.142519 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 3.142$ (Here n^{th} digit is 2 i.e. even \therefore keep unchanged)

e.g. $6.4475 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 6.448$ (Here n^{th} digit is 7 i.e. odd, hence add 1)

SUMMARY

- The standard used for measurement of a physical quantity is called unit of that quantity.
- Requirements of standard unit are that it should be universally accepted, definite, well defined, easily reproducible, invariable, readily available.
- System of units are C.G.S., M.K.S., F.P.S., and S.I. Length, mass and time are the basic quantities preferred in system of units.
- Seven fundamental quantities are length, mass, time, temperature, current, luminous intensity and amount of substance.
- Plane angle and solid angle are the two supplementary quantities.

- Remaining all are derived quantities e.g. volume, density, pressure.
- Accuracy is the agreement of the result of a measurement with the true value of the measured quantity.
- Precision is defined as the repeatability of a measuring process.
- Error is a fault, which may occur even in the most careful observations, which arises due to human limitations and instrumental limitations.
- Errors cannot be completely eliminated but can be reduced.
- Three types of errors are : (1) instrumental, (2) systematic and (3) random errors.
- A significant figure is defined as a figure in any place (in number) which is reasonably trustworthy (meaningful).
- Significant digits : Digits that are meaningful in assigning a true value or realistic value to a result.

Comparison between Vernier caliper and Micrometer screw gauge :

Vernier caliper	Micrometer screw gauge
1. Generally, L.C. = 0.01 cm	1. Generally, L.C. = 0.001 cm i.e. L.C. is less than vernier.
2. Accuracy is less than micrometer.	2. Accuracy is more than vernier.
3. External and internal dimensions both can be measured.	3. Internal dimensions cannot be measured.
4. Generally, range is 0.01 cm to 12 cm.	4. Generally, range is 0.001 cm to 2.5 cm.

Comparison between Fundamental quantity and Derived quantity :

Fundamental quantity	Derived quantity
1. The physical quantities which does not depend on any other quantity are called fundamental quantities.	1. The physical quantities which are derived using two or more fundamental quantities are called derived quantities.
2. The unit of fundamental quantity is called fundamental unit. (Their units are independent.)	2. The unit of derived quantity is called derived unit. (These units are dependent on fundamental units.)
3. According to SI units, there are seven fundamental quantities. e.g. length, mass, time, thermodynamic temperature, electric current, luminous intensity, amount of substance and two supplementary quantities - plane angle and solid angle.	3. The remaining all quantities are derived quantities. Hence, there are large numbers of derived quantities which are formed after combination of different fundamental quantities. e.g. area, volume, velocity, force, energy, torque, viscosity, thermal conductivity etc.

Comparison between Accuracy and Precision :

Accuracy	Precision
1. Accuracy is the agreement of the result of measurement with the true value of the measured quantity.	1. Precision is the repeatability of measuring process.
2. Accuracy depends on least count, range of instrument, human limitations and conditions of surroundings.	2. Precision depends on quality of instrument, human limitations and conditions of surroundings.
3. Accurate measurement may not be precise.	3. Precise measurement may not be accurate.
4. Less is the percentage error then more is the accuracy in measurement.	4. Less is the percentage error then more is the precision in measurement.
5. It can be determined by calibration against standard.	5. It can be determined by comparative test measurement.

Note : For ideal measurement, measurement should be accurate as well as precise.

SOLVED EXAMPLES

Example 1 : Find in significant figures, the surface area and volume of a sphere whose diameter is 12.4 cm.

Solution : Radius of a sphere = $\frac{12.4}{2} = 6.2$ cm
 Surface area of a sphere = $4\pi r^2$
 $= 4 \times 3.14 \times 6.2 \times 6.2 = 482.8$ cm²
 $= 4.8 \times 10^2$ cm² (in significant figures)
 Volume of a sphere = $\frac{4}{3}\pi r^3$
 $= \frac{4}{3} \times 3.14 \times 6.2 \times 6.2 \times 6.2 = 997.79$ cm³

\therefore Volume of a sphere = 1.0×10^3 cm³ (in significant figures)

Example 2 : Find the number of significant figures in following measurements :
 (i) 0.0025, (ii) 0.0250, (iii) 0.002500, (iv) 5.98×10^{24} (BTE April 2007)

- Solution :
 i.e. significant figures
 (i) 0.0025 → 2
 (ii) 0.0250 → 3
 (iii) 0.002500 → 4
 (iv) 5.98×10^{24} → 3

(Note : To understand why it is so, please refer article significant figure in the topic.)

Example 3 : The diameter of rod is measured with micrometer screw gauge for three times as 1.223 cm, 1.224 cm and 1.224 cm. Calculate percentage error in measuring the diameter of rod if zero error of micrometer is + 0.002 cm.

Solution : As given zero error is positive, correction is negative i.e. to obtain corrected reading subtract 0.002 cm from given readings.

Sr. No.	Given reading A cm	Corrected reading cm A	Corrected average reading cm A _m	Absolute error δ_A $\delta_A = A - A_m $	Average absolute error $\delta_{A_{avg}}$	Relative error 'R' $= \frac{\delta_{A_m}}{A_m}$	% error $= R \times 100$ $= \frac{\delta_{A_m}}{A_m} \times 100$
1.	1.223	A ₁ = 1.221		0.0007			
2.	1.224	A ₂ = 1.222	1.2217	0.0003	0.00043	0.00035	0.035
3.	1.224	A ₃ = 1.222		0.0003			

Example 4 : The mass of object is 37.6 ± 0.02 gm. Estimate the percentage error in this measurement. (BTE Dec. 2006)

Solution : Percentage error = $\frac{\text{Average absolute error}}{\text{Average reading}} \times 100$
 $= \frac{0.02}{37.6} \times 100$

\therefore % error = 0.053 %

Example 5 : Which of the following is more accurate reading and why ?

- (a) 2.33 ± 0.002 mm
 (b) 2.55 ± 0.0015 mm.

Solution : We have,

Percentage (%) error = $\frac{\text{Average absolute error}}{\text{Average reading}} \times 100$

(a) % error = $\frac{0.002}{2.33} \times 100 = 0.0858\%$

(b) % error = $\frac{0.0015}{2.55} \times 100 = 0.0588\%$

Percentage error in the second case is less, hence accuracy is more.

Thus 2.55 ± 0.0015 mm is more accurate.

Example 6 : The length of an object measured by vernier caliper is 4.78 cm. If the L.C. of vernier caliper is 0.01 cm, calculate the percentage error.

Solution : Given : $L = 4.78$ cm, $L.C. = 0.01$ cm

$$\text{Percentage error} = \frac{0.01}{4.78} \times 100$$

$$\text{Percentage error} = 0.209\%$$

QUESTIONS

1. Explain need of measurement in engineering and science.
2. Define a unit. What are the requirements of a good unit ?
3. What are the different systems of units ? Explain.
4. Define fundamental and derived quantities.
5. Define fundamental and derived units.
6. State seven fundamental quantities.
7. What do you understand by S.I. system of units ?
8. Define error, accuracy and precision.
9. What are significant figures ?
10. State the rules adopted to determine significant figures.
11. Define absolute error, relative error, percentage error.
12. Differentiate between "precision" and "accuracy" with suitable examples.
13. State the fundamental units of S.I. system.
14. Define pitch of screw gauge and state formula for least count of screw gauge.
15. Define : (a) Percentage error, (b) Significant figure.
16. Name the apparatus you will use in the laboratory :
 - (a) To measure diameter of wire. *vernier*
 - (b) To measure diameter of wooden cylinder. *S*
17. List two supplementary fundamental physical quantities.
18. List four fundamental physical quantities.
19. List four derived physical quantities.
20. List all fundamental physical quantities. State S.I. unit for any two of them.
21. Correct the formula given below :

$$\text{Percentage error} = \frac{\text{Absolute error}}{\text{True error}}$$
22. Define error.

∴ error = $\frac{A - A_c}{A_c} \times 100$
∴ e = $\frac{L.C.}{\text{Length}}$

23. Define accuracy and precision and differentiate between them with suitable example.
24. Define system of units. State different types of system of units and name the fundamental quantity in them with their units.
25. Name the apparatus you will use in the laboratory to measure
 - (a) Internal diameter of tube. *screw*
 - (b) Diameter of wire.
26. Draw neat sketch of vernier caliper and micrometer screw gauge.
27. Draw neat sketch of vernier caliper and state formula for its least count.
28. Draw neat sketch of micrometer screw gauge and state formula for its least count.
29. Define :
 - (a) Absolute error.
 - (b) Average absolute error.
 - (c) Relative error.
 - (d) Percentage (%) error.
30. Classify the following quantities as fundamental and derived quantities : Length, force, temperature, mass, volume, acceleration, area.
31. Differentiate between absolute error and relative error.
32. Define L.C. of vernier.
33. Define L.C. of micrometer.
34. Define least count and range of the instrument.

OBJECTIVE TYPE QUESTIONS

Fill in the blanks with suitable answers from the bracket and rewrite :

- (i) is not a fundamental quantity (Density, Length, Mass, Time)
 - (ii) In both MKS and CGS systems, the unit of is the same. (length, mass, time, none of these)
 - (iii) Lack of proper precautions gives rise to (errors, mistakes).
 - (iv) Errors can be (completely removed by careful calculations, minimized by careful observations, tolerated in any measurements)
 - (v) If the least count of an instrument is made smaller, the accuracy of measurement (decreases, increases, remains unchanged)
 - (vi) The only fundamental unit of the following is (newton, coulomb, radian, candela)
 - (vii) Out of the following measurements, the only measurement containing significant zero is (0.352, 0.0352, 0.03052)
- Ans. : (i) density, (ii) time, (iii) mistakes, (iv) minimized by careful observation, (v) increases, (vi) Candela, (vii) 0.03052

PROBLEMS

1. Express the result of the following in significant figures :
 (i) $325 \times 10^9 \times 0.620$
Ans. 2.02×10^{10}
2. Find in significant figures the area of sphere whose radius is 5.3 cm.
Ans. $3.5 \times 10^2 \text{ cm}^2$
3. Find the number of significant figures in the following measurements :
 (a) 0.031400 (b) 0.03140
 (c) 0.0314 (d) 6.62×10^{-34}
 (e) 6.620×10^{-34} (f) 6.6200×10^{-34}
Ans. (a) 5, (b) 4, (c) 3, (d) 3, (e) 4, (f) 5.
4. Which of the following is more accurate reading and why ?
 (a) $1.75 \pm 0.012 \text{ cm}$ (b) $0.75 \pm 0.007 \text{ cm}$
Ans. (a) % error = 0.686 %, (b) % error = 0.933 %
 % error in the first case is less therefore more accurate, first reading $1.75 \pm 0.012 \text{ cm}$ is more accurate.
5. The length of the object is $18.2 \pm 0.01 \text{ cm}$. Estimate the percentage error in this measurement.
Ans. % error = 0.055 %
6. Diameter of ball measured by micrometer screw gauge is 0.12 cm. If the L.C. of micrometer screw gauge is 0.001 cm, calculate the percentage error.
Ans. Percentage error = 0.83 %
7. The length of the object measured by vernier caliper is 2.34 cm. If the L.C. of vernier is 0.01 cm, calculate the percentage error.
Ans. % error = 0.427 %.
8. A student measures the diameter of the bob three times using vernier caliper. The measurements are 2.35 cm, 2.36 cm, 2.31 cm. Estimate the error in measurement (calculate absolute error and percentage error).
Ans. absolute error = 0.01, 0.02, 0.03
 % error = 0.855 %

Handwritten calculations:
 $\frac{0.1014 \times 100}{18.2 \times 0.01} = 0.55\%$
 $\frac{0.001}{0.12} \times 100 = 0.83\%$
 $\frac{0.01}{2.34} \times 100 = 0.427\%$
 $\frac{0.01}{2.35} \times 100 = 0.425\%$
 $\frac{0.02}{2.36} \times 100 = 0.847\%$
 $\frac{0.03}{2.31} \times 100 = 1.3\%$
 $\frac{0.01 + 0.02 + 0.03}{3} = 0.0167$
 $\frac{0.0167}{2.34} \times 100 = 0.71\%$

GENERAL PROPERTIES OF MATTER

2.1 ELASTICITY

Introduction, elasticity, plastics, rigidity, molecular theory of elasticity, stress, strain, longitudinal (tensile) stress-strain, volume stress-strain, shearing stress-strain, Hooke's law, elastic limit, Young's modulus of elasticity, bulk modulus of elasticity, modulus of rigidity, relation between Y , K and η , behaviour of wire under continuously increasing load, stress-strain diagram of H.T. steel – cast iron, aluminium and concrete, ultimate and breaking stress, factor of safety, applications of elasticity.

2.1.1 Introduction, Elasticity, Plasticity and Plastic Body

Elasticity : When a force is applied on a body, size and shape of a body change and body is said to be deformed. Under deformed condition, the internal force is developed which tries to restore the original (position) size and shape of a body called as internal restoring force. Because of this internal restoring force developed inside a body, the body regains its original size and shape on removal of external deforming force.

Definition : The property on account of which a body regains its original size and shape on removal of external deforming force is called as elasticity.

Plasticity : Some bodies do not oppose the change in size and shape and thus deforms very easily. Such bodies do not regain their original size and shape even after the deforming force is removed. These are called plastic bodies (e.g. clay).

Definition : The property on account of which a body does not regain its original size and shape on removal of applied force is called as plasticity.

Rigidity : A body in which it is not possible to produce any relative displacement of the different particles inside it and hence size and shape of a body remains unchanged even after a large force is applied on it, is called as rigid body. No body is perfectly rigid but stone is taken as a rigid body.

Definition : The property on account of which a body does not change its size and shape even a large force is applied on it, is called as rigidity.

2.1.2 Molecular Theory of Elasticity

1. Consider an elastic body as shown in Fig. 2.1. Molecules are at equal distances from each other. There is a force of attraction between the molecules. Hence, molecules are at equal distances.
2. Consider an external force applied on a body. Because of external force, every molecule starts moving undesirely in the direction of force. They stop moving at a particular position where this applied force is balanced due to equal and opposite force.
3. Due to shifting of molecules, body changes its size and shape and body is said to be deformed.

Definition of deforming force : The force applied on elastic body which is responsible to deform (change size and shape of the body) is called as deforming force.

Under deformed condition, every shifted molecule tries to achieve its original position due to which an internal restoring force is developed inside a body.

Internal restoring force : Under deformed condition, every shifted molecule tries to achieve its original position (because of elastic property). The force which is responsible to restore original size and shape of the body is called as internal restoring force.

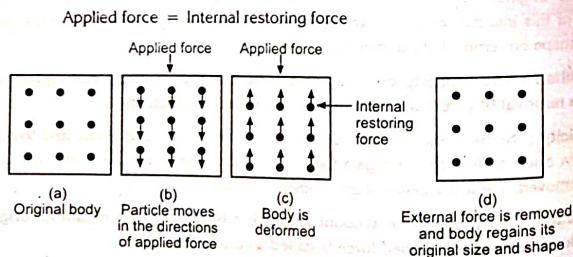


Fig. 2.1

4. Now if external applied force is removed then because of internal restoring force, body regains its original size and shape i.e. *Elasticity*.

Body regains its original size and shape (exactly) on removal of external deforming force if and only if external force is within a certain limit called as *elastic limit*. If external force is too large then the intramolecular structure of body collapses and there will be permanent deformation (i.e. body does not regain its original size and shape on removal of external deforming force) and body loses its elasticity.

2.1.3 Stress, Strain and their Types

Stress : "The internal restoring force per unit cross-sectional area is called as stress".

i.e.
$$\text{Stress} = \frac{\text{Restoring force}}{\text{Cross-sectional area}}$$

But, Restoring force = Applied force

$$\therefore \text{Stress} = \frac{\text{Applied force}}{\text{Cross-sectional area}}$$

$$\text{Stress} = \frac{F}{A}$$

MKS or S.I. unit of stress is N/m^2 and CGS unit is dyne/cm^2 . Dimensions of stress are $[\text{M}^1 \text{L}^{-1} \text{T}^{-2}]$.

Strain : We have seen that when an external deforming force is applied on a body, then size and shape of a body change. i.e. dimension of a body changes.

"The change in dimensions per unit original dimensions is called as strain."

$$\text{Strain} = \frac{\text{Change in dimensions}}{\text{Original dimensions}}$$

Strain is the ratio of two similar quantities. Hence, strain has no unit. Strain is a pure number.

As shown in Fig. 2.2 (a), (b) and (c), when force is applied on a body then length of a body changes or volume changes or shape of a body changes. Accordingly, there are three types of stress and strain.

Three Types of Strain :

1. **Tensile strain (Longitudinal strain)** : Tensile strain is defined as change in length per unit original length.

$$\text{Tensile strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{l}{L}$$

2. **Volume strain** : Volume strain is defined as change in volume per unit original volume.

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{dv}{V}$$

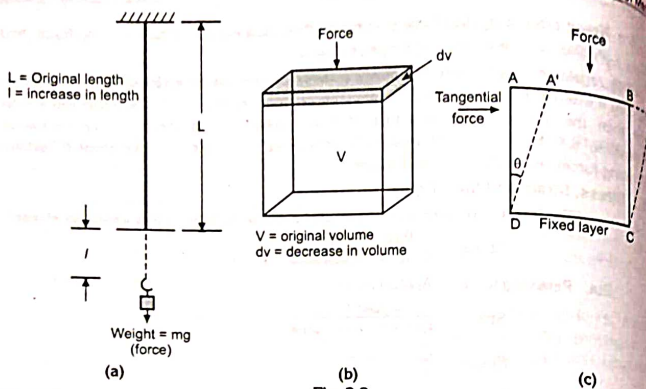


Fig. 2.2

3. Shearing strain (Shear) : Shearing strain is defined as the ratio of lateral displacement of any layer to its distance from fixed layer.

$$\text{Shearing strain} = \frac{\text{Lateral displacement of any layer}}{\text{Distance of the layer from fixed layer}} = \frac{AA'}{AD} = \tan \theta$$

if 'θ' is small then $\tan \theta = \theta$

Three Types of Stress :

1. Tensile stress : The stress measured in connection with the changes in length is called as tensile or longitudinal stress.

$$\text{Tensile stress} = \frac{\text{Applied force}}{\text{Cross-sectional area}} = \frac{Mg}{\pi r^2}$$

MKS unit of tensile stress is N/m^2 and its CGS unit is dyne/cm^2 . The dimensions of tensile stress are $[M^1 L^{-1} T^{-2}]$.

2. Volume stress : If the force is applied and there is change in volume then the corresponding stress is called as volume stress.

$$\text{Volume stress} = \frac{\text{Applied force}}{\text{Area}} = \text{Change in pressure} = dp$$

MKS unit of volume stress is N/m^2 and its CGS unit is dyne/cm^2 . Dimensions are $[M^1 L^{-1} T^{-2}]$.

3. Shearing stress : The stress corresponding to change in shape is called as shearing stress.

$$\text{Shearing stress} = \frac{\text{Tangential force}}{\text{Area}}$$

MKS unit of shearing stress is N/m^2 and its CGS unit is dyne/cm^2 . Its dimensions are $[M^1 L^{-1} T^{-2}]$.

2.1.4 Elastic Limit and Hooke's Law

Elastic limit : "The stress corresponding to the limiting value of the load, which when applied and subsequently released, does not produce permanent deformation is called as elastic limit".

If this limit is crossed, the proportionality is lost and the stress is found to be less than what is expected.

Hooke's law : It states that, "within elastic limit, stress is directly proportional to the strain."

$$\begin{aligned} \therefore \text{Stress} &\propto \text{Strain} \\ \therefore \text{Stress} &= \text{Constant} \times \text{Strain} \\ \therefore \frac{\text{Stress}}{\text{Strain}} &= \text{Constant} \end{aligned}$$

Charles bats

The constant of proportionality is called as modulus of elasticity.

$$\therefore \text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

S.I. unit of modulus of elasticity is N/m^2 . CGS unit is dyne/cm^2 and dimensions of modulus of elasticity are $[M^1 L^{-1} T^{-2}]$.

2.1.5 Types of Modulus of Elasticity

As we have seen when a force is applied on a body, then its length changes or volume changes or shape of a body changes. Accordingly, there are three types of modulus of elasticity.

1. Young's Modulus of Elasticity (Y) :

It is defined as the ratio of tensile stress to tensile strain.

$$\text{We have, } \frac{\text{Stress}}{\text{Strain}} = \text{Constant} = \text{Modulus of elasticity}$$

$$\frac{\text{Tensile stress}}{\text{Tensile strain}} = \text{Young's modulus of elasticity} = Y$$

$$\begin{aligned} \therefore Y &= \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F/A}{l/L} \\ &= \frac{Mg/\pi r^2}{l/L} \end{aligned}$$

$$Y = \frac{Mg L}{\pi r^2 l}$$

where, M = Load attached
 r = Radius of wire
 L = Original length of wire
 l = Extension produced

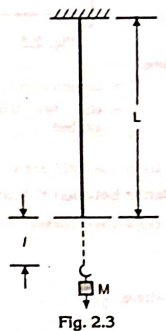


Fig. 2.3

Its S.I. or MKS unit is N/m^2 , CGS unit is dyne/cm^2 and dimensions are $[M^1 L^{-1} T^{-2}]$.

2. Bulk Modulus of Elasticity (K) :

It is defined as the ratio of volume stress to volume strain.

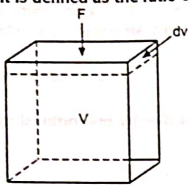


Fig. 2.4

where, V = original volume,
F = applied force,

Bulk modulus of elasticity,

$$K = \frac{\text{Volume stress}}{\text{Volume strain}}$$

$$K = \frac{F/A}{dv/V}$$

$$K = \frac{dp}{dv/V}$$

$$K = \frac{dp \times V}{dv}$$

∴ dv = decrease in volume,

A = area of cross section on which force is applied

3. Modulus of Rigidity (η) :

"It is defined as the ratio of shearing stress to shearing strain."

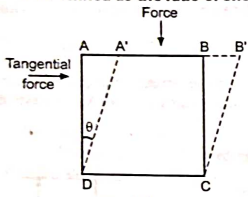


Fig. 2.5

where,

F = tangential force
 A = area on which force is applied

Modulus of rigidity,

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\therefore \eta = \frac{F/A}{\frac{\text{Lateral displacement of layer}}{\text{Its distance from fixed layer}}}$$

$$\therefore \eta = \frac{F/A}{AA'/AD}$$

$$= \frac{F/A}{\tan \theta}$$

$$\frac{AA'}{AD} = \tan \theta$$

$$\therefore \eta = \frac{F}{A \tan \theta}$$

if ' θ ' is small
put $\tan \theta = \theta$

$$\therefore \eta = \frac{F}{A\theta}$$

Its S.I. or MKS unit is N/m^2 , CGS unit is dyne/cm^2 and dimensions are $[M^1 L^{-1} T^{-2}]$.

Relation between Y, K and η :

For a given material, there is certain relation between Y, K and η which is given by,

$$Y = \frac{9\eta K}{3K + \eta}$$

where,

Y = Young's modulus of elasticity of material

K = Bulk modulus

η = Modulus of rigidity

2.1.6 Behaviour of Wire under Continuously Increasing Load or Stress against Strain Diagram

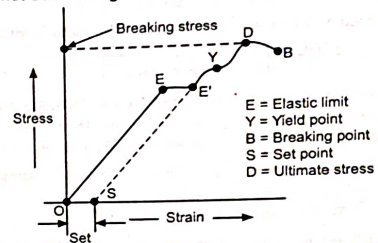


Fig. 2.6

A wire is subjected to increasing load step by step. A graph of stress against strain is as shown. It can be explained as follows.

- OE portion is a straight line which indicates that stress is proportional to strain i.e. wire obeys Hooke's law upto point E.
- The stress corresponding to point E is maximum stress to which wire can be loaded without any permanent elongation. This is called as *elastic limit of wire*.
- EE' portion is curved towards strain axis which shows that increase in strain is more than what is warranted (expected) by corresponding increase in stress i.e. stress is not proportional to strain. i.e. Hooke's law is not obeyed.
- At any point between E and E' if all the load is removed, then some permanent elongation occurs in the wire, this is called set.
- When wire is again subjected to a load, a new straight line SE' (shown by dotted line) is obtained indicating Hooke's law.
- Some portion after point Y is almost parallel to strain axis i.e. strain increases without increase in stress just like wire flows this is called yielding. This is called plastic flow. The point at which this flow begins is called yield point Y.
- With the plastic flow, wire becomes thin and thin. Some weak points called neck are formed in the wire. At weakest point (neck), wire breaks.

The maximum stress upto which wire can be loaded or wire can bear is called as **breaking stress**. The corresponding point in the graph is breaking point B.

Breaking stress : It is the maximum stress at which wire breaks.

Ultimate stress : It is the maximum stress the system is capable of withstanding.

Ultimate stress is defined as the ratio of maximum load that the specimen (system) is capable of withstanding to its original cross-sectional area.

Handwritten notes:
V1 V4
E11, 22, 44
E22

$$\text{Ultimate stress} = \frac{\text{Maximum load the system can withstand}}{\text{Original cross-sectional area}}$$

Working stress : It is the actual practical stress on the system.

Working stress is defined as the ratio of actual load to the original cross-sectional area.

$$\text{Working stress} = \frac{\text{Actual load on the specimen (system)}}{\text{Original cross-sectional area}}$$

Factor of safety : Working stress is the maximum allowed (permitted) stress on the system. For the safety of structure, this working stress should be less than elastic limit of the material. Working stress is determined by dividing the ultimate stress by a number called factor of safety.

Factor of safety is defined as the ratio of ultimate stress to working stress.

$$\text{Factor of safety} = \frac{\text{Ultimate stress (load)}}{\text{Working stress (load)}}$$

or factor of safety is defined as the ratio of maximum load that the structure can bear to the actual load on the structure.

For example, the weighing balance used in grocery shop. Even though the weigh balance can bear a load of 10 kg, the message written on it may be "only for 5 kg".

Q. Which is more elastic steel or rubber? Why?

Ans. Steel is more elastic than rubber : If a body resists deformation and recovers (regains) its original size and shape immediately on the removal of external deforming force then it is called perfectly elastic body.

We have, $Y = \frac{F/A}{l/L}$

$\therefore Y \propto \frac{F}{l}$

where, Y = Young's modulus of elasticity

F = Restoring force

l = Extension produced in the wire

In case of steel, restoring force is more and extension produced is less (approximately 2% of original length) i.e. elasticity is more. Y for steel = $2 \times 10^{11} \text{ N/m}^2$.

On the other hand, in case of rubber, restoring force is less and extension is more (may length get doubled) i.e. elasticity is less. Y for rubber = $1.25 \times 10^7 \text{ N/m}^2$.

SUMMARY

- Elasticity is defined as the property by virtue of which body regains its original size and shape on removal of external deforming force.
- Stress is defined as internal resistive (restoring) force developed per unit cross-sectional area.

- Strain is defined as ratio of change in dimensions to original dimensions.
- Hooke's law states that, within elastic limit, stress is directly proportional to strain.
- Young's modulus, $Y = \frac{\text{Tensile (longitudinal) stress}}{\text{Tensile (longitudinal) strain}}$
- Bulk modulus, $K = \frac{\text{Volume stress}}{\text{Volume strain}}$
- Modulus of rigidity, $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$
- Factor of safety is defined as the ratio of maximum load that the structure can bear to the actual load on the structure. It should always be greater than one.
- Breaking stress** : It is the maximum stress at which wire breaks.
- Ultimate stress** : It is the maximum stress the system is capable of withstanding.
- Working stress** : It is the actual practical stress on the system.

FORMULAE

1. Stress = $\frac{F}{A}$ where, F - Force

2. Longitudinal strain = $\frac{l}{L}$ A - Area

Volume strain = $\frac{\Delta v}{V}$ l - Increase in length

Shearing strain = $\tan \theta$ L - Original length

= θ (if θ is small) Δv - Decrease in volume

V - Original volume

3. $Y = \frac{FL}{A l} = \frac{Mg L}{\pi r^2 l}$ M - Load

r - Radius of wire

4. $K = \frac{FV}{A (\Delta v)} = \frac{PV}{\Delta v}$ P - Pressure applied

5. $\eta = \frac{F}{A \theta}$

6. $Y = \frac{9 K \eta}{3 K + \eta}$

SOLVED EXAMPLES

Example 1 : A wire of length 3 m extends by 3 mm when a force of 2 N is applied to it. Calculate (i) stress produced in it, if $Y = 2 \times 10^{11} \text{ N/m}^2$, (ii) Area of wire. (S - 1999 - 4 M)

Solution : Given : $L = 3 \text{ m}$, $l = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$, $F = 2 \text{ N}$

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\begin{aligned} \therefore \text{Stress} &= Y \times \text{Strain} \\ &= Y \times (l/L) \\ &= 2 \times 10^{11} \times \left(\frac{3 \times 10^{-3}}{3}\right) \end{aligned}$$

$$\boxed{\text{Stress} = 2 \times 10^8 \text{ N/m}^2}$$

$$\begin{aligned} \text{Now, } \text{Stress} &= \frac{\text{Force}}{\text{Area}} \\ \therefore \text{Area} &= \frac{\text{Force}}{\text{Stress}} = \frac{2}{2 \times 10^8} \end{aligned}$$

$$\boxed{\text{Area} = 1 \times 10^{-8} \text{ m}^2}$$

Example 2 : A weight exerts force of 100 N on a steel wire of cross-sectional area 0.02 cm². Find extension produced if the length of wire is 5 m. ($Y = 2 \times 10^{11} \text{ N/m}^2$) (S. 2002 - 2 M)

$$\begin{aligned} \text{Solution : Given : } F &= 100 \text{ N} \\ A &= 0.02 \text{ cm}^2 = 0.02 \times 10^{-4} \text{ m}^2 \\ l &= ? \\ L &= 5 \text{ m} \\ Y &= 2 \times 10^{11} \text{ N/m}^2 \end{aligned}$$

We have,

$$Y = \frac{F/A}{l/L}$$

$$Y = \frac{FL}{Al}$$

$$l = \frac{FL}{YA} = \frac{100 \times 5}{2 \times 10^{11} \times 0.02 \times 10^{-4}}$$

$$\boxed{l = 1.25 \times 10^{-3} \text{ m}}$$

Example 3 : A wire of diameter 3 mm and length 4 m extends by 2.5 mm when a force of 10 N is applied. Find the Young's modulus of material of wire. (W. 2004 - 4 M)

$$\begin{aligned} \text{Solution : Given : } \text{dia} &= 3 \text{ mm} \\ r &= 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m} \\ L &= 4 \text{ m} \\ l &= 2.5 \text{ mm} = 2.5 \times 10^{-3} \text{ m} \\ F &= 10 \text{ N} \\ Y &= ? \end{aligned}$$

We have,

$$Y = \frac{F/A}{l/L} = \frac{FL}{(A)l} = \frac{FL}{(\pi r^2)l}$$

$$= \frac{(10)(4)}{(3.142)(1.5 \times 10^{-3})^2(2.5 \times 10^{-3})}$$

$$\boxed{Y = 2.26 \times 10^9 \text{ N/m}^2}$$

Example 4 : A wire of length 1 m extends by 1 mm when stretched by a load of 1 kg. Find the area of cross section of the wire. (Given $Y = 2 \times 10^{11} \text{ N/m}^2$ and $g = 9.81 \text{ m/s}^2$) (S. 2001 - 4 M)

$$\text{Solution : } L = 1 \text{ m}, l = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}, M = 1 \text{ kg}, A = ?$$

$$Y = \frac{MgL}{Al}$$

$$A = \frac{MgL}{Yl} = \frac{(1) \times (9.8) \times (1)}{(2 \times 10^{11}) \times (1 \times 10^{-3})}$$

$$\boxed{A = 4.9 \times 10^{-8} \text{ m}^2}$$

Example 5 : A copper wire is stretched by 5% of its length. Determine the stress produced in the wire. Given Y for copper = $1.2 \times 10^{11} \text{ N/m}^2$. (W. 1992 - 4 M)

$$\text{Solution : Given : } \text{Since wire is stretched 5\%, take } l = 5 \text{ and } L = 100, \text{ Stress} = ?$$

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = Y \times \text{Strain}$$

$$= Y \times \left(\frac{l}{L}\right)$$

$$= 1.2 \times 10^{11} \times \left(\frac{5}{100}\right)$$

$$\boxed{\text{Stress} = 6 \times 10^9 \text{ N/m}^2}$$

Example 6 : A longitudinal stress of $8 \times 10^7 \text{ N/m}^2$ produces an extension of 1 mm in a wire of length 2 metres. Find Young's modulus of the material of the wire. (S. 1991 - 2 M)

$$\text{Solution : Given : } \text{Stress} = 8 \times 10^7 \text{ N/m}^2, l = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}, L = 2 \text{ m}, Y = ?$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Stress}}{l/L}$$

$$= \frac{\text{Stress} \times L}{l} = \frac{8 \times 10^7 \times 2}{1 \times 10^{-3}}$$

$$\boxed{Y = 1.6 \times 10^{11} \text{ N/m}^2}$$

Example 7 : Equal weights are attached to two wires of same metal. The length and diameter of one wire is 1.5 times that of the other. Calculate ratio of their extensions. (W. 1986 - 4 M)

$$\text{Solution : Given : } 1^{\text{st}} \text{ wire}$$

$$L_1 = 1.5 L_2$$

$$d_1 = 1.5 d_2$$

$$\frac{l_1}{l_2} = ?$$

$$Y = \frac{Mg l_1}{\left(\frac{\pi d_1^2}{4}\right) l_1} \dots (1)$$

$$2^{\text{nd}} \text{ wire}$$

$$\text{Let length} = L_2$$

$$\text{Let diameter} = d_2$$

$$Y = \frac{Mg l_2}{\left(\frac{\pi d_2^2}{4}\right) l_2} \dots (2)$$

Equating (1) and (2), $\frac{L_1}{(d_1)^2 l_1} = \frac{L_2}{d_2^2 l_2}$

$\frac{(1.5 L_2)}{(1.5 d_2)^2 l_1} = \frac{L_2}{d_2^2 l_2}$

$\therefore \frac{1.5}{(1.5)^2} = \frac{l_1}{l_2}$

$\therefore \frac{l_1}{l_2} = \frac{1}{1.5}$

i.e. **Ratio of elongations of two wires is 1 : 1.5**

Example 8 : An unknown weight is attached to the lower end of wire of length 4 m, radius 0.7 mm, extends it by 0.8 mm. If $Y = 2 \times 10^{11} \text{ N/m}^2$, find the unknown weight.

Solution : Given : Weight = $W = mg = F = ?$

- $L = 4 \text{ m}$
- $r = 0.7 \text{ mm} = 0.7 \times 10^{-3} \text{ m}$
- $l = 0.8 \text{ mm} = 0.8 \times 10^{-3} \text{ m}$
- $Y = 2 \times 10^{11} \text{ N/m}^2$

We have,

$Y = \frac{F/A}{l/L}$

$\therefore Y = \frac{FL}{Al}$

$\therefore \frac{Y \times A \times l}{L} = F$

$\therefore F = \frac{Y \times (\pi r^2) \times l}{L}$
 $= \frac{(2 \times 10^{11}) \times (3.142) \times (0.7 \times 10^{-3})^2 \times (0.8 \times 10^{-3})}{4}$

Weight = $F = 61.57 \text{ N}$

QUESTIONS

1. Define the terms : (i) Elasticity, (ii) Stress, (iii) Strain, (iv) Elastic limit, (v) Yield point, (vi) Breaking point.
2. State and explain Hooke's law.
3. Explain : (i) Young's modulus, (ii) Bulk modulus, (iii) Modulus of rigidity.
4. Explain : (i) Longitudinal (Tensile) strain, (ii) Volume strain, (iii) Shearing strain.
5. Explain behaviour of wire under continuously increasing load.
6. Define factor of safety.
7. State whether rubber is more elastic or steel, why ?
8. Define ultimate and breaking stress.

9. Define factor of safety and elastic limit.
10. Define Young's modulus, Bulk modulus and Rigidity modulus of elasticity. State relation between them.
11. Distinguish between elastic bodies and plastic bodies.
12. Define :
 - (i) Stress,
 - (ii) Strain,
 - (iii) Restoring force,
 - (iv) Deforming force.
13. Explain elastic limit and yield point on stress-strain diagram.
14. State and explain Hooke's law. Define modulus of rigidity. Write down its S.I. unit.
15. State Hooke's law of elasticity and define Young's modulus of elasticity.
16. Define deforming force and restoring force. State the S.I. unit of stress.
17. State Hooke's law of elasticity and define elastic limit.
18. Define :
 - (i) Young's modulus 'Y',
 - (ii) Bulk modulus 'K',
 - (iii) Modulus of rigidity 'η'.

19. Correct the equation $Y = \frac{9 K \eta}{3(K + \eta)}$

B.T.E. QUESTIONS

1. Out of steel and rubber, which is more elastic and why ? (S. 2006 – 2 M)
2. Explain the behaviour of wire under continuously increasing load with the help of graph. (S. 2006, S. 2005, S. 2003, W. 2002 – 4 M)
3. Define stress and factor of safety. (W. 2004 – 2 M)
4. Define stress and strain. (S. 2005 – 2 M)
5. Define rigidity and stress. (W. 2005 – 2 M)
6. State and explain Hooke's law of elasticity. Hence define elastic limit. (W. 2005 – 4 M)
7. Define the terms :
 - (a) Young's modulus (S. 2004 – 2 M)
 - (b) Yield point. (S. 2002 – 4 M)
8. Define modulus of rigidity. Give its units. (S. 2003 – 3 M)
9. Define : (1) Elasticity, (2) Plasticity. (W. 2002 – 2 M)
10. Define the term breaking stress.

PROBLEMS

- Calculate the strain produced in a wire under tension when its stress is $12 \times 10^5 \text{ kg/cm}^2$. Y for material is $2 \times 10^6 \text{ kg/cm}^2$. (S. 1986 - 2 M)
Ans. 0.6.
- A wire of diameter 4 mm and of length 2 m extends by 1.99 mm applying a force of 10 N. Find Young's modulus of material of the wire. (S. 1990 - 4 M)
Ans. $7.996 \times 10^8 \text{ N/m}^2$.
- Calculate the strain produced in a material if the stress is 2000 kg/cm^2 and $Y = 2 \times 10^6 \text{ kg/cm}^2$. (W. 1989 and S. 1986 - 4 M)
Ans. 10^{-3} .
- A wire of length 1.5 m extends by 1.5 mm when a force is applied to it. Calculate the stress produced in it. Given $Y = 2 \times 10^{11} \text{ N/m}^2$. (W. 1989 - 4 M)
Ans. $2 \times 10^5 \text{ N/m}^2$. (S. 2007 - 4 M)
- A wire of length 1 m extends by 2 mm when stress acting on it is found to be 4000 kg/cm^2 . Find Young's modulus of material of the wire. (W. 1987 - 4 M)
Ans. $1.96 \times 10^{11} \text{ N/m}^2$.
- A steel wire of length 314 cm and diameter 1 mm has its upper end tied to a beam. If a load of 2 kg is attached to the lower end, find the extension produced in the wire. $Y = 1.96 \times 10^{11} \text{ N/m}^2$. (S. 1985 - 4 M)
Ans. 0.4 mm.
- A weight exerts force of 120 N on steel wire of cross-sectional area 0.02 cm^2 . Find extension produced if the length of wire is 5 m ($Y = 2 \times 10^{13} \text{ N/m}^2$). (S. 1992 - 2 M)
Ans. $l = 1.5 \times 10^{-5} \text{ m}$
- Equal weights are attached to two wires of the same metal. The length and radius of one wire is twice the length and radius of the other wire. Calculate the ratio of their extensions. (S. 2002 - 4 M)
Ans. $l_1 : l_2 = 1 : 2$.
- Find the weight attached to the lower end of the wire of length 1.5 m, radius 0.3 mm extends it by 0.6 mm. If $Y = 2 \times 10^{11} \text{ N/m}^2$.
Ans. Weight = $F = 22.62 \text{ N}$.

2.2 SURFACE TENSION

Introduction, molecular theory of surface tension, angle of contact and its significance, capillarity, shape of liquid surface in a capillary tube, surface tension of liquid in a capillary tube, effect of impurity and temperature on surface tension of liquid, applications of surface tension.

2.2.1 Introduction

Liquids do not have definite shape but they have definite volume. Therefore, liquids take the shape of a container and have free surfaces. The well-defined free surfaces of liquids have an interesting property which we are going to study in this chapter.

Free surface of liquids behaves like stretched elastic membrane. It has tendency to contract and occupy minimum surface area like stretched elastic membrane. This property is called *surface tension*.

The property of surface tension can be demonstrated by following examples :

- A needle, if gently placed on the surface of water, floats even though it is heavier than water.
- Insects like 'water spider' can walk on water surface, as if they are walking on stretched elastic membrane.
- Small drops of water and mercury are spherical in shape because sphere is the only shape which has minimum surface area.
- Hail stones and planets are spherical because in the process of their formation they take spherical shape since they were in the liquid form before freezing.
- A painting brush when dipped into water and taken out, their bristles are pulled together as water surface formed on the bristles contracts.
- The following experiment illustrates the property of surface tension. A thread is tied to wire frame and immersed in soap solution and taken out so that film of soap solution is formed with the loose thread in the film. If the film is broken (using pin) on one side of the thread then the *remaining film contracts* and pulls the thread (film of soap solution behaves like surface of liquid).



Fig. 2.7

Before going to *molecular theory* of surface tension, we will go through some definitions.

Molecular force : Every molecule attracts another molecule. This force of attraction is called molecular force.

Adhesive force : It is the intramolecular force of attraction between two different types of molecules e.g. force of attraction between glass molecule and water molecule.

Cohesive force : The molecules of liquid exert force of attraction on one another called cohesive force.

Molecular range : The maximum distance upto which cohesive force can act is called molecular range.

Sphere of influence : The imaginary sphere, surrounding a molecule, in which force of attraction is present is called the sphere of influence of that molecule.

OR

The imaginary sphere drawn with molecule as a centre and molecular range as a radius is called sphere of influence.

2.2.2 Laplace's Molecular Theory of Surface Tension

Consider three molecules M_1, M_2, M_3 of liquid as shown in Fig. 2.8.

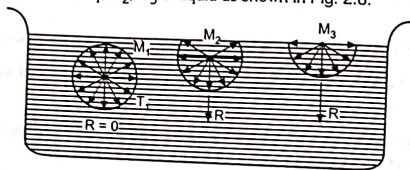


Fig. 2.8

Molecule M_1 : Its sphere of influence is totally inside liquid i.e. it is equally surrounded by neighbouring molecules. Therefore, molecule M_1 will be attracted by other molecules in its sphere of influence equally in all directions and the resultant force of attraction is zero.

Molecule M_2 : Its part of sphere of influence lies outside the liquid. Means it is not equally surrounded by neighbouring molecules. The molecules in the lower part of sphere are more. Therefore, it will get attracted downwards and resultant force acting on molecule M_2 is in downward direction.

Molecule M_3 : Its half part of sphere of influence lies outside the liquid. Therefore there are no liquid molecules on the upperside, hence no upward force. Therefore, molecule M_3 experiences more downward resultant force than M_2 .

Thus, in short, molecule M_1 experiences zero resultant force, M_2 experiences downward resultant force, M_3 experiences more downward resultant force. Thus, molecules which lie on the surface of liquid (surface film) experience downward resultant force and are being pulled inside the liquid. To balance this downward force, molecules come closer to each other which reduce the surface area of liquid. This shows that there is a downward force of attraction on the liquid surface and it is under constant tension.

Potential Energy of Molecules : Molecules which are below surface have no potential energy. But the molecules on the surface possess potential energy due to their position. As the natural tendency is to attain the position of minimum potential energy, these molecules are attracted downwards inside the liquid. As a result of this, the number of molecules on the surface of liquid comes closer to each other and hence its surface area is reduced or surface of liquid contracts producing tension.

Thus, the surface tension is defined as the property of liquids by virtue of which the surface of a liquid is under constant tension due to the tendency to contract and occupy minimum surface area.

This is the reason for - Why hail stones and planets are spherical in shape, also small drops of mercury or water are spherical in shape. Because sphere is the only shape which has minimum surface area. This property of surface tension is used to prepare ball bearings or bullets which are spherical in shape.

Unit of Surface Tension :

Surface tension can be defined as the force of contraction per unit length in the free surface of liquid.

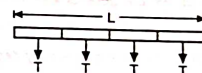


Fig. 2.9

Its S.I. unit is N/m and CGS unit is dyne/cm.

T is the force of contraction on unit length, therefore force F acting on total length 'L' is

$$F = T \times L$$

2.2.3 Angle of Contact and Its Significance

Consider water in one glass container and mercury in another container.

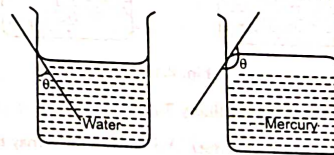


Fig. 2.10

The water level creeps up at the wall of container but mercury level depresses down at the wall of a container. Draw tangent to the curved part of the liquid surface at the point of contact.

The angle made by the tangent to the curved part of liquid surface at the point of contact with wall of a container measured through the liquid is called angle of contact θ .

Its unit is radian.

In case of water in the glass container, θ is acute because force of adhesion is stronger than cohesion i.e. water can stick to glass and therefore liquid creeps up at the wall of a container.

In case of mercury in glass container, θ is obtuse because force of cohesion is stronger i.e. mercury does not stick to glass container and therefore liquid depresses down at the wall of a container.

2.2.4 Capillarity or Capillary Action

Consider a capillary tube (i.e. a glass tube with narrow bore) dipped in a liquid. It is observed that the liquid rises in the capillary above the general level of liquid in the beaker if the angle of contact is acute e.g. water rises up inside the capillary; and it is depressed down the general level of liquid if the angle of contact is obtuse e.g. mercury depresses down inside the capillary. This rise or fall of liquid inside the capillary is called as capillarity. [Refer Fig. 2.11] e.g. Rise of oil upto wick end of oil lamp, i.e. the rise of oil in oil lamp upto the tip of wick is due to capillary action.

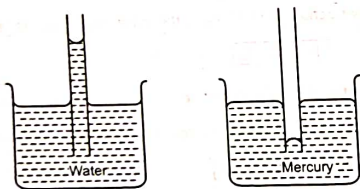
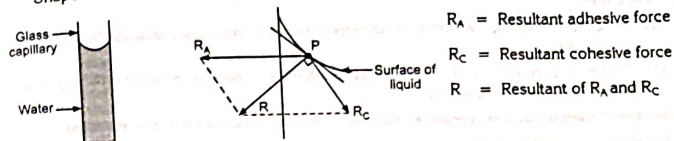


Fig. 2.11

2.2.5 Shape of Liquid Surface In a Capillary Tube

It is observed that shape of liquid surface in a capillary tube may be plane or concave or convex depending upon the nature of liquid and container.

Shape of water surface in a glass capillary tube is concave as shown in Fig. 2.12 (a).



(a) Shape of water surface in glass capillary is concave

(b) Diagram explaining why water surface is concave in glass capillary

Fig. 2.12

Explanation : 'P' is a water molecule near the glass wall. Molecule 'P' experiences two forces at a time.

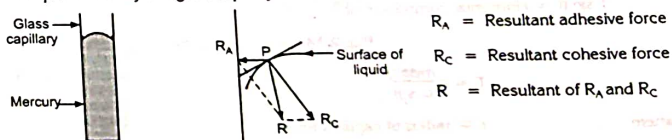
- (1) Force of adhesion i.e. force of attraction between water molecule 'P' and glass molecule.
- (2) Force of cohesion i.e. force of attraction between water molecule 'P' and other water molecules near to it.

In case of water in glass container, force $R_A > R_C$ (water sticks to glass).

R is the resultant of R_A and R_C as shown in Fig. 2.12 (b).

In order to balance this resultant force, the surface of liquid should be perpendicular to 'R'. Hence, the surface of water gets concave shape as shown.

Shape of mercury in a glass capillary tube is convex as shown in Fig. 2.13 (a).



(a) Shape of mercury surface in glass capillary is convex

(b) Diagram explaining why mercury surface is convex in glass capillary

Fig. 2.13

SOLVED EXAMPLES

Example 1 : A capillary tube of diameter 1 mm is dipped in water. How far will the water rise in tube if surface tension of water is $7 \times 10^{-2} \text{ N/m}$?

(W. 1986, W. 1987 and S. 1990 - 4 M)

Solution : Given : diameter = 1 mm
 radius $r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$

Surface tension $T = 7 \times 10^{-2} \text{ N/m}$

Find : $h = ?$

Assume density $d = 1000 \text{ kg/m}^3$
 $\theta = 0^\circ$

We have,

$$T = \frac{rhdg}{2 \cos \theta}$$

$$h = \frac{2T \cos \theta}{rdg}$$

$$= \frac{2(7 \times 10^{-2}) \cos 0}{(0.5 \times 10^{-3})(1000)(9.8)}$$

$$h = 2.857 \times 10^{-2} \text{ m}$$

\therefore Liquid will rise by $2.857 \times 10^{-2} \text{ m}$ in a tube.

Example 2 : A capillary tube of diameter 1 mm is dipped in water. How far will the water rise in the tube if surface tension of water is $7.2 \times 10^{-2} \text{ N/m}$? Density of water = $1 \times 10^3 \text{ kg/m}^3$.

(W. 2005 - 4 M)

Solution : Given : diameter = 1 mm

$r = 0.5 \text{ mm}$
 $= 0.5 \times 10^{-3} \text{ m}$

$h = ?$
 $T = 7.2 \times 10^{-2} \text{ N/m}$

$d = 1 \times 10^3 \text{ kg/m}^3$

$\theta = 0^\circ$ (\because water)

We have,

$$T = \frac{rhdg}{2 \cos \theta}$$

$$h = \frac{2T \cos \theta}{rdg}$$

$$= \frac{2 \times (7.2 \times 10^{-2}) \cos 0}{(0.5 \times 10^{-3})(1 \times 10^3)(9.8)}$$

$$h = 0.0293 \text{ m}$$

Example 3 : A capillary tube of diameter 0.2 mm is dipped into a liquid of density $0.85 \times 10^3 \text{ kg/m}^3$ and angle of contact 24° . If the liquid rises by 41 mm in the tube, find the surface tension of liquid.

Solution : Given : diameter = 0.2 mm

$r = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$

$d = 0.85 \times 10^3 \text{ kg/m}^3$

$\theta = 24^\circ$

$h = 41 \text{ mm} = 41 \times 10^{-3} \text{ m}$

$T = ?$

$$T = \frac{rhdg}{2 \cos \theta}$$

$$= \frac{(0.1 \times 10^{-3})(41 \times 10^{-3})(0.85 \times 10^3)(9.81)}{2 \cos 24^\circ}$$

$$T = 0.0187 \text{ N/m}$$

Example 4 : A liquid rises up by a height of 5.2 cm in a capillary tube of diameter 0.82 mm. How high will it rise in another tube of radius 0.025 cm? (S. 2005 - 4 M)

Solution : Given :

$h_1 = 5.2 \text{ cm}$

$h_2 = ?$

$\text{dia}_1 = 0.82 \text{ mm}$

$r_2 = 0.025 \text{ cm}$

$r_1 = 0.41 \text{ mm}$

$= 0.041 \text{ cm}$

We have,

$$r_1 h_1 = r_2 h_2$$

$$h_2 = \frac{r_1 h_1}{r_2}$$

$$= \frac{(0.041)(5.2)}{(0.025)}$$

$$h_2 = 8.53 \text{ cm}$$

Example 5 : A liquid of density $1.1 \times 10^3 \text{ kg/m}^3$ and surface tension $31.5 \times 10^{-3} \text{ N/m}$ rises to a height of 0.15 cm in a tube of diameter 0.82 mm. Find the angle of contact of the liquid. (W. 2005 - 4 M)

Solution : Given :

$d = 1.1 \times 10^3 \text{ kg/m}^3$

$T = 31.5 \times 10^{-3} \text{ N/m}$

$h = 0.15 \text{ cm}$

$= (0.15 \times 10^{-2}) \text{ m}$

$$\text{diameter} = 0.82 \text{ mm}$$

$$\therefore r = 0.41 \text{ mm} \\ = (0.41 \times 10^{-3}) \text{ m}$$

$$\theta = ?$$

$$\text{We have, } T = \frac{rhdg}{2 \cos \theta}$$

$$\therefore 2 \cos \theta = \frac{rhdg}{T}$$

$$\therefore \cos \theta = \frac{rhdg}{2T} \\ = \frac{(0.41 \times 10^{-3}) (0.15 \times 10^{-2}) (1.1 \times 10^3) (9.8)}{2 \times (31.5 \times 10^{-3})}$$

$$\cos \theta = 0.1052$$

$$\therefore \theta = \cos^{-1}(0.1052)$$

$$\therefore \theta = 83.96^\circ$$

Example 6 : Find the surface tension of mercury if mercury is depressed by 8.22 mm in a capillary tube, if diameter = 2.25 mm, angle of contact = 140° , density of mercury = $13.6 \times 10^3 \text{ kg/m}^3$.

$$\text{Solution : Given : } -h = -8.22 \text{ mm} \\ = -8.22 \times 10^{-3} \text{ m}$$

$$\text{diameter} = 2.25 \text{ mm}$$

$$\therefore r = 1.125 \text{ mm} = 1.125 \times 10^{-3} \text{ m}$$

$$\theta = 140^\circ$$

$$d = 13.6 \times 10^3 \text{ kg/m}^3$$

$$T = ?$$

$$\text{We have, } T = \frac{rhdg}{2 \cos \theta} \\ = \frac{(1.125 \times 10^{-3}) (-8.22 \times 10^{-3}) (13.6 \times 10^3) (9.8)}{2 \cos 140^\circ}$$

$$\boxed{T = 0.804 \text{ N/m}}$$

Example 7 : A liquid rises through a height of 4 cm in a capillary tube of radius 0.4 mm. How far will it rise in a capillary tube of radius 0.8 mm?

$$\text{Solution : Given : } \begin{array}{ll} \text{radius, } r_1 = 0.4 \text{ mm} & r_2 = 0.8 \text{ mm} \\ & = 0.04 \text{ cm} & = 0.08 \text{ cm} \\ \text{rise of liquid, } h_1 = 4 \text{ cm} & h_2 = ? \end{array}$$

$$r_1 h_1 = r_2 h_2$$

$$h_2 = \frac{r_1 h_1}{r_2}$$

$$= \frac{(0.04) \times (4)}{(0.08)}$$

$$\boxed{h_2 = 2 \text{ cm}}$$

\therefore Liquid will rise by 2 cm in a capillary tube of radius 0.8 mm.

Example 8 : A capillary tube of diameter 0.5 mm is dipped in a liquid of density 800 kg/m^3 and surface tension 0.028 N/m . Calculate the rise if angle of contact is 16.3° .

$$\text{Solution : Given : } \text{diameter} = 0.5 \text{ mm}$$

$$\text{radius } r = 0.25 \text{ mm} = 0.25 \times 10^{-3} \text{ m}$$

$$d = 800 \text{ kg/m}^3$$

$$\theta = 16.3^\circ$$

$$T = 0.028 \text{ N/m}$$

Find :

$$h = ?$$

$$T = \frac{rhdg}{2 \cos \theta}$$

\therefore

$$h = \frac{2T \cos \theta}{rdg}$$

$$= \frac{2 \times 0.028 \times \cos 16.3^\circ}{(0.25 \times 10^{-3}) \times (800) \times (9.81)}$$

\therefore

$$\boxed{h = 0.028 \text{ m}}$$

\therefore Liquid will rise by 0.028 m in a capillary tube.

Example 9 : Pure water rises to a height of 2.5 cm in a capillary tube of diameter 1 mm. Find the surface tension if density of water is 1000 kg/m^3 . (S. 1999 and S. 2002 - 4 M)

$$\text{Solution : Given : } h = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$$

$$\text{diameter} = 1 \text{ mm}$$

\therefore

$$r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

$$T = ?$$

$$\theta = 0 \text{ (for pure water)}$$

$$T = \frac{rhdg}{2 \cos \theta}$$

$$= \frac{(0.5 \times 10^{-3}) \times (2.5 \times 10^{-2}) \times (1000) \times (9.8)}{2 \cos 0}$$

$$\boxed{T = 0.061 \text{ N/m}}$$

QUESTIONS

- Define surface tension of a liquid.
- Explain the phenomenon of surface tension with the help of Laplace's molecular theory.
- Explain the molecular theory of surface tension with respect to the potential energy of surface molecules.
- State unit of surface tension.
- Define angle of contact. On what factors it depends ?
OR Explain the significance of acute and obtuse angle.
- State the formula for surface tension.
- Define :
 - Cohesive force,
 - Adhesive force,
 - Sphere of influence,
 - Angle of contact,
 - Capillary action.
- State the effect of temperature and impurity on surface tension of the liquid.
- Define surface tension. State the relation between surface tension, capillary rise and radius of capillary tube. State the meaning of symbols used in it.
- Explain with neat sketch how liquid form concave and convex surfaces in a tube.
- Why the shape of liquid drop is spherical ?
- What is the effect of organic impurity and temperature on surface tension of liquid ?
- Explain why surface of water is concave in glass capillary tube.
- Explain why surface of mercury is convex in glass capillary tube.
- Define adhesive and cohesive molecular forces of attraction. Give one example of each force. (W. 2004 - 4 M)
- Define surface tension of a liquid and state its S.I. unit. (S. 2002 - 2 M)

PROBLEMS

- A liquid rises to a height of 2 cm in a tube of diameter 2.5 mm. How far will it rise in a tube of diameter 1.5 mm ? (S. 1987 - 4 M)
Ans. 3.33 cm.
- A glass capillary tube of diameter 0.02 cm is dipped into a liquid of density $0.85 \times 10^3 \text{ kg/m}^3$. The liquid rises to a height of 14 cm. The angle of contact is 15° . Calculate the surface tension of the liquid. (S. 1984 - 4 M)
Ans. 0.06 N/m.
- The liquid rises to a height of 2.25 cm in a capillary tube of certain diameter, when the surface tension of the liquid is $6.45 \times 10^{-2} \text{ N/m}$. Density of the liquid is $0.9 \times 10^3 \text{ kg/m}^3$ and angle of contact is 2° . Calculate the diameter of tube. (W. 1993 and S. 1988 - 4 M)
Ans. $r = 0.0649 \text{ cm}$ and diameter = 0.1298 cm
- In a capillary tube of diameter 0.1 cm, the mercury level is depressed by 1.25 cm. Find surface tension of mercury.
Density of mercury = $13.6 \times 10^3 \text{ kg/m}^3$
Angle of contact = 128.67° (S. 1992 and S. 2003 - 4 M)
Ans. 0.66 N/m.
- A capillary tube of radius 0.05 cm is dropped in water. How far will the water rise in the tube ? Given surface tension of water is $7 \times 10^{-2} \text{ N/m}$, angle of contact is 0° , density of water is 10^3 kg/m^3 , $g = 9.8 \text{ m/s}^2$. (S. 2001 - 4 M)
Ans. $h = 2.857 \text{ cm}$
- A capillary tube of radius 0.06 cm is dipped in pure water. How far will the water rise in the tube if surface tension of water is $7.0 \times 10^{-2} \text{ N/m}$? Density of water = $1 \times 10^3 \text{ kg/m}^3$. (S. 1997 - 4 M)
Ans. 2.38 cm.
- A liquid of density 1200 kg/m^3 and surface tension 0.036 N/m rises to a height of 1.8 mm in a tube of radius 0.3 mm. Find the angle of contact of the liquid.
Ans. $\theta = 84.93^\circ$.

2.3 VISCOSITY

Introduction, Viscosity, Velocity gradient, Newton's law of viscosity, Coefficient of viscosity, Stoke's law of viscosity, Buoyant force, Streamline flow, Turbulent flow, Critical velocity, Reynold's number, Effect of temperature and adulteration on viscosity of liquid, Applications of viscosity.

2.3.1 Introduction

Liquids like honey, glycerine take more time to flow on surface than that of water and kerosene, etc.

i.e. Honey, glycerine are more viscous than that of water and kerosene. In other words, glycerine/honey are less mobile, hence more viscous and water/kerosene are more mobile, hence less viscous.

2.3.2 Viscosity

Consider flow of liquid on the horizontal plane surface. Imagine liquid is made up of number of horizontal layers. It is observed that different liquid layers move with different velocities.

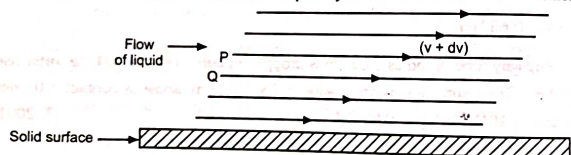


Fig. 2.16

The bottom-most layer liquid molecules have minimum speed on account of adhesion and speed of layer increases from bottom to top, then top-most layer has maximum speed.

Consider layers P and Q. Layer P which has more speed, tries to accelerate layer Q. At the same time, layer Q which has less speed, tries to decrease speed of layer P and thus because of different tendencies of different layers, a frictional force (fluid friction) is created between two layers called as viscous force which tries to oppose the relative motion

between different layers i.e. tries to reduce the difference between their speeds called viscosity.

Definition of viscosity : Viscosity is the property of liquid on account of which liquid tries to oppose the relative motion between its different layers.

Velocity gradient :

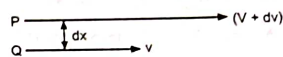


Fig. 2.17

Consider any two layers P and Q with velocities $(v + dv)$ and v respectively. Here 'dv' is the change in velocity of layers and 'dx' is the change in vertical distance between the layers.

Then $\frac{dv}{dx}$ is called velocity gradient.

Definition of velocity gradient : Velocity gradient is defined as, the change in velocity per unit change in (vertical) distance of liquid layers.

$$\text{Velocity gradient} = \frac{dv}{dx}$$

Unit of velocity gradient is per second i.e. 1/sec.

2.3.3 Newton's Law of Viscosity

The law states that the viscous force 'F' developed between two liquid layers is

- (1) directly proportional to surface area 'A' of liquid layer,
- (2) directly proportional to velocity gradient.

Thus, $F \propto A$

and $F \propto \frac{dv}{dx}$

\therefore Combining, $F \propto A \times \frac{dv}{dx}$

$$F = \eta A \frac{dv}{dx}$$

where η is called coefficient of viscosity, ' η ' is constant for a given liquid. It changes from liquid to liquid.

$$\eta = \frac{F}{A \times \frac{dv}{dx}} \quad \text{i.e.} \quad \frac{\text{newton}}{\text{m}^2 \times \frac{\text{m/s}}{\text{m}}}$$

$$= \text{Ns/m}^2 \text{ is the SI unit of } \eta.$$

or dyne-s/cm² is the CGS unit of η .

or $1 \text{ poise} = 1 \text{ dyne-s/cm}^2 = \frac{1}{10} \text{ N-s/m}^2$

Definition of 1 poise : The coefficient of viscosity ' η ' is said to be 1 poise if 1 dyne force is developed between two liquid layers of 1 cm² area for unit velocity gradient.

In order to define ' η ', if $A = 1$, $\frac{dv}{dx} = 1$, then $\eta = F$.

Definition of coefficient of viscosity ' η ' : Coefficient of viscosity ' η ' of a liquid is defined as the viscous force developed between two liquid layers of unit surface area for unit velocity gradient.

2.3.4 Flow of Liquid through a Tube - Streamline Flow and Turbulent Flow

We have seen different layers of liquid flow with different velocities. In case of flow of liquid through a tube, the liquid layer which flows along axis has maximum velocity and layers which are in contact with wall of tube has minimum velocity.

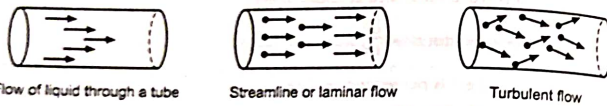


Fig. 2.18

Streamline or Laminar flow :

Streamline flow is the flow of the liquid in which every particle of liquid moves in the same direction (i.e. parallel or inline) of flow of liquid.

When liquid flows steadily, each particle follows its earlier particle. In streamline flow, the velocity at every point within the liquid remains constant. The flow remains streamline so long as the velocity is below certain velocity called critical velocity. e.g. Flow of river water in summer season is slow and steady i.e. streamline.

Critical Velocity :

The value of velocity of flow of liquid upto which flow is streamline is called critical velocity ' v_c '.

If velocity of flow of liquid increases and crosses critical velocity, then particle starts moving in random direction i.e. flow becomes turbulent.

Thus, critical velocity is defined as the velocity of flow of liquid at which streamline flow changes into turbulent flow.

Turbulent flow :

The flow of liquid in which every particle is not moving in line and they move in random direction is called turbulent flow.

This happens when speed of flow of water is more than certain value (i.e. more than critical velocity). e.g. Flow of river water after very heavy rain i.e. during flood. Velocity is given by the relation

$$v = \frac{\eta R}{\rho r}$$

where

- v - Velocity of flow of liquid
- η - Coefficient of viscosity of liquid
- R - Reynold's number
- ρ - Density of liquid
- r - Radius of the tube

Significance of Reynold's number :

On the basis of experiments, scientist Reynold observed that, Reynold's number determines the nature of flow of liquid through a tube. He observed that for a tube of radius 1 cm,

when R is less than 2000, then liquid flow is streamline.

when R is in between 2000 to 3000, the liquid flow is unstable.

when R is greater than 3000, the liquid flow is turbulent.

2.3.5 Free Fall of Spherical Body through Viscous Medium and Stoke's Law

When a body falls under gravitation, through a column of liquid, different liquid layers move with different speeds. Liquid layers which are in contact with body has maximum velocity and liquid layer in contact with wall of container has zero velocity. Because of different speeds of different layers, a viscous force which opposes the relative motion is developed.

Practically, it is observed that after covering certain distance the body, which is freely falling, attains constant velocity called as terminal velocity ' v '.

Terminal velocity : The constant velocity with which a body falls through liquid column is called terminal velocity.

Stoke's law : Stoke's law states that the force of viscosity experienced by a small metal sphere falling freely through a viscous medium, with terminal velocity is directly proportional to

- (1) radius of metal sphere ' r '
- (2) terminal velocity ' v '
- (3) coefficient of viscosity of liquid ' η '

i.e. $F \propto \eta r v$

$$F = 6\pi \eta r v \dots \text{Stoke's law formula}$$

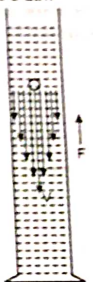


Fig. 2.19

'η' by Stoke's Method :

Consider a metal sphere placed on the surface of liquid taken in glass jar. It is observed that after covering certain distance, metal sphere attains a constant velocity called *terminal velocity*.

Metal sphere falling freely through a liquid experiences three forces :

- (1) Weight of the metal sphere in the downward direction.
- (2) Force of viscosity in the upward direction.
- (3) Upthrust force (force of Buoyancy) in the upward direction. Buoyant force or upthrust force is the force with which liquid lifts the body dipped into it.

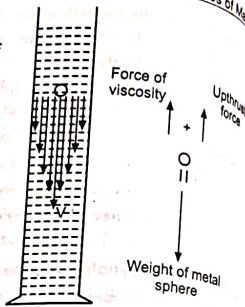


Fig. 2.20

By Archemedi's principle,

$$\begin{aligned} \text{Upthrust force} &= \text{Loss of weight of body in liquid} \\ &= \text{Weight of displaced liquid} \end{aligned}$$

Since metal sphere falls with constant velocity, the total upward force is equal to downward force.

$$\text{Force of viscosity} + \text{Upthrust force} = \text{Weight of metal sphere}$$

$$\therefore \text{Total upward force} = \text{Downward force}$$

$$\therefore [\text{Force of viscosity}] + [\text{Upthrust force}] = [\text{Weight of the metal sphere}]$$

$$\therefore [6\pi\eta rv] + [\text{Weight of the displaced liquid}] = [\text{Weight of the metal sphere}]$$

$$\therefore \eta = \frac{2r^2g(d-\rho)}{9v}$$

$$\text{or } v = \frac{2r^2g(d-\rho)}{9\eta}$$

where

- η - coefficient of viscosity of liquid
- r - radius of metal sphere
- d - density of metal (heavier)
- ρ - density of liquid (lighter)
- v - terminal velocity

SUMMARY

- The property of liquid due to which liquid tries to oppose the relative motion between two layers is called viscosity.
- Newton's law of viscosity states that the force of viscosity between two layers is directly proportional to surface area of liquid layer and velocity gradient.
- Velocity gradient is defined as change in velocity per unit change in vertical distance of liquid layers.
- Coefficient of viscosity is defined as the tangential force of viscosity developed between two liquid layers of unit surface area for unit velocity gradient.
- Stoke's law states that the force of viscosity experienced by a small metal sphere falling freely through liquid with terminal velocity is directly proportional to : (1) radius of sphere, (2) terminal velocity, and (3) coefficient of viscosity of liquid.

FORMULAE

$$(1) F = \eta A \frac{dv}{dx} \quad \dots \text{ Newton's formula}$$

- where F - Force of viscosity
- η - Coefficient of viscosity
- A - Area of liquid layer
- $\frac{dv}{dx}$ - Velocity gradient

$$(2) F = 6\pi\eta rv \quad \dots \text{ Stoke's formula}$$

- where F - Force of viscosity
- η - Coefficient of viscosity
- r - Radius of metal sphere
- v - Terminal velocity

$$(3) \eta = \frac{2r^2g(d-\rho)}{9v}$$

- where, r - Radius of metal sphere
- d - Density of metal (heavier)
- ρ - Density of liquid (lighter)
- v - Terminal velocity

$$(4) v = \frac{R\eta}{\rho r}$$

- where, v - Velocity of flow of liquid
- R - Reynold's number
- η - Coefficient of viscosity
- ρ - Density of liquid
- r - Radius of the tube (pipe)

SOLVED EXAMPLES

Example 1 : A spherical ball of radius 0.15 cm takes 6 seconds to travel a distance of 80 cm through a viscous liquid. If the density of ball is $8 \times 10^3 \text{ kg/m}^3$ and that of liquid is $1.2 \times 10^3 \text{ kg/m}^3$, find the viscosity of the liquid. (W. 2004-4)

Solution : Given :

$$r = 0.15 \text{ cm} = 0.15 \times 10^{-2} \text{ m}$$

$$v = \frac{\text{distance}}{\text{time}} = \frac{80 \text{ cm}}{6 \text{ sec}} = (13.33) \text{ cm/s} = (0.133) \text{ m/s}$$

$$d = 8 \times 10^3 \text{ kg/m}^3$$

$$\rho = 1.2 \times 10^3 \text{ kg/m}^3$$

$$\eta = ?$$

$$\eta = \frac{2}{9} \frac{r^2 g (d - \rho)}{v}$$

$$= \frac{2}{9} \frac{(0.15 \times 10^{-2})^2 (9.8) (8 \times 10^3 - 1.2 \times 10^3)}{(0.133)}$$

$$\boxed{\eta = 0.25 \text{ Ns/m}^2}$$

Example 2 : A spherical ball of radius 2.2 mm and density $8 \times 10^3 \text{ kg/m}^3$ falls through liquid of density $1.3 \times 10^3 \text{ kg/m}^3$. Find terminal velocity. (S. 2005-4)

(Given : η for liquid = 0.45 Ns/m^2 **)**

Solution : Given :

$$r = 2.2 \text{ mm} = 2.2 \times 10^{-3} \text{ m}$$

$$d = 8 \times 10^3 \text{ kg/m}^3$$

$$\rho = 1.3 \times 10^3 \text{ kg/m}^3$$

$$\eta = 0.45 \text{ Ns/m}^2$$

$$v = ?$$

We have,

$$\eta = \frac{2}{9} \frac{r^2 g (d - \rho)}{v}$$

$$v = \frac{2}{9} \frac{r^2 g (d - \rho)}{\eta}$$

$$v = \frac{2}{9} \times \frac{(2.2 \times 10^{-3})^2 (9.8) (8 \times 10^3 - 1.3 \times 10^3)}{0.45}$$

$$\therefore \boxed{v = 0.157 \text{ m/s}}$$

Example 3 : A metal plate of area 0.25 m^2 rests on a layer of oil 0.003 m thick with coefficient of viscosity 1.56 Ns/m^2 . If the plate is moved with a velocity of 0.05 m/s, calculate the horizontal force acting on the plate. (S. 2002-4)

Solution : Given :

$$A = 0.25 \text{ m}^2$$

$$dx = 0.003 \text{ m}$$

$$\eta = 1.56 \text{ Ns/m}^2$$

$$dv = 0.05 \text{ m/s}$$

$$F = ?$$

We have,

$$F = \eta A \frac{dv}{dx}$$

$$= (1.56) (0.25) \frac{(0.05)}{(0.003)}$$

$$\boxed{F = 6.5 \text{ N}}$$

Example 4 : A plate of metal having 100 sq. cm. area rests on a layer of paraffin oil 2 mm thick. If the horizontal force required to move the plate with velocity 3 cm/s is 0.24 newton, find the coefficient of viscosity.

Solution :

$$A = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$$

$$dx = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$dv = 3 \text{ cm/s} = 3 \times 10^{-2} \text{ m/s}$$

$$F = 0.24 \text{ N}$$

$$\eta = ?$$

By Newton's law of viscosity,

$$F = \eta A \times \frac{dv}{dx}$$

\therefore

$$\eta = \frac{F}{A \times \frac{dv}{dx}}$$

$$\eta = \frac{(0.24)}{(100 \times 10^{-4}) \frac{(3 \times 10^{-2})}{(2 \times 10^{-3})}}$$

$$\boxed{\eta = 1.6 \text{ Ns/m}^2}$$

Example 5 : Find the velocity of flow of a liquid of viscosity 0.02 Ns/m^2 and density 1.2 gm/cm^3 through a pipe of radius 0.05 m; Reynold's number = 2000.

Solution : Given :

$$\eta = 0.02 \text{ Ns/m}^2$$

$$\rho = 1.2 \text{ gm/cm}^3 = 1.2 \times 10^3 \text{ kg/m}^3$$

$$r = 0.05 \text{ m}$$

$$R = 2000$$

$$v = ?$$

$$v = \frac{\eta R}{\rho r}$$

$$= \frac{(0.02)(2000)}{(1.2 \times 10^3)(0.05)}$$

$$v = 0.667 \text{ m/s}$$

Example 6 : A liquid flows through a pipe of diameter 10 cm with a speed of 0.5 m/s. Density of liquid is $0.8 \times 10^3 \text{ kg/m}^3$ and the coefficient of viscosity is 0.4 Ns/m^2 . Determine Reynold's number and state whether the flow is turbulent or streamline. (S. 2001-4)

Solution : Given : diameter = 10 cm

$$\text{radius } r = 5 \text{ cm} = 0.05 \text{ m}$$

$$v = 0.5 \text{ m/s}$$

$$\rho = 0.8 \times 10^3 \text{ kg/m}^3$$

$$\eta = 0.4 \text{ Ns/m}^2$$

$$R = ?$$

$$v = \frac{\eta R}{\rho r}$$

$$\therefore R = \frac{v \rho r}{\eta} = \frac{(0.5) \times (0.8 \times 10^3) \times (0.05)}{0.4}$$

$$R = 50$$

$\therefore R < 2000$, the liquid flow is streamlined.

Example 7 : A steel ball of density $8 \times 10^3 \text{ kg/m}^3$ falls vertically in a tall jar containing oil of density $1.5 \times 10^3 \text{ kg/m}^3$ and acquires a terminal velocity 0.2 m/s. If radius of the ball is 2 mm, find coefficient of viscosity of oil. ($g = 9.81 \text{ m/s}^2$) (S. 2002-4)

Solution : Given : $d = 8 \times 10^3 \text{ kg/m}^3$

$$\rho = 1.5 \times 10^3 \text{ kg/m}^3$$

$$v = 0.2 \text{ m/s}$$

$$r = 2 \text{ mm} = 0.002 \text{ m}$$

$$\eta = ?$$

$$\eta = \frac{2}{9} \frac{r^2 g (d - \rho)}{v}$$

$$= \frac{2}{9} \times \frac{(0.002)^2 (9.81) (8 \times 10^3 - 1.5 \times 10^3)}{0.2}$$

$$\eta = 0.28 \text{ Ns/m}^2$$

Example 8 : Calculate the viscous force on a steel ball of radius 2 mm, falling freely in a tall jar containing oil with a terminal velocity 0.2 m/s, coefficient of viscosity of oil is 0.28 Ns/m^2 .

Solution : $F = ?$

$$r = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$v = 0.2 \text{ m/s}$$

$$\eta = 0.28 \text{ Ns/m}^2$$

We have Stoke's formula for viscous force.

$$F = 6\pi\eta r v$$

$$= 6 \times 3.142 \times (0.28) (2 \times 10^{-3}) (0.2)$$

$$F = 2.11 \times 10^{-3} \text{ N}$$

Example 9 : Assuming the Reynold's number to be 1000, calculate the critical velocity for glycerine in the pipe of diameter 2 cm. Density and viscosity of glycerine are $1.36 \times 10^3 \text{ kg/m}^3$ and 0.85 Ns/m^2 respectively. (S. 1987 and W. 2002 - 4 M)

Solution : Given : $R = 1000$

$$\text{diameter} = 2 \text{ cm}$$

$$r = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$$

$$\text{density, } \rho = 1.36 \times 10^3 \text{ kg/m}^3$$

$$\eta = 0.85 \text{ Ns/m}^2$$

$$v = v_c = ?$$

$$v = v_c = \frac{R\eta}{\rho r} = \frac{(1000) \times (0.85)}{(1.36 \times 10^3) \times (1 \times 10^{-2})}$$

$$\therefore v_c = 62.5 \text{ m/s}$$

Example 10 : An air bubble of radius 1 cm rises steadily through solution of density $1.75 \times 10^3 \text{ kg/m}^3$ at the steady velocity of 0.35 m/s. Calculate coefficient of viscosity of solution neglecting density of air.

Solution : Given : $r = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$

density of liquid, $d = 1.75 \times 10^3 \text{ kg/m}^3$

$v = 0.35 \text{ m/s}$

$$\eta = \frac{2r^2g(d-\rho)}{9v}$$

$$= \frac{2(1 \times 10^{-2})^2 \times (9.8) \times (1.75 \times 10^3 - 0)}{9 \times 0.35}$$

$$\therefore \eta = 1.08 \text{ Ns/m}^2$$

QUESTIONS

1. Explain the property of viscosity of liquid.
2. State Newton's law of viscosity.
3. Define coefficient of viscosity and state its unit.
4. Define velocity gradient.
5. State Stoke's law and state formula for terminal velocity.
6. Define terminal velocity.
7. What is streamline flow and turbulent flow?
8. Define critical velocity.
9. Explain significance of Reynold's number.
10. State Stoke's law of viscosity and state the formula for coefficient of viscosity. (W. 2004-4M)
11. Define critical velocity and streamline flow.
12. State Newton's law of viscosity and hence define coefficient of viscosity. Give its S.I. unit.
13. Define velocity gradient and state its unit.
14. What is Reynold's number? State its significance.
15. Define velocity gradient and terminal velocity.
16. State Newton's law of viscosity. Define velocity gradient. State the formula for viscous force, hence define 1 poise.

PROBLEMS

1. A spherical liquid drop of diameter 0.02 cm is falling freely through air. Find the terminal velocity of the drop. The density of the liquid is 500 kg/m^3 and the viscosity of air is $2 \times 10^{-5} \text{ Ns/m}^2$. Neglect density of air.

Ans. 0.54 m/s.

2. With what terminal velocity will an air bubble of radius 0.1 cm rise through water? Coefficient of viscosity of water is 10^{-2} poise. Neglect density of bubble.

Ans. 218 cm/s.

3. A spherical steel ball of radius 0.3 mm falls vertically through water. Find the coefficient of viscosity of water.

Given : terminal velocity acquired by the ball = $9.8 \times 10^{-2} \text{ m/s}$,

density of water = $1 \times 10^3 \text{ kg/m}^3$ and density of the ball = $1.5 \times 10^3 \text{ kg/m}^3$.

Ans. $1 \times 10^{-3} \text{ Ns/m}^2$.

4. Calculate the viscous force on raindrop of diameter 4 mm falling with a constant velocity of 4 m/s through air. Coefficient of viscosity of air is 1.8×10^{-4} poise.

Ans. 0.27 dyne.

5. A spherical ball of diameter 3 mm and density $7.8 \times 10^3 \text{ kg/m}^3$ falls vertically through the oil of tall jar and attains a terminal velocity of 0.231 m/s. If density of oil is $1.2 \times 10^3 \text{ kg/m}^3$, find coefficient of viscosity of oil. (S. 1998 - 4 M)

Ans. $\eta = 0.14 \text{ Ns/m}^2$.

6. A spherical steel ball of diameter 6 mm and density $7.2 \times 10^3 \text{ kg/m}^3$ falls with terminal velocity of 0.24 m/s through glycerine. Find the coefficient of viscosity of glycerine. **Given :** density of glycerine = $1.26 \times 10^3 \text{ kg/m}^3$ and $g = 9.8 \text{ m/s}^2$. (S. 1990 - 4 M)

Ans. $\eta = 0.485 \text{ Ns/m}^2$

7. A force of 8 N is required to move a solid horizontal surface over a liquid of surface area 0.25 m^2 with a velocity of 0.05 m/s. If the thickness of the liquid layer is 2 mm, calculate the coefficient of viscosity. (S. 1989 - 4 M)

Ans. $\eta = 1.28 \text{ Ns/m}^2$.

8. A sphere of density $8 \times 10^3 \text{ kg/m}^3$ falls vertically in a jar containing oil of density $1.5 \times 10^3 \text{ kg/m}^3$ and acquires a terminal velocity of 0.25 m/s . If the radius of a sphere is 2 mm , find the coefficient of viscosity of oil. (W. 2001 - 4 M)
- Ans. $\eta = 0.2265 \text{ Ns/m}^2$.
9. Assuming the Reynold's number to be 1200, calculate the critical velocity for liquid in the pipe of radius 1.2 cm . Density and viscosity of liquid are 1300 kg/m^3 and 0.8 Ns/m^2 respectively.
- Ans. $v_c = 61.54 \text{ m/s}$.
10. A liquid flows through a pipe of radius 4 cm with a speed of 12 m/s . The density of liquid is $0.85 \times 10^3 \text{ kg/m}^3$ and coefficient of viscosity is 0.6 Ns/m^2 . Determine Reynold's number and state whether the flow is turbulent or streamline.
- Ans. $R = 680$ which is less than 2000, hence flow is streamlined.

HEAT

3.1 TRANSMISSION OF HEAT AND EXPANSION OF SOLIDS

Conduction, Convection, Radiation, Good and bad conductor, Thermal conductivity and coefficient of thermal conductivity, Temperature gradient, Coefficient of linear expansion, Coefficient of aerial (surface) expansion, Cubical expansion, Relation between α , β , and γ .

3.1.1 Three Modes of Transmission of Heat

Energy always flows from higher energy level to lower energy level. Heat is a form of energy which flows from a body at higher temperature to a body at lower temperature. There are three modes of transfer of heat.

1. **Conduction** : It is a process of transfer of heat from a part of body at higher temperature to a part of body at lower temperature *without bodily (actual) movement of particles*. It takes place through solids.

2. **Convection** : It is a process of transfer of heat from part of body at higher temperature to part of body at lower temperature with *bodily (actual) movement of particles*. It takes place in case of liquids.

3. **Radiation** : It is the process of transfer of heat in which heat is transferred from one place to other directly without the necessity of intervening medium e.g. we get heat from sun without affecting the intervening medium. Heat radiations can pass through vacuum.

Good conductor : The material through which heat conducts easily and speedily is called good conductor of heat. e.g. copper, iron, aluminium. e.g. Good conducting material is used as heat sink in electronic circuits which absorbs heat and protects the components from overheating. Spiral tube of good conductor is used in electric heater for speedy conduction of heat. MICA is a good conductor of heat, but bad conductor of electricity, hence used in electric iron.

Bad conductor : The material through which heat does not conduct (flow) is called a bad conductor of heat. e.g. wood, wool, plastic. e.g. Bad conducting material like thermocol is used in ice box in which ice melts slowly.

In this topic, we will study conduction in more details.

3.1.2 Conduction in Detail

It is the process of transfer of heat in which heat is transferred from a part of body at high temperature to a part of body at lower temperature without bodily movement of particles (molecules) from one place to the other.

When one end of metal rod is heated, other end also gets heated after some time.

Let us assume that the rod is divided into different compartments P, Q, R, S.

Before heating, molecules in all compartments are vibrating about their mean position with same amplitude. If now end A is heated, then molecules in compartment 'P' receive heat and start vibrating with more amplitude. These molecules collide with molecules in the next compartment 'Q'. Now molecules in compartment 'Q' receive vibrational energy and they also start vibrating with more amplitude and this process continues. Thus, heat travels through the rod. In this process, the individual molecules do not travel from one end to other end. They only vibrate about their mean positions and vibrations are passed on from one end to other.

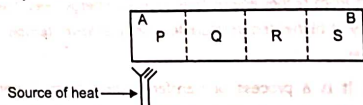


Fig. 3.1

3.1.3 Thermal Conductivity and Coefficient of Thermal Conductivity (Law of Thermal Conductivity)

Factors Affecting the Conduction of Heat :

Suppose AB is a bar of metal, of cross-sectional area A as shown in Fig. 3.2. Consider the bar to be in steady state.

No amount of heat is lost to the surroundings by means of radiation.

Consider two planes C and D in the bar.

- Let,
- Q = the amount of heat flowing from C to D
 - d = distance between C and D or distance between two thermometers
 - θ_1 = temperature of plane C
 - θ_2 = temperature of plane D
 - $\theta_1 > \theta_2$

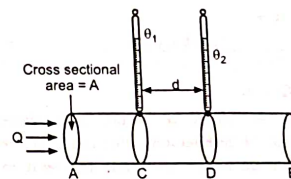


Fig. 3.2

Then, amount of heat flowing (Q) from C to D at steady state is directly proportional to

1. Cross-sectional area 'A' of rod,
2. Temperature difference between two planes i.e. $(\theta_1 - \theta_2)$,
3. Time 't' for which heat flows, and inversely proportional to
1. Distance 'd' between two planes or distance between two thermometers.

Thus,

$$Q \propto A$$

$$Q \propto (\theta_1 - \theta_2)$$

$$Q \propto t$$

$$Q \propto \frac{1}{d}$$

∴ Combining,

$$Q \propto \frac{A \times (\theta_1 - \theta_2) t}{d}$$

$$Q = \text{Constant} \times \frac{A \times (\theta_1 - \theta_2) t}{d}$$

$$Q = K \times \frac{A \times (\theta_1 - \theta_2) \times t}{d} \quad \dots (3.1)$$

where K is the constant of proportionality which is called coefficient of thermal conductivity. 'K' depends on material of a bar.

Coefficient of Thermal Conductivity (K) :

We have,

$$Q = K \frac{A (\theta_1 - \theta_2) t}{d} \quad \dots \text{from equation (3.1)}$$

If $A = 1$

$(\theta_1 - \theta_2) = 1$

$t = 1$

then

$$Q = \frac{K \times 1 \times 1 \times 1}{1}$$

$$Q = K$$

Thus, coefficient of thermal conductivity is defined as amount of heat conducted in one second, in steady state of temperature through unit cross-sectional area of an element of material of unit thickness with unit temperature difference between its opposite faces.

Unit of 'K' :

$$K = \frac{Q \times d}{A (\theta_1 - \theta_2) t} = \frac{\text{cal} \times \text{m}}{\text{m}^2 (\text{°C}) \text{sec}}$$

$$= \frac{\text{cal}}{\text{m} \text{°C} \text{sec}}$$

$$= \frac{\text{cal}}{\text{cm} \text{°C} \text{sec}}$$

... CGS unit

$$= \frac{\text{kcal}}{\text{m} \text{°C} \text{sec}}$$

... MKS unit

$$= \frac{\text{watt}}{\text{m} \text{°K}}$$

... S.I. unit

Temperature Gradient :

The factor $\frac{(\theta_1 - \theta_2)}{d}$ in equation (3.1) is called temperature gradient.

Here

 θ_1 - temperature at plane C θ_2 - temperature at plane D

d - distance between C and D

The temperature gradient is defined as change in temperature per unit length of rod. The unit of temperature gradient is °C/m or °K/m.

Using

$$\text{T.G.} = \frac{(\theta_1 - \theta_2)}{d}$$

Equation (3.1) becomes

$$Q = K \times A \times \text{T.G.} \times t$$

If $A = 1$, $\text{T.G.} = 1$, and $t = 1$

then

$$K = Q$$

Thus, coefficient of thermal conductivity (K) of material can also be defined as amount of heat flowing through a rod of unit area, in one second for unit temperature gradient at steady state.

3.1.4 Expansion of Solids

Whenever solid is heated, it expands. If solid is in thin rod form, then after heating, its length increases. If it is in thin sheet form, then its area increases and if it is in cube form, then after heating, its volume increases.

Accordingly, there are three types of coefficient of expansion.

(a) Linear expansion or Coefficient of linear expansion (α) : Consider a metal rod of length L_0 .

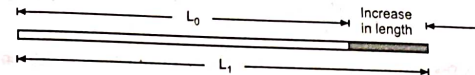


Fig. 3.3

Let,

 L_0 = Original length of rod at 0°C L_t = New length at t°C $(L_t - L_0)$ = Increase in length (shaded portion) $(t - 0)$ = Increase in temperature

Then, coefficient of linear expansion is given by

$$\alpha = \frac{(L_t - L_0)}{L_0 \times t}$$

Definition of coefficient of linear expansion 'α' : The coefficient of linear expansion 'α' of a material is defined as increase in length per unit original length at 0°C, per unit increase in temperature.

Unit of α is /°C.

We have seen that if a metal rod or metal bar is heated, then its length increases. Because of this, some gap or space is kept between the two rails. i.e. while laying railway lines, small gaps are left between two consecutive rails which provide scope for expansion.

(b) Aerial expansion : Coefficient of aerial expansion or surface expansion or superficial expansion (β) : Consider thin metal sheet PQRS of area A_0 .

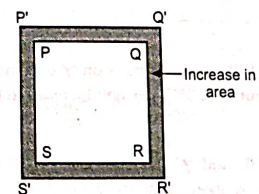


Fig. 3.4

Let, A_0 = Original area (PQRS) at 0°C
 A_t = New area (P'Q'R'S') at $t^\circ\text{C}$
 $(A_t - A_0)$ = Increase in area (shaded portion)
 $(t - 0)$ = Increase in temperature

Then coefficient of aerial expansion ' β ' is given by

$$\beta = \frac{(A_t - A_0)}{A_0 \times t}$$

Definition : The coefficient of aerial expansion ' β ' of a material is defined as increase in area per unit original area at 0°C per unit increase in temperature.

Unit of β is $^\circ\text{C}^{-1}$.

(c) Cubical expansion : Coefficient of cubical (volume) expansion (γ) : Consider metal cube of volume V_0 .

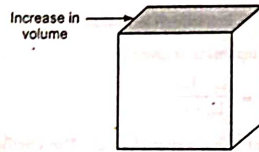


Fig. 3.5

Let V_0 = Original volume at 0°C
 V_t = New volume at $t^\circ\text{C}$
 $(V_t - V_0)$ = Increase in volume
 $(t - 0)$ = Increase in temperature

Then, coefficient of cubical expansion γ is given by

$$\gamma = \frac{(V_t - V_0)}{V_0 t}$$

Definition : The coefficient of cubical expansion ' γ ' of a material is defined as increase in volume per unit original volume at 0°C , per unit increase in temperature.

Unit of γ is $^\circ\text{C}^{-1}$.

(d) Relation between α , β , and γ : We have seen α , β and γ . For a given material coefficient of linear expansion ' α ' is always constant. Similarly, for a given material, ' β ' and ' γ ' are constant. Also it is observed that for a given material, α , β and γ are interrelated with each other.

For a given material (e.g. aluminium),

β is twice of $\alpha \rightarrow (\beta = 2\alpha)$

and γ is thrice of $\alpha \rightarrow (\gamma = 3\alpha)$

Thus for a given material, relation between α , β and γ is as under :

Ratio $\alpha : \beta : \gamma$ is 1 : 2 : 3.

i.e. $\alpha : \beta : \gamma$
 1 : 2 : 3

SUMMARY

- Conduction is a process of transfer of heat from a part of body at higher temperature to a part of body at lower temperature without bodily (actual) movement of particles.
- Convection is a process of transfer of heat from a part of body at higher temperature to a part of body at lower temperature with bodily (actual) movement of particles.
- Radiation is the process of transfer of heat in which heat is transferred from one place to other directly without the necessity of intervening medium.
- Amount of heat flowing through a material is directly proportional to (1) its cross-sectional area, (2) temperature difference, (3) time and inversely proportional to distance between its ends.
- Coefficient of thermal conductivity ' K ' of material is defined as heat conducted in one second in steady state through unit cross-sectional area for unit temperature gradient.
- Temperature gradient is defined as change in temperature per unit length of rod.
- Coefficient of linear expansion ' α ' of material is defined as increase in length per unit original length at 0°C , per unit increase in temperature.
- Coefficient of aerial expansion ' β ' of material is defined as increase in area per unit original area at 0°C per unit increase in temperature.
- Coefficient of cubical expansion ' γ ' of material is defined as increase in volume per unit original volume at 0°C , per unit increase in temperature.
- For a given material, the ratio of $\alpha : \beta : \gamma$ is 1 : 2 : 3.
- Small gap is kept between two consecutive rails for providing scope for expansion.
- A number of small gaps are kept in a long concrete bridge for providing scope for expansion.

IMPORTANT FORMULAE

1. $Q = \frac{KA(\theta_1 - \theta_2)t}{d}$ where, Q = Amount of heat conducted
 A = Cross-sectional area
 d = Thickness of material

2. Temperature gradient = $\frac{(\theta_1 - \theta_2)}{d}$ where,

θ_1 and θ_2 = Temperatures of two faces or
 d = Distance between two
 (thermometers) faces

3. $\alpha = \frac{(L_1 - L_0)}{L_0 t}$ where,

α = Coefficient of linear expansion

$\beta = \frac{(A_1 - A_0)}{A_0 t}$

β = Coefficient of aerial expansion

$\gamma = \frac{(V_1 - V_0)}{V_0 t}$

γ = Coefficient of volume expansion

L_0, A_0, V_0 - Length, area, volume at 0°C

L_1, A_1, V_1 - Length, area, volume at $t^\circ\text{C}$

4. $\beta/\alpha = 2$

$\gamma/\alpha = 3$

or $\alpha : \beta : \gamma = 1 : 2 : 3$

SOMETHING TO BE NOTICED (RESEMBLANCE) :

See the similarity between information and definitions of the following with the pressure coefficient of expansion :

Pressure coefficient of expansion,

Volume coefficient of expansion,

Linear coefficient of expansion,

Aerial coefficient of expansion,

SOLVED EXAMPLES

EXAMPLES ON CONDUCTIVITY :

Example 1 : Find the quantity of heat conducted in 5 minutes across a silver sheet of size $40\text{ cm} \times 30\text{ cm}$ of thickness 3 mm . If its two faces are at temperatures of 40°C and 25°C . K for silver = $0.1\text{ kcal/m}^\circ\text{C s}$. (W. 2001 - 4 M)

Solution : Given :

$Q = ?$

$t = 5\text{ min} = (5 \times 60)\text{ sec}$

$A = 40\text{ cm} \times 30\text{ cm}$

$= 1200\text{ cm}^2$

$= 0.12\text{ m}^2$

$d = 3\text{ mm}$

$= 3 \times 10^{-3}\text{ m}$

$(\theta_1 - \theta_2) = (40 - 25) = 15^\circ\text{C}$

We have,

$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$

$= \frac{(0.1)(0.12)(15)(5 \times 60)}{(3 \times 10^{-3})}$

$Q = 18000\text{ kcal}$

[Note : Here unit of K is given in $\text{kcal/m}^\circ\text{C sec}$. Therefore, answer of ' Q ' will be in kcal .]

Example 2 : A nickel plate of thickness 4 mm has temperature difference of 32°C between its faces. It transmits 200 kcal per hour through an area of 5 cm^2 . Calculate the conductivity of nickel. (S. 2006 - 4 M)

Solution : Given :

$d = 4\text{ mm}$

$= 4 \times 10^{-3}\text{ m}$

$(\theta_1 - \theta_2) = 32^\circ\text{C}$

$Q = 200\text{ kcal}$

$A = 5\text{ cm}^2$

$= 5 \times 10^{-4}\text{ m}^2$

$K = ?$

$t = 1\text{ hr}$

$= (60 \times 60)\text{ sec}$

We have,

$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$

$K = \frac{Q \cdot d}{A(\theta_1 - \theta_2)t}$

$= \frac{(200)(4 \times 10^{-3})}{(5 \times 10^{-4})(32)(60 \times 60)}$

$K = 0.0139\text{ kcal/m}^\circ\text{C sec}$

Example 3 : A metal rod of length 0.20 m has one of its ends at 20°C , while the other is at 50°C . Find the temperature gradient. (S. 2002 - 2 M)

Solution : Given :

$d = 0.20\text{ m}$ (i.e. distance between two thermometers)

$\theta_1 = 20^\circ\text{C}$

$\theta_2 = 50^\circ\text{C}$

Temperature gradient = ?

$$\text{Temperature gradient} = \frac{(\theta_2 - \theta_1)}{d} = \frac{50 - 20}{0.2}$$

$$\text{Temperature gradient} = 150^\circ\text{C/m}$$

Example 4 : If the thickness of the plate is 8 cm and temperature of the two faces are 100°C and -20°C, find the temperature gradient. (Dec. 2010)

Solution : Temperature difference = $\theta_2 - \theta_1 = (100 - (-20)) = 120^\circ\text{C}$

Distance between two points at which temperature difference is known = thickness = 8 cm

We have

$$\text{Temperature gradient} = \frac{\text{Temperature difference}}{\text{Distance}} = \frac{120^\circ\text{C}}{8 \text{ cm}}$$

$$\text{Temperature gradient} = 15^\circ\text{C/cm}$$

Example 5 : A copper rod 19 cm long and area of cross-section 0.79 cm², which is thermally insulated, is heated at one end to a temperature of 100°C while the other end is kept at 30°C. Calculate the amount of heat which will flow in 5 min. along the rod if K for copper is 380 W/m²K. (S. 2002 - 4 M)

Solution : Given : d = 19 cm (length of rod or distance between two thermometers) = 0.19 m

$$A = 0.79 \text{ cm}^2 = 0.79 \times 10^{-4} \text{ m}^2$$

$$\theta_1 = 100^\circ\text{C}$$

$$\theta_2 = 30^\circ\text{C}$$

$$Q = ?$$

$$t = 5 \text{ min} = 5 \times 60 = 300 \text{ sec.}$$

$$K = 380 \text{ W/m}^2\text{K}$$

We have,
$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} = \frac{380 \times 0.79 \times 10^{-4} \times (100 - 30) \times 300}{0.19}$$

$$Q = 3318 \text{ J}$$

OR
$$Q = \frac{3318}{4.2}$$

$$Q = 790 \text{ cal.}$$

Example 6 : The inside and outside temperatures of a glass window pan are 24°C and 18°C respectively. The window is 3 mm thick. Find its area if 50 kcal of heat escapes in a minute. (W. 2003 - 4 M)

Solution : Given : $(\theta_1 - \theta_2) = (24 - 18)^\circ\text{C}$
d = 3 mm

$$A = ?$$

$$Q = 50 \text{ kcal}$$

$$t = 1 \text{ minute} = 60 \text{ sec.}$$

Note : There is a discrepancy in the question paper, value of 'K' is not given.

K for glass is not given. $Q = \frac{KA(\theta_1 - \theta_2)t}{d}$

$$\therefore A = \frac{Q \times d}{K(\theta_1 - \theta_2)t}$$

$$\therefore A = \frac{(50)(3 \times 10^{-3})}{K(24 - 18)(60)}$$

$$\therefore A = \frac{4.16 \times 10^{-4}}{K} \text{ m}^2$$

Example 7 : Find the amount of heat conducted in one hour by a window pane 60 cm x 30 cm and thickness 3 mm, if the difference between temperatures is 5°C and K = 0.0002 kcal/m²°C sec.

Solution : Given :

$$Q = ?$$

$$t = 1 \text{ hr} = 60 \times 60 \text{ sec}$$

$$A = 60 \text{ cm} \times 30 \text{ cm} = 0.6 \times 0.3 \text{ m}^2 = 0.18 \text{ m}^2$$

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$(\theta_1 - \theta_2) = 5^\circ\text{C}$$

We have,

$$Q = \frac{KA(\theta_1 - \theta_2)t}{d} = \frac{0.0002 \times 0.18 \times (5) \times 60 \times 60}{3 \times 10^{-3}}$$

$$\therefore Q = 216 \text{ kcal}$$

Example 8 : Heat is conducted through a slab of two layers of different metals of conductivities 0.1 and 0.15 CGS units respectively. The thickness of the layers is 4 cm and 6 cm respectively. If the temperatures of the outer faces are 100°C and 0°C respectively, find the temperature of the interface.

Solution :

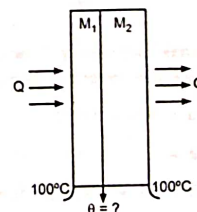


Fig. 3.6

Handwritten calculations for Example 8:

$$Q = \frac{K_1 A (\theta_1 - \theta_2)}{d_1} = \frac{K_2 A (\theta_2 - \theta_3)}{d_2}$$

$$\frac{0.1 \times A \times (100 - \theta)}{4} = \frac{0.15 \times A \times (\theta - 0)}{6}$$

$$0.1 \times (100 - \theta) = 0.15 \times \theta$$

$$100 - \theta = 1.5 \theta$$

$$100 = 2.5 \theta$$

$$\theta = \frac{100}{2.5} = 40^\circ\text{C}$$

Metal 1
 $K_1 = 0.1$ CGS
 $d_1 = 4$ cm

Temperature difference between its faces
 $= (100 - \theta)^\circ\text{C}$

Since they are in contact, cross-sectional area of them will be equal.

\therefore Heat conducted through M_1 is

$$Q = \frac{K_1 A (100 - \theta) t}{d_1}$$

and heat conducted through M_2 is

$$Q = \frac{K_2 A (\theta - 0) t}{d_2}$$

But heat conducted through two metals is same because layers are in contact.

$$\frac{K_1 A (100 - \theta) t}{d_1} = \frac{K_2 A (\theta - 0) t}{d_2}$$

$$\frac{K_1 (100 - \theta)}{d_1} = \frac{K_2 (\theta - 0)}{d_2}$$

$$\frac{(0.1) \times (100 - \theta)}{4} = \frac{(0.15) \times (\theta - 0)}{6}$$

$$(6) \times (0.1) \times (100 - \theta) = (4) \times (0.15) \times (\theta - 0)$$

$$60 - 0.6\theta = 0.6\theta$$

$$60 = 1.2\theta$$

$$\theta = \frac{60}{1.2} = 50^\circ\text{C}$$

\therefore Temperature of interface is $\theta = 50^\circ\text{C}$

EXAMPLES ON EXPANSION :

Example 9 : A thin rod of iron has coefficient of linear expansion $\alpha = 0.12 \times 10^{-4}/^\circ\text{C}$. Calculate the coefficient of superficial expansion of thin sheet of iron.

Solution : Given : $\alpha = 0.12 \times 10^{-4}/^\circ\text{C}$

$\beta = ?$

We have,

$$\beta = 2\alpha = 2 \times 0.12 \times 10^{-4}/^\circ\text{C}$$

$$\beta = 0.24 \times 10^{-4}/^\circ\text{C}$$

Example 10 : The length of metal rod at 15°C is 50 cm and at 90°C is 50.15 cm. Calculate the coefficient of linear expansion. (S. 2001 - 4 M)

Solution : $L_1 = 50$ cm, $t_1 = 15^\circ\text{C}$, $L_2 = 50.15$ cm, $t_2 = 90^\circ\text{C}$

$$(L_2 - L_1) = (50.15 - 50) = 0.15 \text{ cm}$$

$$(t_2 - t_1) = 90 - 15 = 75^\circ\text{C}$$

$$(L_2 - L_1) \propto L_1$$

$$(L_2 - L_1) \propto (t_2 - t_1)$$

$$(L_2 - L_1) \propto L_1 (t_2 - t_1)$$

$$(L_2 - L_1) = \alpha L_1 (t_2 - t_1)$$

$$\alpha = \frac{(L_2 - L_1)}{L_1 (t_2 - t_1)} = \frac{(0.15)}{50 (75)}$$

Coefficient of linear expansion, $\alpha = 4 \times 10^{-5}/^\circ\text{C}$

Example 11 : The length of copper rod at 0°C is 90 cm. When heated to 100°C , it increases by 0.14 cm. Find the coefficient of linear expansion. (S. 2005 - 4 M)

Solution : Given :

$$L_0 = 90 \text{ cm at } 0^\circ\text{C}$$

$$(L_t - L_0) = 0.14 \text{ cm}$$

$$L_t = 90.14 \text{ cm at } t = 100^\circ\text{C}$$

$$\alpha = ?$$

$$\alpha = \frac{(L_t - L_0)}{L_0 t}$$

$$= \frac{0.14}{90 \times 100}$$

$$= \frac{0.14}{9000}$$

$$\alpha = 1.56 \times 10^{-5}/^\circ\text{C}$$

Example 12 : A sheet of copper has area 80 cm^2 at 50°C . What will be its area when heated to 90°C ? The coefficient of linear expansion of copper is $16 \times 10^{-6}/^\circ\text{K}$. (S. 2003 - 4 M)

Solution :

$$A_1 = 80 \text{ cm}^2 \quad \dots \text{ at } t_1 = 50^\circ\text{C}$$

$$A_2 = ? \quad \dots \text{ at } t_2 = 90^\circ\text{C}$$

Coefficient of linear expansion,

$$\alpha = 16 \times 10^{-6}/^\circ\text{K}$$

∴ Coefficient of aerial expansion,

$$\beta = 2\alpha = 2 \times 16 \times 10^{-6} / ^\circ\text{C}$$

$$\beta = \frac{(A_2 - A_1)}{A_1 (t_2 - t_1)}$$

$$2 \times 16 \times 10^{-6} = \frac{(A_2 - 80)}{80 (90 - 50)}$$

$$0.1024 = (A_2 - 80)$$

$$A_2 = 80 + 0.1024$$

$$A_2 = 80.1024 \text{ cm}^2$$

Example 13 : A copper sheet at 30°C is heated to 80°C , its area becomes 60.09 cm^2 . What is its initial area? Coefficient of linear expansion of copper is 16×10^{-6} per $^\circ\text{C}$.

Solution : Area is $A_1 = ?$ at $t_1 = 30^\circ\text{C}$

$$A_2 = 60.09 \text{ cm}^2 \text{ at } t_2 = 80^\circ\text{C}$$

Coefficient of linear expansion, $\alpha = 16 \times 10^{-6} / ^\circ\text{C}$

∴ Coefficient of aerial expansion,

$$\beta = 2\alpha = 2 \times 16 \times 10^{-6} / ^\circ\text{C}$$

We have,

$$(A_2 - A_1) = \beta \times A_1 \times (t_2 - t_1)$$

∴

$$A_2 = A_1 + \beta A_1 (t_2 - t_1)$$

$$A_2 = A_1 [1 + \beta (t_2 - t_1)]$$

∴

$$A_1 = \frac{A_2}{[1 + \beta (t_2 - t_1)]}$$

$$= \frac{60.09}{[1 + (2 \times 16 \times 10^{-6}) \times (50)]}$$

$$A_1 = 59.99 \text{ cm}^2 = 60 \text{ cm}^2$$

Example 14 : The length of an aluminium rod is 2 m at room temperature. When the rod is heated to 120°C , its length increases by 0.5 cm . Find the room temperature. (W. 2002-4)

($\alpha = 25 \times 10^{-6} / ^\circ\text{C}$).

Solution : Given :

$$L_1 = 2 \text{ m} = 200 \text{ cm} \dots \text{ at } t_1 = ? \text{ (room temperature)}$$

$$L_2 = 2 \text{ m} + 0.5 \text{ cm} \dots \text{ at } t_2 = 120^\circ\text{C}$$

$$= 200.5 \text{ cm}$$

$$\alpha = 25 \times 10^{-6} / ^\circ\text{C}$$

$$\alpha = \frac{L_2 - L_1}{L_1 (t_2 - t_1)}$$

$$25 \times 10^{-6} = \frac{200.5 - 200}{200 (120 - t_1)}$$

$$(120 - t_1) = \frac{200.5 - 200}{200 \times (25 \times 10^{-6})}$$

$$(120 - t_1) = 100$$

$$(120 - 100) = t_1$$

$$t_1 = 20^\circ\text{C}$$

Example 15 : The coefficient of cubical expansion of a metal is $46 \times 10^{-6} / ^\circ\text{C}$. Find the coefficient of aerial expansion.

Solution : Given :

$$\gamma = 46 \times 10^{-6} / ^\circ\text{C}, \beta = ?$$

We have $\beta : \gamma$ is $2 : 3$.

i.e.

$$\frac{\beta}{\gamma} = \frac{2}{3}$$

$$\beta = \frac{2}{3} \times \gamma = \frac{2}{3} \times (46 \times 10^{-6})$$

$$\beta = 30.66 \times 10^{-6} / ^\circ\text{C}$$

Example 16 : How much gap should be left in between the two rails if each rail is 14 m long and the maximum rise in temperature is 25°C ? The coefficient of linear expansion of material is $20 \times 10^{-6} / ^\circ\text{C}$.

Solution : Given :

$$L_0 = 14 \text{ m}$$

$$(t - 0) = 25^\circ\text{C}$$

$$\alpha = 20 \times 10^{-6} / ^\circ\text{C}$$

$$\text{Gap} = (L_t - L_0) = ?$$

We have,

$$\alpha = \frac{(L_t - L_0)}{L_0 (t - 0)}$$

∴

$$\alpha \times L_0 \times t = (L_t - L_0)$$

∴

$$(L_t - L_0) = \alpha \times L_0 \times t$$

$$= 20 \times 10^{-6} \times 14 \times 25$$

$$= 70000 \times 10^{-6}$$

$$= 0.007 \text{ m}$$

∴

$$\text{Gap should be } 0.007 \text{ m}$$

Example 17 : An iron rod has length of 90 cm at 20°C temperature. What would be its length at 80°C? Coefficient of linear expansion, $\alpha = 0.000012$ °C.

Solution : Given :

$$l_1 = 90 \text{ cm}, \quad t_1 = 20^\circ\text{C}$$

$$l_2 = ?, \quad t_2 = 80^\circ\text{C}$$

We have,

$$\alpha = \frac{l_2 - l_1}{l_1 \times (t_2 - t_1)}$$

$$1.2 \times 10^{-5} = \frac{l_2 - 90}{90 \times (80 - 20)}$$

$$l_2 = (1.2 \times 10^{-5} \times 90 \times 60) + 90$$

$$l_2 = 90.0648 \text{ cm}$$

Example 18 : The length of an aluminium rod is 2 metre at room temperature. When the rod is heated at 125°C, its length increases by 0.5 cm. Find the room temperature. The coefficient of linear expansion of aluminium is 25×10^{-6} °C.

Solution : Given :

Room temperature, $t_1 = ?$

$$L_1 = 2 \text{ m}$$

Final

$$t_2 = 125^\circ\text{C}$$

Let length be $L_2 = (L_2 - L_1)$

$$L_2 - L_1 = 0.5 \text{ cm}$$

$$L_2 - L_1 = 0.005 \text{ m}$$

Coefficient of linear expansion,

$$\alpha = 25 \times 10^{-6} \text{ }^\circ\text{C}$$

$$\alpha = \frac{(L_2 - L_1)}{L_1 (t_2 - t_1)}$$

$$(t_2 - t_1) = \frac{(L_2 - L_1)}{L_1 \alpha}$$

$$t_2 = t_1 + \frac{(L_2 - L_1)}{L_1 \alpha}$$

$$t_1 = t_2 - \frac{(L_2 - L_1)}{L_1 \alpha}$$

$$t_1 = 125 - \frac{(0.005)}{(2) (25 \times 10^{-6})}$$

$$= 125 - 100$$

$$t_1 = 25^\circ\text{C}$$

$$\therefore \text{Room temperature} = 25^\circ\text{C}$$

QUESTIONS

1. Explain conduction of heat on the basis of molecular theory.
2. Define temperature gradient and mention its unit.
3. Define coefficient of thermal conductivity of a material. State its unit and dimensions.
4. Define :
 - (a) Coefficient of linear expansion,
 - (b) Coefficient of aerial expansion,
 - (c) Coefficient of cubical expansion.
5. State the factors affecting conduction of heat and state relation between them.
6. Define coefficient of cubical expansion of solid. Give its S.I. units.
7. Define temperature gradient. State its unit.
8. Explain the modes of transfer of heat and define temperature gradient.
9. State the relation between coefficient of linear expansion, aerial expansion and cubical expansion of solid.
10. State the law of thermal conductivity of heat and define coefficient of thermal conductivity. State its S.I. units.
11. "While laying railway lines, small spaces are left between consecutive rails." Give reason.

PROBLEMS

1. A glass sheet of area 1 m^2 has thickness 2 mm. Its opposite faces are at 35°C and 20°C respectively. If coefficient of thermal conductivity of glass is $0.2 \text{ cal/m }^\circ\text{C sec}$, find the quantity of heat conducted in half hour.

$$\text{Ans. } Q = 27 \times 10^5 \text{ cal.}$$

2. Find out the temperature difference between the two faces of a metal of thickness 50 mm through every square meter of which heat is streaming from a furnace on one side to boiling water on the other side at the rate of 140 kcal per minute. (K of metal is $15 \text{ cal/m} \cdot \text{s }^\circ\text{C}$)

$$\text{Ans. } (\theta_1 - \theta_2) = 7.8^\circ\text{C.}$$

3. A rectangular brass plate of area 120 cm^2 and of thickness 8 mm has its opposite faces maintained at 100°C and 70°C respectively. How much heat will flow through the plate in one minute? (K for brass = $109.2 \text{ W/m}^\circ\text{K}$)
 Ans. $2.95 \times 10^5 \text{ J}$
4. Heat is conducted through a composite slab of two different metals, having thermal conductivities 0.3 and 0.6 MKS units and thickness of 4 cm and 6 cm respectively. If the outer faces of the slab are at 101°C and 59°C respectively, find the temperature of the interface.
 Ans. 77°C
5. Find the quantity of heat conducted in 10 minutes across a silver sheet of size $40 \text{ cm} \times 30 \text{ cm}$, thickness 6 mm , if its two faces are at temperatures 40°C and 25°C . K for silver is $0.1 \text{ kcal/m}^\circ\text{C sec}$.
 Ans. $Q = 18000 \text{ kcal}$ (S. 1997 - 4 M)
6. The thickness of plate is 10 cm . The temperatures of the two faces of element are 100°C and 60°C . Find the temperature gradient.
 Ans. 4°C/cm (W. 2004 - 2 M)
7. How much heat will be conducted in one hour through a layer of an ice 5 cm thick covering a pool of area 20 m^2 , if water below it is at 0°C and air above it is at -10°C ? K for ice is $0.0005 \text{ kcal/m}^\circ\text{C sec}$.
 Ans. $Q = 7200 \text{ kcal}$ (S. 1990 - 4 M)
8. The thickness of the plate is 8 cm . The temperatures of the two faces are 100°C and -20°C . Find the temperature gradient.
 Ans. 15°C/cm
9. A window pane is $100 \text{ cm} \times 30 \text{ cm}$. Its thickness is 3 mm . If the difference between inside and outside temperatures of the window is 5°K , find the amount of heat conducted in one hour. (Given : $K = 0.238 \text{ cal/m}^\circ\text{Cs}$)
 Ans. $Q = 4.286 \times 10^5 \text{ cal}$ (S. 1985 - 4 M)
10. Calculate the coefficient of linear expansion of material of rod if its length is 1 m at 0°C and 1.09 m at 65°C .
 Ans. $1.384 \times 10^{-3}/^\circ\text{C}$
11. A thin rod of copper has coefficient of linear expansion $\alpha = 1.56 \times 10^{-5}/^\circ\text{C}$. Calculate the coefficient of superficial expansion of thin sheet of copper.
 Ans. $3.12 \times 10^{-5}/^\circ\text{C}$
12. A sheet of copper has an area 30 cm^2 at 0°C . Find its area if temperature rises to 50°C . The coefficient of linear expansion of copper is $16 \times 10^{-6}/^\circ\text{C}$.
 [Hint : Use $\beta = 2\alpha$ in the formula]
 Ans. 30.048 cm^2
13. The length of copper rod at 10°C is 80 cm . When heated to 100°C , it increases by 0.12 cm . Find the coefficient of linear expansion of copper.
 Ans. $\alpha = 1.66 \times 10^{-5}/^\circ\text{C}$

3.2 GAS LAWS AND SPECIFIC HEAT OF GASES

Introduction, Boyle's law, Charles' law, Gay Lussac's law, gas equation, universal gas constant (molar constant), Normal (standard) temperature, pressure, absolute zero, absolute scale of temperature, specific heat of gases, relation between C_p and C_v , thermodynamic variable, first law of thermodynamics, isothermal process, isobaric process, isochoric process, adiabatic process.

3.2.1 Introduction

Compare solids, liquids and gases. In case of *solids*, the molecules are closely bound to each other by intramolecular forces of attraction. In case of *liquids*, distance between the molecules is comparatively more. In case of *gases*, distance between the molecules is very large and hence there is very small (negligible) force of attraction between the gas molecules.

Therefore, if a very large pressure is applied on solids and liquids, then there is a very small change in volume. But in case of *gases*, even a small pressure is applied on it, then there is appreciably more change in volume.

In short, gases are very much sensitive to changes in pressure, volume and temperature. Hence, it is very necessary to study the relation between pressure, volume and temperature of gases. This relation between P , V and T is given by gas laws.

Before going into the details of gas laws, we must know about pressure. All the gas molecules are free to move in all possible directions. While moving, they collide on each other and also on walls and piston of container, which creates pressure. The number of hits in one second determines the pressure. The temperature of gas determines the speed of gas molecules in motion. If the temperature of gas is kept constant, then the speed of gas molecules remains constant.

3.2.2 Gas Laws

Boyle's Law :

Consider a gas enclosed in a cylinder. Gas molecules are free to move and they collide on walls of container and on piston which creates pressure.

As shown in Fig. 3.7 in the 1st position, distance ' d_1 ' to be covered for molecules is more. Therefore number of hits in one second are less i.e. pressure is less. But in 2nd position, as piston is pushed in, the *volume* of gas decreases and distance ' d_2 ' to be covered by molecules decreases. Temperature remaining constant, speed of molecules is also constant. Thus, now

speed of molecules is same but distance ' d_2 ' to be covered is less. Therefore, number of hits in one second increases i.e. pressure increases. Thus, ' V ' decreases, and ' P ' increases.

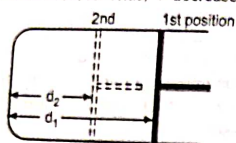


Fig. 3.7

Thus, at constant temperature, pressure of gas is inversely proportional to its volume.

Statement of Boyle's law : Boyle's law states that for fixed mass of a gas, temperature of a gas remaining constant, its pressure is inversely proportional to its volume.

$$\text{i.e. } P \propto \frac{1}{V} \quad (\text{at constant temperature})$$

$$\therefore P = \text{constant} \times \frac{1}{V}$$

$$\therefore P \cdot V = \text{constant}$$

Thus, at constant temperature, product of pressure and volume of fixed mass of gas remains constant.

$$\text{i.e. } P_1 \cdot V_1 = P_2 \cdot V_2 = \text{constant}$$

i.e. product of old pressure and old volume is equal to product of new pressure and new volume.

Charles' Law :

We know that when a gas is heated, it expands.

Statement of Charles' law : Charles' law states that for fixed mass of a gas, pressure of gas remaining constant, its volume is directly proportional to the absolute temperature.

$$\text{Thus, } V \propto T \quad (\text{at constant pressure})$$

Above effect can be observed, if gas enclosed in cylinder with movable piston is heated. It is seen that as temperature increases, the number of hits in one second increases and thus piston gets pushed out and pressure remains same because of motion of piston. Thus, pressure remaining same, volume increases with temperature. Thus, if pressure of fixed mass of gas is constant, then its volume is directly proportional to its absolute temperature.

Gay Lussac's Law :

Piston of cylinder containing gas is not allowed to move i.e. volume is kept constant and if gas is heated, then the molecules inside it start moving with more speed. Therefore, number of hits in one second increases i.e. pressure increases.

Thus, as temperature of gas increases, its pressure also increases at constant volume.

Statement of Gay Lussac's law : Gay Lussac's law states that, for fixed mass of a gas, volume of a gas remaining constant, its pressure is directly proportional to the absolute temperature.

$$\text{i.e. } P \propto T \quad (\text{as constant volume})$$

3.2.3 General Gas Equation and Specific and Universal Gas Constant

We have seen that for gases

$$\text{According to Gay Lussac's law, } P \propto T \quad \dots (3.2)$$

$$\text{According to Charles' law, } V \propto T \quad \dots (3.3)$$

Combining equations (3.2) and (3.3), we can write,

$$PV \propto T$$

$$PV = \text{Constant} \times T$$

If mass of gas is measured in kg or gm then constant used is 'K'.

$$PV = K \times T$$

Ideal gas equation ... (3.4)

where, K is a gas constant, dependent on mass of a gas. Equation (3.4) is called as Ideal gas equation.

For 1 kg of gas, K is called specific gas constant. If mass of gas is 1 kg-mole or 1 gram-mole, then constant K will have same value for all gases. Therefore, K is replaced by R called universal gas constant.

$$PV = RT$$

General gas equation ... (3.5)

where, R is Universal gas constant. Equation (3.5) is called General gas equation.

One gram-mole : The mass of 1 gram-mole is different for different gases.

e.g. Consider Cl_2 molecule, its molecular weight is 71.1 gm, therefore 71 gm of Cl_2 means 1 gram-mole of Cl_2 .

Similarly, 32 gm of O_2 means 1 gram-mole of O_2 .

Molar Constant or Universal Gas Constant :

We have, $PV = \text{Constant} \times T$

For 1 gm-mole of a gas, above equation becomes,

$$PV = RT$$

where R is Universal gas constant. R is called universal gas constant because value of R is same for all gases. The reason is that one gram molecule of any gas occupies same volume under NTP condition.

Thus value of 'R' is same for all gases and $R = 8314.91$ S.I. units.

Unit of 'R' : $PV = RT$

$$\therefore \frac{PV}{T} = R$$

$$\therefore R = \frac{(P)(V)}{(T)} = \frac{(N/m^2)(m^3/kg \cdot \text{mole})}{(^{\circ}K)}$$

$$= N \cdot m^2 / K \cdot \text{kg} \cdot \text{mole} \text{ Or } J^{\circ}K \text{ kg-mole}$$

Thus, $R = 8314.91$ J/ $^{\circ}K$ kg-mole and unit of K is J/kg $^{\circ}K$.

It is observed that the product of the molecular weight (M) and the characteristic gas constant (R) is constant for all gases and this product $M \times R$ denoted by R is called molar constant or universal gas constant.

A universal gas constant 'R' can be defined as a constant 8314.91 J/ $^{\circ}K$ kg mole equal to product PV of 1 kg molecule of an ideal gas divided by absolute temperature T.

3.2.4 Standard or Normal Temperature and Pressure (NTP or STP)

Standard or normal pressure denotes a pressure exerted by a column of mercury exactly 76 cm high at 0 $^{\circ}C$ and is equal to the average pressure of the atmosphere at sea level and is called one atmosphere. STP or NTP denotes a temperature of 0 $^{\circ}C$ or 273 $^{\circ}K$ and pressure of one atmosphere.

i.e. under NTP condition, temperature = 0 $^{\circ}C$ or 273 $^{\circ}K$ and pressure = 76 cm of Hg or 1 atmospheric pressure.

3.2.5 Concept of Absolute Zero and Absolute Scale of Temperature (May 2011)

If we go on cooling a gas then its pressure and volume decreases. It is observed that at -273 $^{\circ}C$, pressure and volume of a gas theoretically become zero.

i.e. -273 $^{\circ}C$ is minimum possible temperature for all gases.

This temperature at which pressure and volume of gas (theoretically) become zero is called absolute zero temperature,

i.e. -273 $^{\circ}C = 0^{\circ}A = 0^{\circ}K$

Absolute Scale of Temperature (Kelvin Scale of Temperature) :

In normal degree celsius scale, the starting point is 0 $^{\circ}C$. But for gases we have seen that -273 $^{\circ}C$ is the minimum possible temperature. Therefore, a new scale which starts from -273 $^{\circ}C$ is called absolute or Kelvin scale of temperature.

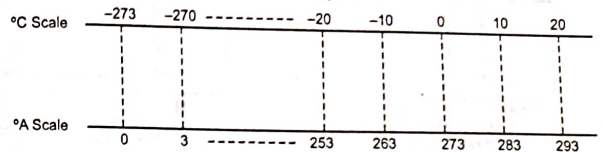


Fig. 3.8

Thus, $x^{\circ}C = (x + 273)^{\circ}A$ or $= (x + 273)^{\circ}K$

3.2.6 Two Specific Heats of Gas and Relation Between Them

Specific heat of a gas (C) : It is the amount of heat required to increase the temperature of 1 kg of a gas by 1 $^{\circ}C$.

Specific heat of a gas at constant volume (C_v) : It is the amount of heat required to increase the temperature of 1 kg of a gas by 1 $^{\circ}C$ at constant volume.

In this case, whatever may be heat supplied is used to increase only temperature of gas because volume of gas is kept constant.

Specific heat of a gas at constant pressure (C_p) : It is the amount of heat required to increase the temperature of 1 kg of a gas by 1 $^{\circ}C$ at constant pressure (Refer Fig. 3.9)

In this condition, pressure is kept constant but volume of gas increases i.e. piston is pushed out. Some work is done in pushing this piston. Thus heat supplied under this condition is used to increase the temperature of gas and also to increase the volume of a gas. Thus in the case of C_p, some additional heat is to be supplied which is being used for expansion.

Thus $C_p > C_v$

Specific heat of a gas at constant pressure is always greater than specific heat at constant volume.

Gases are very much sensitive for their changes in pressure, volume and temperature. If any one parameter changes, then there is a considerable change in the other two parameters. Hence gases have two specific heats. i.e. specific heat at constant pressure and specific heat at constant volume.

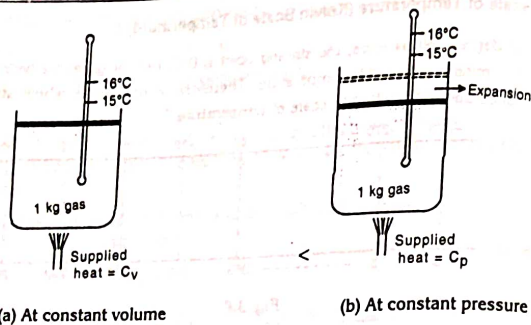


Fig. 3.9

RELATION BETWEEN C_p AND C_v OR MAYOR'S RELATION :

' C_v ' is the amount of heat required to increase the temperature of 1 kg of a gas by 1 °C at constant volume.

Consider 1 kg of a gas heated at constant volume [Refer Fig. 3.9 (a)]. In this case, since the volume of a gas is kept constant, the heat supplied is used to increase the temperature only.

' C_p ' is the amount of heat required to increase the temperature of 1 kg of a gas by 1 °C at constant pressure.

Consider 1 kg of a gas heated at constant pressure [Refer Fig. 3.9 (b)]. In this case, pressure of a gas is constant but volume increases. Therefore heat supplied in this case is used to

1. increase the temperature and
2. increase the volume of a gas.

Thus in case of C_p , some additional heat is required for expansion.

$$C_p = C_v + \text{heat required to increase the volume of a gas}$$

$$C_p = C_v + H$$

$$\therefore C_p - C_v = H$$

Relation between C_p and C_v is given by

$$C_p - C_v = \frac{R}{J}$$

where R = universal gas constant when mass of gas is 1 kg molé.

But here 1 kg of gas is considered.

$$\therefore C_p - C_v = \frac{R}{MJ}$$

where M – Molecular weight

or
$$C_p - C_v = \frac{R'}{J}$$

where $R' = \frac{R}{M}$

Its unit is J/kg °K.

Ratio of specific heats :

We have seen that for a particular gas, C_p and C_v are always constant. Also C_p is greater than C_v .

$\therefore \frac{C_p}{C_v}$ is also constant.

$$\frac{C_p}{C_v} = \gamma \dots \text{constant}$$

$$\frac{C_p}{C_v} > 1 \text{ because } C_p > C_v$$

$$\therefore \gamma > 1$$

- Value of γ
- = 1.66 for monoatomic gases
 - = 1.41 for diatomic gases
 - = 1.31 for triatomic gases

3.2.7 Isothermal and Adiabatic Changes (Expansion) of Gas

Isothermal changes : If volume of gas is changed keeping its temperature constant, then change is called isothermal change.

Such changes can be achieved by taking gas in container which is good conductor of heat and changing the volume very slowly. Because of good conductivity of container, inside heat can be given to the surroundings and also slow changes allow sufficient time for exchange of heat. Thus exchange of heat takes place and temperature remains constant.

Here we can use Boyle's law relation i.e. $PV = \text{Constant}$.

Since temperature is constant, $P_1V_1 = P_2V_2$

e.g. (1) Melting of solid, (2) Boiling of water.

Adiabatic changes : If volume of gas is changed such that its temperature also changes, then the changes are called adiabatic changes.

Such changes can be achieved by taking gas in container of bad conductivity of heat and changing the volume rapidly. Because of bad conductivity of container, heat will not be given to the surroundings and also rapid changes do not allow sufficient time for exchange of heat. Thus exchange of heat does not take place and therefore temperature of gas changes.

Here we cannot use the relation $PV = \text{constant}$ because temperature of gas is not constant.

New relation we can use is $PV^{\gamma} = \text{constant}$.

e.g. Bursting of cycle rubber tube.

Isothermal expansion : If the volume of gas increases, its temperature may decrease. If temperature decreases, the heat is given to the gas from surroundings. If the volume decreases, temperature increases. Then the gas gives heat to the surroundings and temperature of gas remains constant. Such changes in volume of gas are called isothermal changes.

Definition : Isothermal expansion is defined as expansion of a gas while its temperature remains constant.

The exchange of heat by the gas with the surroundings is slow, therefore, isothermal changes are slow. Boyle's law relation is valid i.e. $PV = \text{constant}$ because temperature is constant.

Adiabatic expansion : Now the gas can be thermally insulated from the surroundings. The exchange of heat is not possible. Therefore, the temperature of gas changes with changes in the volume. Such changes in volume of gas are adiabatic changes.

Definition : Adiabatic expansion is defined as expansion of gas, while its temperature changes.

Boyle's law relation is not valid. Since temperature is changing, new valid relation is $PV^{\gamma} = \text{constant}$.

SUMMARY

- Pressure and volume of a gas becomes zero at -273°C .
- The temperature at which volume and pressure of a gas becomes zero is called absolute zero temperature. Thus, $-273^{\circ}\text{C} = 0^{\circ}\text{A} = 0^{\circ}\text{K}$
- Specific heat of a gas at constant pressure is always greater than specific heat at constant volume.

Isothermal Change (process)	Adiabatic Change (process)
1. Gas volume is changed by keeping temperature constant.	1. Gas volume and also its temperature changes.
2. For this, changes in volume are made very slowly.	2. Changes in volume are made very quickly.
3. Exchange of heat is allowed.	3. Exchange of heat is not allowed.
4. Boyle's law i.e. relation $PV = \text{constant}$ is valid.	4. Boyle's law i.e. relation $PV = \text{constant}$ is not valid. New valid relation is $PV^{\gamma} = \text{constant}$
5. Expansion of gas takes place.	5. Compression of gas takes place.
6. e.g. (1) melting of solid, (2) boiling of water.	6. e.g. bursting of cycle rubber tube.

IMPORTANT INFORMATION AND CONVERSIONS

- While solving problems on Boyle's law, Gay Lussac's law and Charles' law or $\frac{PV}{T} = \text{constant}$, T should be in $^{\circ}\text{K}$.

$$(x)^{\circ}\text{C} = (x + 273)^{\circ}\text{K}$$

- 76 cm of mercury (Hg) = 1 atmospheric pressure
= $1.013 \times 10^5 \text{ N/m}^2$
= 1.013 bar
- 1 bar = $1 \times 10^5 \text{ N/m}^2$
- 4.2 joule = 1 cal
- 4200 joule = 1 kcal
- 1 kcal = 1000 cal
- 1 joule = 1×10^7 ergs

- NTP condition \rightarrow Normal Temperature Pressure
 $\downarrow \qquad \qquad \downarrow$
 $0^{\circ}\text{C} \quad 76 \text{ cm of Hg}$
 i.e. $273^{\circ}\text{K} \quad 1 \text{ atmospheric}$

- Joule's constant $J = 4.2 \text{ J/cal}$
or $J = 4200 \text{ J/kcal}$
- If $C_p - C_v = \frac{R}{M}$ formula is used, then answer of C_p or C_v will be in $\text{J/kg}^{\circ}\text{K}$
and if $C_p - C_v = \frac{R}{M}$ formula is used then answer of C_p or C_v will be in $\text{cal/kg}^{\circ}\text{K}$
- If 'K' is given in $\text{W/m}^{\circ}\text{C sec}$, then answer of 'Q' will be in joules.

IMPORTANT FORMULAE

$$1. P_1 V_1 = P_2 V_2$$

$$2. \frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$3. \frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$4. \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

where, P_1 = Initial pressure

V_1 = Initial volume

T_1 = Initial absolute temperature (i.e. in $^{\circ}\text{K}$)

P_2 = New pressure

V_2 = New volume

5. $W = P(V_2 - V_1)$ $T_2 =$ New temperature in $^{\circ}\text{K}$
6. $\frac{C_p}{C_v} = \gamma$
7. $C_p - C_v = \frac{R}{M}$ in MKS unit i.e. cal/kg $^{\circ}\text{K}$
8. $C_p - C_v = R$ in S.I. units i.e. J/kg $^{\circ}\text{K}$
9. $C_p - C_v = \frac{R}{M}$ where $M =$ Molecular weight
10. Isothermal elasticity, $K = P$
11. Adiabatic elasticity, $K = \gamma P$

SOLVED EXAMPLES**EXAMPLES ON GAS LAWS :**

Example 1 : A flask containing dry air is corked up at 37°C , the pressure is 75 cm of Hg. The cork blows up when the pressure becomes 100 cm of mercury. Find the temperature at which cork blows up.

Solution :	Initial	Final
	$P_1 = 75$ cm of Hg	$P_2 = 100$ cm of Hg
	$t_1 = 37^{\circ}\text{C}$	$t_2 = ?$
\therefore	$T_1 = (273 + 37)^{\circ}\text{K}$	$\therefore T_2 = ?$
	$= 310^{\circ}\text{K}$	

Here volume of gas is constant.

\therefore We can use Gay Lussac's law,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

$$\therefore T_2 = T_1 \times \frac{P_2}{P_1}$$

$$\therefore T_2 = (310) \times \frac{(100)}{(75)}$$

$$\therefore \boxed{T_2 = 413.33^{\circ}\text{K}}$$

Or $t_2 = 413.33 - 273$

$$\boxed{t_2 = 140.33^{\circ}\text{C}}$$

Example 2 : Certain mass of a gas occupies 30 cm^3 at 27°C and 780 mm of pressure. Find its volume at 57°C and 690 mm of pressure. (S. 2003, W. 2005 and S. 2006 - 4M)

Solution : Given : Initial

$$P_1 = 780 \text{ mm}$$

$$V_1 = 30 \text{ cm}^3$$

$$t_1 = 27^{\circ}\text{C}$$

$$\therefore T_1 = (27 + 273)$$

$$= 300^{\circ}\text{A}$$

Final

$$P_2 = 690 \text{ mm}$$

$$V_2 = ?$$

$$t_2 = 57^{\circ}\text{C}$$

$$T_2 = (57 + 273)$$

$$= 330^{\circ}\text{A}$$

We have,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore V_2 = \frac{P_1 V_1 T_2}{T_1 P_2}$$

$$\therefore V_2 = \frac{780 \times 30}{300} \times \frac{330}{690}$$

$$\boxed{V_2 = 37.3 \text{ cm}^3}$$

Example 3 : A glass bulb having capacity 100 ml contains air at 20°C and at pressure of 76 cm of mercury. When the bulb is immersed in boiling water, its volume increases by 10%. Find pressure of air inside the bulb when it is kept in boiling water at 100°C . (S. 2002 - 4 M)

Solution : Given : Initial

$$V_1 = 100 \text{ ml}$$

$$t_1 = 20^{\circ}\text{C}$$

$$\therefore T_1 = 20 + 273 = 293^{\circ}\text{K}$$

$$P_1 = 76 \text{ cm of mercury}$$

Final

$$V_2 = 110 \text{ ml } (\because \text{Vol. increases by } 10\%)$$

$$t_2 = 100^{\circ}\text{C}$$

$$\therefore T_2 = 100 + 273 = 373^{\circ}\text{K}$$

$$P_2 = ?$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore P_2 = \frac{P_1 V_1 T_2}{T_1 V_2}$$

$$= \frac{76 \times 100}{293} \times \frac{373}{110} = 87.96$$

$$\boxed{P_2 = 87.96 \text{ cm of mercury}}$$

Example 4 : The volume of a gas at 0°C is 100 ml. Find its volume at 100°C if its pressure remains constant. (S. 2002 - 2 M)

Solution : Given :	$t_1 = 0^\circ\text{C}$	$t_2 = 100^\circ\text{C}$
\therefore	$T_1 = 273 + 0$	$\therefore T_2 = 100 + 273$
	$= 273^\circ\text{K}$	$= 373^\circ\text{K}$
	$V_1 = 100 \text{ ml}$	$V_2 = ?$

P constant

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\therefore V_2 = \frac{T_2}{T_1} \times V_1$$

$$= \frac{373}{273} \times 100$$

$V_2 = 136.63 \text{ ml}$

Example 5 : A gas at 17°C and pressure of 60 cm of mercury has volume 2 litres. Find its volume at 27°C and pressure of 70 cm of mercury. (S. 2001 - 4 M)

Solution : Given :

$P_1 = 60 \text{ cm}$	$P_2 = 70 \text{ cm}$
$V_1 = 2 \text{ lit}$	$V_2 = ?$
$t_1 = 17^\circ\text{C}$	$t_2 = 27^\circ\text{C}$
$\therefore T_1 = (17 + 273) = 290^\circ\text{A}$	$\therefore T_2 = (27 + 273) = 300^\circ\text{A}$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore V_2 = \frac{P_1 V_1}{T_1} \times \frac{T_2}{P_2} = \frac{(60) \times (2)}{(290)} \times \frac{(300)}{(70)}$$

$V_2 = 1.77 \text{ lit.}$

Example 6 : 100 ml of air is measured at 20°C. If the temperature of air is raised to 50°C what will be its volume? Pressure remaining constant. (S. 2004 - 2 M)

Solution :

Given :	$V_1 = 100 \text{ ml}$	$V_2 = ?$
	$t_1 = 20^\circ\text{C}$	$t_2 = 50^\circ\text{C}$
\therefore	$T_1 = (20 + 273)$	$\therefore T_2 = (50 + 273)$
	$= 293^\circ\text{A}$	$= 323^\circ\text{A}$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\therefore V_2 = \frac{V_1 \times T_2}{T_1}$$

$$= \frac{(100) (323)}{(293)}$$

$V_2 = 110.24 \text{ ml}$

Example 7 : A given mass of a gas exerts a pressure of 60 cm of mercury at -73°C. Find the pressure exerted at 27°C if volume of the gas remains constant. (S. 2003 - 4 M)

Solution :	$P_1 = 60 \text{ cm of mercury}$	$P_2 = ?$
	$t_1 = -73^\circ\text{C}$	$t_2 = 27^\circ\text{C}$
\therefore	$T_1 = (-73 + 273)$	$\therefore T_2 = (27 + 273)$
	$T_1 = 200^\circ\text{A}$	$T_2 = 300^\circ\text{A}$

V = constant.

By Gay Lussac's law,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore P_2 = \frac{P_1}{T_1} \times T_2$$

$$= \frac{(60)}{(200)} \times (300)$$

$P_2 = 90 \text{ cm of mercury}$

Example 8 : Calculate the temperature in °C required to change 10 litres of helium at 100°K and 0.4 atmosphere pressure to 25 litres at 0.8 atmosphere pressure. (W. 2002 - 4 M)

Solution : Given :	$P_1 = 0.4 \text{ atm}$	$P_2 = 0.8 \text{ atm}$
	$V_1 = 10 \text{ litres}$	$V_2 = 25 \text{ litres}$
	$T_1 = 100^\circ\text{K}$	$T_2 = ?$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore T_2 = \frac{P_2 V_2}{P_1 V_1} \times T_1$$

$$= \frac{(0.8) (25)}{(0.4) (10)} \times 100$$

$$\therefore T_2 = 500^\circ\text{K}$$

$$\therefore t_2 = 500 - 273$$

$t_2 = 227^\circ\text{C}$

Example 9 : A gas has a volume of 0.6 m^3 at 27°C and pressure of 78 cm of mercury. Find its volume at N.T.P. (W. 2004 - 4 M)

Solution : Given : $V_1 = 0.6 \text{ m}^3$
 $t_1 = 27^\circ\text{C}$
 $\therefore T_1 = (27 + 273)^\circ\text{K}$
 $= 300^\circ\text{K}$
 $P_1 = 78 \text{ cm of Hg}$

$V_2 = ?$
 Since at NTP
 $t_2 = 0^\circ\text{C}$
 $\therefore T_2 = 273^\circ\text{K}$
 $P_2 = 76 \text{ cm of Hg}$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore V_2 = \frac{P_1 V_1}{T_1} \times \frac{T_2}{P_2} = \frac{(78)(0.6)}{(300)} \times \frac{(273)}{(76)}$$

$$\therefore \boxed{V_2 = 0.56 \text{ m}^3}$$

Example 10 : What will be the effect on volume of an enclosed gas, if the pressure of the gas is made twice, keeping the temperature constant?

Solution : Suppose initial volume of the gas is V_1 and initial pressure is P_1 . The final pressure is $2P_1$ when temperature is kept constant.

Boyle's law states that,

$$P_1 V_1 = P_2 V_2$$

$$\therefore V_2 = \frac{P_1 V_1}{P_2} = \frac{P_1 \times V_1}{2P_1} = \frac{V_1}{2}$$

\therefore The volume of gas becomes half.

Example 11 : To what temperature a gas at 0°C must be heated at constant pressure so that its volume is doubled? [B.T.E. Dec. 85]

Solution :

Given :	Initial	Final
	$t_1 = 0^\circ\text{C}$	$t_2 = ?$
\therefore	$T_1 = (0 + 273) = 273^\circ\text{K}$	$\therefore T_2 = ?$
Let volume be V_1 .		$V_2 = 2 \times V_1$ (given)

From Charles' law,

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$\frac{V_1}{2V_1} = \frac{T_1}{T_2}$$

$$T_2 = T_1 \times 2$$

$$= 273 \times 2$$

$$\boxed{T_2 = 546^\circ\text{K}}$$

or, $t_2 = (546 - 273)$

$$\boxed{t = 273^\circ\text{C}}$$

\therefore Thus gas must be heated upto 546°K or 273°C .

Example 12 : Volume of certain quantity of gas at N.T.P. is 12 litres. What will be the pressure exerted by same quantity of gas when enclosed in a gas cylinder of 10 litres at 273°C ?

Solution : Given :

Initial	Final
$P_1 = 76 \text{ cm of Hg}$ (\therefore normal pressure)	$P_2 = ?$
$V_1 = 12 \text{ litres}$	$V_2 = 10 \text{ litres}$
$t_1 = 0^\circ\text{C}$	$t_2 = 273^\circ\text{C}$
$\therefore T_1 = 273^\circ\text{K}$ (\therefore normal temp)	$\therefore T_2 = 546^\circ\text{K}$

From general gas equation, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$P_2 = \frac{P_1 V_1 T_2}{T_1 V_2} = \frac{(76) \times (12) \times (546)}{(273) \times (10)}$$

$$\therefore \boxed{P_2 = 182.4 \text{ cm of Hg}}$$

Example 13 : A certain quantity of gas is enclosed at 27°C . Its temperature is increased till its volume becomes one and half times and pressure is doubled. Calculate the necessary temperature for this.

Solution : Given :

Initial	Final
$P_1 =$ Initial pressure	$P_2 = 2P_1$
$V_1 =$ Initial volume	$V_2 = 1.5 V_1$
$t_1 = 27^\circ\text{C}$	$t_2 = ?$
$\therefore T_1 = 300^\circ\text{K}$	$T_2 = ?$

From general gas equation, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$T_2 = \frac{P_2 V_2}{P_1 V_1} \times T_1$$

$$= \frac{(2P_1) \times (1.5 V_1) \times (300)}{P_1 V_1} = 2 \times 1.5 \times 300$$

$$T_2 = 900^\circ\text{K}$$

$$t_2 = 627^\circ\text{C}$$

Or

EXAMPLES ON SPECIFIC HEAT :

Example 14 : The difference between the two specific heats of gas is 1600 and their ratio is 1 : 6. Find C_p and C_v .

Solution : Given : $C_p - C_v = 1600$... (1)

$$\frac{C_p}{C_v} = \frac{6}{1} \quad \dots (2)$$

$\therefore C_p > C_v$ (always)

From equation (2), $C_p = 6 C_v$

Putting in equation (1), $6C_v - C_v = 1600$

$$5C_v = 1600$$

$$C_v = \frac{1600}{5}$$

$$C_v = 320 \text{ units}$$

Putting in equation (2), $C_p = 6 \times 320$

$$C_p = 1920 \text{ units}$$

Example 15 : The ratio γ for a gas is 1.4 and $\frac{R}{M} = 0.0718$. Find values of C_p and C_v .

Solution : Given : $\gamma = 1.4$

i.e. $\frac{C_p}{C_v} = 1.4$... (1)

$$\frac{R}{M} = 0.0718$$

but $C_p - C_v = \frac{R}{M}$

$\therefore C_p - C_v = 0.0718$... (2)

From equation (1), $C_p = 1.4 C_v$

Putting in equation (2),

$$1.4 C_v - C_v = 0.0718$$

$$C_v (1.4 - 1) = 0.0718$$

$$C_v = \frac{0.0718}{0.4}$$

$$C_v = 0.1795 \text{ J/kg } ^\circ\text{K}$$

Putting in equation (1), $C_p = 1.4 \times (0.1795)$

$$C_p = 0.2513 \text{ J/kg } ^\circ\text{K}$$

Example 16 : The ratio of C_p to C_v for a gas is 1.4 and $R/M = 0.0714$. Calculate the values of C_p and C_v .

Solution : Solve it same as above.

$$C_v = 0.1785 \text{ J/kg } ^\circ\text{K}$$

and

$$C_p = 0.2499 \text{ J/kg } ^\circ\text{K}$$

Example 17 : For hydrogen the difference between the principal specific heats C_p and C_v is $4000 \text{ J/kg } ^\circ\text{K}$. If the ratio of specific heats is 1.4, calculate C_p and C_v .

Solution : Given : $C_p - C_v = 4000 \text{ J/kg } ^\circ\text{K}$... (1)

$$\frac{C_p}{C_v} = 1.4 \quad \dots (2)$$

From equation (2), $C_p = 1.4 C_v$

Putting in equation (1),

$$1.4 C_v - C_v = 4000$$

$$C_v (1.4 - 1) = 4000$$

$$C_v = \frac{4000}{0.4}$$

$$C_v = 10,000 \text{ J/kg } ^\circ\text{K}$$

Putting in equation (2),

$$C_p = 1.4 \times 10,000$$

$$C_p = 14,000 \text{ J/kg } ^\circ\text{K}$$

Example 18 : For a certain gas, the difference between its specific heats is $140 \text{ Joules/kg } ^\circ\text{K}$. If the ratio C_p/C_v is 1.2, find the specific heats. Calculate C_p and C_v .

Solution : Given : $C_p - C_v = 140 \text{ joules/kg } ^\circ\text{K}$

and $\frac{C_p}{C_v} = 1.2$

$\therefore C_p = 1.2 C_v$

Now, $C_p - C_v = 140$

becomes $1.2 C_v - C_v = 140$

$\therefore 0.2 C_v = 140 \therefore C_v = 700 \text{ joules/kg } ^\circ\text{K}$

$\therefore C_p - 700 = 140$

$\therefore C_p = 840 \text{ joules/kg } ^\circ\text{K}$

Example 19 : The difference between two specific heats of gas is $0.055 \text{ J/kg } ^\circ\text{K}$. Find J if $R = 235.4 \text{ MKS units}$. (W. 1983, W. 1992 and S. 2005 - 4 M)

Solution : Given : $C_p - C_v = 0.055 \text{ J/kg } ^\circ\text{K}$

$R = 235.4 \text{ MKS}$

$J = ?$

We have,

$C_p - C_v = \frac{R}{J}$

$\therefore J = \frac{R}{C_p - C_v}$

$\therefore J = \frac{235.4}{0.055}$

$\therefore J = 4280 \text{ joules/kcal}$

Example 20 : The ratio of C_p to C_v for a gas is 1.4 and $\frac{R}{M} = 0.0124$. Find the values of C_p and C_v .

Solution : Given : $\frac{C_p}{C_v} = 1.4 \dots (1)$

$\frac{R}{M} = 0.0124$

We have,

$C_p - C_v = \frac{R}{M}$

$\therefore C_p - C_v = 0.0124 \dots (2)$

From equation (1),

\therefore Equation (2) becomes $C_p = 1.4 C_v$ Put this value in equation (2)

$1.4 C_v - C_v = 0.0124$

$C_v (1.4 - 1) = 0.0124$

$C_v (0.4) = 0.0124$

$\therefore C_v = \frac{0.0124}{0.4}$

$C_v = 0.031 \text{ J/kg } ^\circ\text{K}$

Putting this value in equation (1),

$\therefore \frac{C_p}{0.031} = 1.4$

$\therefore C_p = 1.4 \times 0.031$

$\therefore C_p = 0.0434 \text{ J/kg } ^\circ\text{K}$

Note : If in the above example, we use the formula

$C_p - C_v = \frac{R}{M}$

$\therefore (C_p - C_v) J = \frac{R}{M}$

Taking $J = 4.2 \text{ J/cal}$, then answer of C_p and C_v will be having unit $\text{cal/kg } ^\circ\text{K}$.

QUESTIONS

- State Boyle's law, Charles' law, Gay Lussac's law.
- State general gas equation. State S.I. unit of universal gas constant.
- Define absolute zero and Kelvin's scale of temperature.
- Define the terms :
 - Specific heat at constant volume.
 - Specific heat at constant pressure
 Why does a gas have two specific heats ?
- State relation for difference in specific heats of a gas. State relation between C_p and C_v . State Mayor's relation. OR OR
- Explain why $C_p > C_v$.
- State :
 - Charles' law.
 - Gay Lussac's law.
- State general gas equation. Hence define universal gas constant.
- Define specific heat of a gas at constant pressure and constant volume. State the relation between C_p and C_v .
- State three gas laws and write general gas equation.
- Distinguish between isothermal and adiabatic changes of gas.
- Define two principal specific heats of a gas. State relation between them. State their S.I. units.
- Explain concept of absolute zero.
- Distinguish between a gas constant and universal gas constant.
- Define universal gas constant R in the equation $PV = RT$. State its value in S.I. system.
- Define coefficient of linear expansion and aerial expansion and state relation between them.

17. State four distinguishing points between isothermal and adiabatic processes.
18. Differentiate between isothermal and adiabatic expansion of a gas.
19. State Gay Lussac's law, Charles' law, isothermal process and adiabatic process.
20. Define specific heat of a gas at constant pressure (C_p) and specific heat of a gas at constant volume (C_v). Explain why C_p is greater than C_v .

PROBLEMS

1. 100 ml of air is measured at 20°C . If temperature of air is raised to 50°C , what will be its volume? Pressure of air remains constant.
Ans. 110.24 ml.
2. 100 ml of air is measured at 20°C . If the temperature of air is raised by 50°C , what will be its volume? Pressure of air remains constant.
[Note: See difference between problems (1) and (2) carefully]
Ans. 117.06 ml.
3. A gas at 25°C has its temperature raised so that its volume doubles, while its pressure remains constant. Find its final temperature.
Ans. $T_2 = 596^\circ\text{K}$ or $t_2 = 323^\circ\text{C}$.
4. Express 100°C , $97^\circ\text{C} - 18^\circ\text{C}$, on the Kelvin scale of temperature.
5. Calculate the rise in temperature required to increase the pressure of a gas 3 times at constant volume if initial temperature is 0°C .
Ans. 819°K or 546°C .
6. A balloon has maximum capacity of holding 2000 litres of gas. It is filled with 1800 litres of gas at 27°C and 74 cm of Hg and is allowed to rise in air. It bursts at some time at temperature 37°C . Find its pressure at which the burst occurs.
Ans. 68.82 cm of Hg.
7. The volume of a certain quantity of gas at N.T.P. is 24 litres. What will be the pressure exerted by the same quantity of gas when enclosed in a gas cylinder of capacity 20 litres at 27°C ?
Ans. 100.22 cm of Hg or 1.318 atm.
8. A certain mass of a gas occupies 40 cm^3 at 28°C and 780 mm of pressure. Find its volume at 57°C and 780 mm.
[B.T.E. May 87]
Ans. 43.85 c.c.

9. A gas at 13°C is heated at constant pressure till its volume is doubled. What is its final temperature?
[B.T.E. Dec. 88]
Ans. 572°K or 299°C
10. The difference between the two specific heats of gas is $0.0537\text{ kcal/kg }^\circ\text{K}$. Find J if $R = 230\text{ MKS units}$.
Ans. 4283 J/kcal.
11. A certain mass of a gas is enclosed at 27°C . Its temperature is increased till its volume becomes one and half times and the pressure is doubled. Calculate the necessary temperature for this.
Ans. 900°K or 627°C .
12. A certain mass of air at 20°C has a volume of 200 ml. It is heated at 50°C keeping pressure constant. What would be the new volume?
Ans. 220.47 ml.
13. A glass contains air at a pressure of 76 cm of Hg and temperature 27°C . It is placed in the oil bath at temperature 127°C . What will be the pressure inside? Assume that the volume of the bulb increases by 10%.
Ans. $P_2 = 92.12\text{ cm of Hg}$.
14. Calculate the temperature required to change 10 litres of helium at 100°K and 0.2 atmospheric pressure to 20 litres at 4 atmospheric pressure. (S. 1998 - 4 M)
Ans. $T_2 = 4000^\circ\text{K}$ or $t_2 = 3727^\circ\text{C}$.
15. A flask containing dry air is corked up at 27°C , the pressure is 76 cm of mercury. The cork blows up when the pressure becomes 1.5 atmosphere. Find the temperature at which cork blows up. (S. 1996 - 4 M)
Ans. $T_2 = 450^\circ\text{K}$ or $t_2 = 177^\circ\text{C}$.
16. Certain mass of gas occupies 40 cc at 27°C and 780 mm of pressure. Find its volume at 47°C and 680 mm of pressure. (W. 1995 - 4 M)
Ans. $V_2 = 48.94\text{ cc}$
17. A glass bulb contains air at a pressure of 76 cm of Hg and temperature of 27°C . It is placed in the oil at temperature 127°C . What will be the pressure inside? Assume that the volume of the bulb increases by 5%.
Ans. $P_2 = 96.51\text{ cm of Hg}$.

18. To what temperature a gas at 0°C must be heated at constant volume so that its pressure is doubled?
 Ans. $T_2 = 546^{\circ}\text{K}$ or $t_2 = 273^{\circ}\text{C}$.
 (W. 1985 - 4 M)

19. A glass bulb contains air at pressure of 76 cm of Hg at 27°C when its volume is 100 cc. It is then placed in oil of temperature of 327°C . What will be the pressure inside, when the volume of the bulb becomes 152 cc?
 Ans. $P_2 = 100$ cm of Hg.
 (S. 1993 - 4 M)

20. To what temperature a gas at 27°C must be heated at constant volume, so that its pressure is trebled?
 Ans. $T_2 = 900^{\circ}\text{K}$ or $t_2 = 627^{\circ}\text{C}$.
 (S. 1993 - 4 M)



LIGHT, LASER AND SOUND

4.1 PROPERTIES OF LIGHT

Reflection, Refraction, Snell's law, Physical significance of refractive index, Definition of dispersion, Polarization and diffraction of light along with ray diagram, Principle of superposition of waves, Interference of light, Constructive and destructive interference.

4.1.1 Reflection

When a ray of light is allowed to fall on a reflecting surface such as mirror, it is observed that light reflects in the same medium as shown.

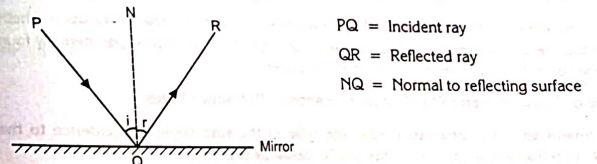


Fig. 4.1

Angle made by incident ray with normal is called angle of incidence i.e. $\angle PQN$.

Angle made by reflected ray with normal is called angle of reflection i.e. $\angle NQR$.

Laws of reflection :

- (1) The angle of incidence is equal to the angle of reflection, $i = r$.
- (2) The incident ray, normal and reflected ray, lie in one plane.

(4.1)