# Integration

It is an Inverse process of differentiation is called Integration.

It is also called Anti-derivative or primitive.

It is denoted by (I)

Stigma (sum)

$$I = \int f(x)\,dx = F(x) + c$$

Sign of Integration Integrated

Integration of f(x) w.r.t dx

Integral

Integration constant or Arbitrary constant



$\frac{d\,2x}{dx} = 2$ $\qquad \int 2\,dx = 2x + c$

$\qquad \int 2\,dx = 2x + c$

$\frac{d(2x+4)}{dx} = 2$

$\frac{d(2x-5)}{dx} = 2$ $\qquad \int 2\,dx = 2x + c$.

$\frac{d}{dx}x^4 = 4x^3$ $\qquad \int 4x^3 = 4\frac{x^4}{4} + c$

1. $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$ $\qquad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

2. $\frac{d}{dx} \log x = \frac{1}{x}$ $\qquad$ $\int \frac{1}{x} dx = \log|x| + c$

3. $\frac{de^x}{dx} = e^x$ $\qquad$ $\int e^x dx = e^x + c$

4. $\frac{d}{dx}(a^x \cancel{\log a}) = a^x \log a$ $\qquad$ $\int a^x \log a \, dx = \log a \frac{a^x}{\log a} = a^x + c$

5. $\frac{d\sin x}{dx} = \cos x$ $\qquad$ $\int \cos x \, dx = \sin x + c$

$\frac{d\cos x}{dx} = -\sin x$ $\qquad$ $\int \sin x \, dx = -\cos x + c$

$\frac{d\tan x}{dx} = \sec^2 x$ $\qquad$ $\int \sec^2 x \, dx = \tan x + c$

$\frac{d(-\cot x)}{dx} = \operatorname{cosec}^2 x$ $\qquad$ $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\frac{d\sec x}{dx} = \sec x \tan x$ $\qquad$ $\int \sec x \tan x \, dx = \sec x + c$

$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$ $\qquad$ $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

$\frac{d\sin^{-1} x}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

$\frac{d\tan^{-1} x}{dx} = \frac{1}{1+x^2}$ $\qquad$ $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

$\frac{d\sec^{-1} x}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + c.$

# Method of Integration

## BY SUBSTITUTION

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c,$$

let $(ax+b) = t$

$$a = \frac{dt}{dx} \Rightarrow a\,dx = dt$$

$$\therefore \int (ax+b)^n dx = \int t^n \frac{dt}{a} = \frac{1}{a} \frac{t^{n+1}}{n+1}$$

$$\Rightarrow \frac{1}{a} \frac{(ax+b)^{n+1}}{n+1} + c.$$



$\int \sin x\,dx = -\cos x + c$

$\int \cos x\,dx = \sin x + c$

$\int \tan x\,dx = -\log|\cos x| + c$

$\int \cot x\,dx = \log|\sin x| + c$

$\int \sec x\,dx = \log|\sec x + \tan x| + c$

$\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$

Rule 1. if the form of Integrated are

$\frac{e^{\tan^{-1}x}}{1+x^2}$, $\frac{x^2}{e^{x^3}}$, $e^{2x+3}$ ----. etc, then

put $t =$ ----- . the power of e, differeink it, find dx after put dx in eqn (1) then, solve it.

$\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

$I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

$t = \tan^{-1}x$

$\frac{dt}{dx} = \frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$

$dx = dt(1+x^2)$.

$= \int \frac{e^t}{\cancel{1+x^2}} dt\,\cancel{(1+x^2)}$

$= \int e^t dt = e^t + c$

$= e^{\tan^{-1}x} + c$



3: if the angle of any trigometric function without $x$, then put $t$ = angle of trigonometric function.

$\int \sin(ax+b)\,dx$

let $t = ax+b$ then $\frac{dt}{dx} = a$

$\int \sin t \frac{dt}{a} = \frac{1(-\cos t)}{a} = -\frac{\cos(ax+b)}{a} + c.$

3: $\int \tan^2(2x-3)\,dx$, $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$. $\int \cos 2x\,dx$.

3: if the form of Integral are $\{f(x)\}^n$ or $\frac{1}{\{f(x)\}^n} dx$ then put $t = f(x)$.

$\int \frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8} dx$, $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$, $\int \sqrt{\sin 2x} \cos 2x\,dx$

$$I = \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}\, dx$$

put $t = \tan^{-1} x^4$

$$\frac{dt}{dx} = \frac{d\tan^{-1}x^4}{dx^4} \cdot \frac{dx^4}{dx} = \frac{4x^3}{1+x^8}$$

$$\int \frac{\cancel{x^3} \sin t \; dt \; \cancel{(1+x^8)}}{\cancel{(1+x^8)} \; 4\cancel{x^3}} = \frac{1}{4}(-\cos t) + c$$

$$I = \frac{-\cos(\tan^{-1} x^4)}{4} + c.$$

Rule 4. if the form of integration is $I = \int \frac{f'(x)}{f(x)}\, dx$

$$\boxed{I = \int \frac{f'(x)}{f(x)}\, dx = \log|f(x)| + c}$$



$$\int \frac{\cos x - \sin x}{\cos x + \sin x}\, dx = \log|\sin x + \cos x| + c$$

$$\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx = -\log|\cos x| + c$$

$$\int \cot x\, dx = \int \frac{\cos x}{\sin x}\, dx = \log \sin x + c$$

$$\int \frac{x}{x^2-1}\, dx = \int \frac{2x}{x^2-1}\, dx = \log|x^2-1| + c$$

$$\int \frac{\cos x}{1+\sin x}\, dx = \log|1+\sin x| + c.$$

5. if the form of Integration is $I = \int \frac{f'(x)}{\sqrt{f(x)}} dx$ is $2\sqrt{f(x)} + c$ then put $t = f(x)$.

$$\boxed{I = \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c}$$

$$\int \frac{\sin x}{\sqrt{1-\cos x}} dx = 2\sqrt{1-\cos x} + c.$$

Soln:- let $t = 1-\cos x$

$dt/dx = \sin x$

$$\int \frac{\sin x}{\sqrt{t}} \frac{dt}{\sin x} = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + c = 2\sqrt{1-\cos x} + c$$

6. $$\boxed{I = \int [f(x)]^n f'(x) dx = \frac{f(x)^{n+1}}{n+1} + c}$$

$$\int \frac{\log x}{x} dx = \int \log x \cdot \frac{1}{x} dx = \frac{(\log x)^2}{2} + c.$$

$$\int x^n dx = \int x^n \cdot 1 dx = \frac{x^{n+1}}{n+1} + c$$

Q. $\int \tan x \, dx = -\log|\cos x| + c = \log|\sec x| + c$

Q. $\int \cot x \, dx = \log \sin x + c = -\log|\operatorname{cosec} x| + c$

Q. $\int \sec x \, dx = \log|\sec x + \tan x| + c = \log|\tan(\pi/4 + x/2)| + c$

Q. $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c = \log|\tan \frac{x}{2}| + c.$

Rule-7. $\int \sin^m x \cos^n x \, dx$ ; $m, n \in N$.

- one of them is odd; then power of even of trigonomerty that trigono. Substiuent 't'.

$$\int \sin^3 x \cos^2 x \, dx \qquad t = \cos x.$$

$$\int \sin^3 x \, \frac{t^2 \, dt}{-\sin x} = -\int \sin^2 x \, t^2 \, dt$$

$$-\int (1 - \cos^2 x) \, t^2 \, dt$$

$$-\int (1 - t^2) \, t^2 \, dt = -\frac{t^3}{3} + \frac{t^4}{4} + c$$

$$\frac{\cos^4 x}{4} - \frac{\cos^3 x}{3} + c.$$

- Both of them is odd; then any trigono-substiuent 't'

$$\int \sin^3 x \cos^3 x \, dx, \quad \int \sin x \cos x \, dx$$

- Both of them is even; Not use substiuent that Integration is use to trigono-metric Identities.

$$\int \sin^2 x \cos^2 x \, dx = \int \frac{4 \sin^2 x \cos^2 x \, dx}{4} = \int \frac{\sin^2 2x \, dx}{4}$$

m & n are rational numbers.

$\frac{m+n-2}{2}$ is negative Integer; then substituent · $\tan x = t$, $\cot x = t$.

$$\int \frac{dx}{\sin x \cos^3 x}$$

$m = -1$

$n = -3$

$\frac{m+n-2}{2} = \frac{-1-3-2}{2} = -3$

let $t = \tan x$ so, $\frac{dt}{dx} = \sec^2 x$

$$\int \frac{dt}{\sec^2 x \sin x \cos^3 x} = \int \frac{dt}{\sin x \cos x} = \int \frac{dt}{\frac{\sin x \cos x \times \cos x}{\cos x}}$$

$$\int \frac{dt}{\tan x \cos^2 x} = \int \frac{\sec^2 x \, dx}{\tan x} = \log|\tan x| + C.$$



$\int \tan^m x \cot^n x \, dx$ — $m, n \in N$

$$\int \frac{dx}{\sin^3 x \cos^3 x}.$$ let $t = \tan x$; so that $\frac{dt}{dx} = \sec^2 x$.

$$\int \frac{1/\cos^6 x}{\sin^3 x \cos^3 x / \cos^6 x} dx = \int \frac{\sec^6 x}{\tan^3 x} dx = \int \frac{\cancel{\sec^6 x}\ \sec^4 x}{t^3} \frac{dt}{\cancel{\sec^2 x}}$$

$$\int \frac{(1+\tan^2 x)^2}{t^3} dt = \int \frac{(1+t^2)^2}{t^3} dt$$

$$\int \frac{1+t^4+2t^2}{t^3} dt = \int \frac{1}{t^3} dt + \int t\,dt + 2\int \frac{2}{t^3} dt$$

$$-\frac{1}{2}\frac{1}{t^2} + \frac{t^2}{2} + \log t = \frac{\tan^2 x}{2} - \frac{1}{2\tan^2 x} + \log|\tan x| + c$$

Rule-8 $\int \tan^m x \sec^n x \, dx$ ; $m, n \in N$

→ if the form of Integration is
$I = \int \tan^m x \sec^n x \, dx$ ; where $m, n \in N$
then, see in the first the even power of $\sec x$.

I) if the power of $\sec x$ is even ; put $t = \tan x$
$\int \tan^3 x \sec^2 x \, dx$ $t = \tan x$.

II) if the power of $\sec x$ is odd ; then see power of $\tan x$.

$\int \tan^3 x \sec^3 x \, dx$ ; $\int \tan^2 x \sec x \, dx$

- if the power of $\tan x$ is odd ; the put $t = \sec x$.
  $\int \tan^3 x \sec^3 x \, dx$ ; $t = \sec x$.

- if the power of $\tan x$ is even ; then Express $\tan^2 x = \sec^2 x - 1$.
  $\int \tan^2 x \sec x \, dx$

III) if the power of $\sec x$ is odd and power of $\tan x$ is zero (0). then, Apply by part Rule.
$\int \sec x \, dx$ ; $\int \sec^3 x \, dx$.

## BY PART'S RULE

| I | L | A | T | E |
|---|---|---|---|---|
| Inverse Trigo. | Logarithm | Algebra | Trigonometry | Exponential |
| $\sin^{-1}x$ | $\log x$ | $x$ | $\sin x$ | $e^x$ |
| $\cos^{-1}x$ | $\log \sin x$ | $x^2$ | $\cos x$ | $e^{\sin x}$ |
| $\tan^{-1}x$ | $\log a^x$ | $3x^3$ | $\tan x$ | $e^{\log x}$ |

$$I = \int u \cdot v\,dx = u\int v\,dx - \int\left\{\frac{du}{dx}\cdot\int v\,dx\right\}dx$$

$$I = \int \mathrm{I}\cdot\mathrm{II}\,dx = \mathrm{I}\int \mathrm{II}\,dx - \int\left\{\frac{d\mathrm{I}}{dx}\cdot\int \mathrm{II}\,dx\right\}dx$$

$$\int x\log x = \log x\int x\,dx - \int\left\{\frac{d\log x}{dx}\cdot\int x\,dx\right\}dx$$

$$= \log x\,\frac{x^2}{2} - \int \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2}\,dx$$

$$= \log x\cdot\frac{x^2}{2} - \frac{1}{2}\,\frac{x^2}{2} + c$$

$$\boxed{I = \frac{x^2}{2}\log x - \frac{1}{4}x^2 + c}$$

I-I. $\boxed{\int e^x\left[f(x) + f'(x)\right]dx = e^x f(x) + c}$

$$\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx = \frac{e^x}{x} + c.$$

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R-II. $\boxed{\int e^{kx}\{kf(x)+f'(x)\}dx = e^{kx}f(x)+c}$

Ex:- $\int e^{2x}(-\sin x + 2\cos x)dx = e^{2x}\cos x + c$

Solⁿ:- $\int e^{2x}(2\cos x - \sin x)dx$

$\int e^{2x}\cdot 2\cdot \cos 2x\,dx - \int e^{2x}\sin x\,dx$

$2\left[\cos x\int e^{2x}dx - \int\left\{\frac{d\cos x}{dx}\cdot\int e^{2x}dx\right\}dx\right] - \int e^{2x}\sin x\,dx$

$2\left[\cos x\frac{e^{2x}}{2} + \int \sin x\frac{e^{2x}}{2}dx\right] - \int e^{2x}\sin x\,dx$

$\frac{\cancel{2}\cos x\, e^{2x}}{\cancel{2}} + \frac{\cancel{2}}{\cancel{2}}\cancel{\int e^{2x}\sin x\,dx} - \cancel{\int e^{2x}\sin x\,dx}$

$e^{2x}\cos x + c$.

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# Some special Integrals

1. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c.$

2. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

3. $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

proof:- $\int \frac{dx}{a^2x^2} = \int \frac{dx}{(a+x)(a-x)} = \int \frac{2a}{2a(a+x)(a-x)} dx$

$\frac{1}{2a} \int \frac{a+x+a-x}{(a+x)(a-x)} dx = \frac{1}{2a} \cdot \int \frac{\cancel{a+x}}{(\cancel{a+x})(a-x)} dx + \int \frac{\cancel{a-x}}{(a+x)(\cancel{a-x})} dx$

$\frac{1}{2a} \int \frac{1}{a-x} dx + \int \frac{1}{a+x} dx$

$\frac{1}{2a} \left[-\log|a-x| + \log|a+x|\right] = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right|$

$$\boxed{I = \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c}$$

Ex:- $\int \frac{dx}{1-4x^2} = \frac{1}{4}\int \frac{dx}{1/4 - x^2} = \frac{1}{4}\int \frac{dx}{(\frac{1}{2})^2 - x^2}$

$\frac{1}{4}\left[\frac{1}{2\times\frac{1}{2}} \log\left|\frac{1/2 + x}{1/2 - x}\right|\right] + c = \frac{1}{4}\log\left|\frac{1+2x}{1-2x}\right|$

Remark → if we have an integral of the form $\int \frac{dx}{ax^2+bx+c}$ then we put the denominator in the form $[(x+a)^2 \pm B^2]$ and then integrate.



Ex:- $\int \frac{dx}{x^2+6x+13} = \int \frac{dx}{x^2+6x+9+4} = \int \frac{dx}{(x+3)^2+2^2}$

$= \frac{1}{2}\tan^{-1}\left(\frac{x+3}{2}\right) + c.$

Ex:- $\int \frac{dx}{x^2+8x+20} = \int \frac{dx}{(x+4)^2+2^2} = \frac{1}{2}\tan^{-1}\frac{x+4}{2} + c$

Ex:- $\int \frac{dx}{9x^2-12x+8} = \frac{1}{9}\int \frac{dx}{x^2 - \frac{12x}{9} + \frac{8}{9}} = \frac{1}{9}\int \frac{dx}{x^2 - \frac{4}{3}x + \frac{8}{9}}$

$\frac{1}{9}\int \frac{dx}{x^2 + \frac{12x}{9} + \frac{8}{9} + (\frac{2}{3})^2 - (\frac{2}{3})^2} = \frac{1}{9}\int \frac{dx}{(x-\frac{2}{3})^2 + (\frac{2}{3})^2}$

$\frac{1}{9}\times\frac{3}{2} \tan^{-1}\left(\frac{x-2/3}{2/3}\right) = \frac{1}{6}\tan^{-1}\frac{3x-2}{2} + c$

Rule $\int \frac{dx}{a\sin^2x + b\cos^2x}$ or $\int \frac{dx}{a\sin^2x + b}$ or $\int \frac{dx}{a\sin^2x + \cos^2x + b}$

Or $\int \frac{dx}{(a\sin x + b\cos x)^2}$ or $\int \frac{dx}{a + b\cos^2x}$

Step I Divide by $\cos^2x$ on numerator as well as Denominator.

Step II if In Denominator $\sec^2x$ to convert into the term of $\tan x$ i.e $\sec^2x = 1 + \tan^2x$

step III After that put $t = \tan x$.

Rule. $\int \frac{dx}{a\sin x + b\cos x + c}$ Replace $\sin x = \frac{2\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

6. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$

7. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log|x + \sqrt{x^2-a^2}| + c$

8. $\int \frac{dx}{\sqrt{x^2+a^2}} = \log|x + \sqrt{x^2+a^2}| + c.$



proof:- put $x = a\sin\theta$

$\frac{dx}{d\theta} = a\cos\theta \Rightarrow dx = a\cos\theta d\theta$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{a\cos\theta\, d\theta}{\sqrt{a^2-a^2\sin^2\theta}} = \int \frac{a\cos\theta\, d\theta}{a\cos\theta}$

$\int d\theta = \theta + c = \sin^{-1}\frac{x}{a} + c$ $\left[\begin{array}{l} x = a\sin\theta \\ \frac{x}{a} = \sin\theta \end{array}\right]$

proof:- put $x = a\sec\theta$, $dx = a\sec\theta\tan\theta\, d\theta$

$\int \frac{a\sec\theta\tan\theta\, d\theta}{\sqrt{a^2\sec^2\theta - a^2}} = \int \frac{\cancel{a}\sec\theta\cancel{\tan\theta}\, d\theta}{\cancel{a\tan\theta}}$

$\int \sec\theta d\theta = \log|\sec\theta + \tan\theta| + c$

$= \log\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right| + c$

$= \log|x + \sqrt{x^2-a^2}| + c.$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right| + c$$

II. $$\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + c$$

Proof $$I = \int \sqrt{a^2-x^2}\,dx = \int \sqrt{a^2-x^2}\cdot 1\,dx$$

$$= \sqrt{a^2-x^2}\int 1\,dx - \int\left\{\frac{d\sqrt{a^2-x^2}}{d(a^2-x^2)}\frac{d(a^2-x^2)}{dx}\int 1\,dx\right\}dx$$

$$= x\sqrt{a^2-x^2} - \int \frac{-2x}{2\sqrt{a^2-x^2}}\cdot x\,dx$$

$$= x\sqrt{a^2-x^2} + \int \frac{x^2}{\sqrt{a^2-x^2}}\,dx$$

$$= x\sqrt{a^2-x^2} + \int \frac{x^2+a^2-a^2}{\sqrt{a^2-x^2}}\,dx$$

$$= x\sqrt{a^2-x^2} + \int \frac{a^2}{\sqrt{a^2-x^2}}\,dx - \int \frac{a^2-x^2}{\sqrt{a^2-x^2}}\,dx$$

$$I = x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a} - \int \sqrt{a^2-x^2}\,dx$$

$$I = x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a} - I$$

$$2I = x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a}$$

$$\boxed{I = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c}$$

Remark: if the form of Integration is

$$I = \int \frac{dx}{ax^2+bx+c} \text{ or } \int \frac{dx}{\sqrt{ax^2+bx+c}} \text{ or } \int \sqrt{ax^2+bx+c}$$

then, we put the denominator in the form $[(x+a)^2 \pm \beta^2]$ and then Integrate.

Ex:- $\int \frac{dx}{16-2x-2x^2}$ or $\int \frac{dx}{9x^2+6x+5}$ or $\int \frac{dx}{\sqrt{10-8x-2x^2}}$

$\int \frac{dx}{\sqrt{x^2-3x+2}}$ or $\int \frac{dx}{\sqrt{2x-x^2}}$ or $\int \sqrt{2x^2+3x+4}\, dx$

$\int \sqrt{x^2+4x+6}\, dx$ or $\int \sqrt{x^2-8x+3}\, dx$ or $\int \frac{dx}{(x-1)(x-3)}$

Sol^n:- $\int \frac{dx}{\sqrt{2x-x^2}} = +\int \frac{dx}{\sqrt{-(x^2-2x)}} = \int \frac{dx}{\sqrt{-(x^2-2x+1^2-1^2)}}$

$\int \frac{dx}{\sqrt{-[(x-1)^2-1^2]}} = \int \frac{dx}{1^2-(x-1)^2} = \sin^{-1}(x-1)+c$

Task - if the Form of Integration are :—

$$I = \int \frac{px+q}{ax^2+bx+c}\,dx \text{ or, } \int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx \text{ or}$$

$$\int (px+q)\sqrt{ax^2+bx+c}\,dx \quad , \quad p, q \in R.$$

Then,

let $px+q = A\frac{d}{dx}(ax^2+bx+c) + B$ but $p \neq 0$

$px+q = A(2ax+b) + B$

$px+q = 2ax \cdot A + Ab + B$

$px = 2ax \cdot A$ ; $q = Ab + B$

$\boxed{A = \frac{p}{2a}}$ $\quad B = q - Ab$

$\boxed{B = q - \frac{pb}{2a}}$



i) $\int \frac{3x+1}{2x^2-3x+3}\,dx$ ii) $\int \frac{5x-2}{1+2x+3x^2}$ iii) $\int \frac{x+2}{2x^2+6x+5}\,dx$

iv) $\int \frac{5x+3}{\sqrt{x^2+4x+10}}\,dx$ v) $\int \frac{6x+3}{\sqrt{(x-5)(x-4)}}\,dx$ vi) $\int x\sqrt{x+x^2}\,dx$

vii) $\int (x+3)\sqrt{3-4x-x^2}\,dx.$

Ex:- $I = \int \frac{5x-2}{3x^2+2x+1}\,dx$ - - - (i).

let $5x-2 = A\frac{d}{dx}(3x^2+2x+1)+B$

$= A(6x+2)+B$

$5x-2 = 6Ax+2A+B$

$5x = 6Ax$ ; $-2 = 2A+B$

$A = \frac{5}{6}$ ; $-2 = 2\times\frac{5}{6}+B$

$B = \frac{-6-5}{3} = \frac{-11}{3}$

Now, $I = \int \frac{5x-2}{3x^2+2x+1}\,dx$

$I = \int \frac{A(6x+2)+B}{3x^2+2x+1} = \int \frac{\frac{5}{6}(6x+2)+\left(\frac{-11}{3}\right)}{3x^2+2x+1}$

$\int \frac{5}{6}\frac{(6x+2)}{3x^2+2x+1}\,dx + \int \frac{-11/3}{3x^2+2x+1}\,dx$

$\frac{5}{6}\int \frac{6x+2}{3x^2+2x+1}\,dx - \frac{11}{3}\int \frac{dx}{3\left(x^2+\frac{2x}{3}+\frac{1}{3}\right)}$

$\frac{5}{6}\log|3x^2+2x+1| - \frac{11}{9}\int \frac{dx}{x^2+2x\cdot\frac{1}{3}+\left(\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2+\frac{1}{3}}$

$$\frac{5}{6}\log|3x^2+2x+1| - \frac{11}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}$$

$$\frac{5}{6}\log|3x^2+2x+1| - \frac{11}{9}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{3}\right)^2}$$

$$\frac{5}{6}\log|3x^2+2x+1| - \frac{11}{9}\left[\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\frac{\left(x+\frac{1}{3}\right)}{\frac{\sqrt{2}}{3}}\right]$$

$$\frac{5}{6}\log|3x^2+2x+1| - \frac{11}{3\sqrt{2}}\tan^{-1}\frac{3x+1}{\sqrt{2}} + c.$$

# Partial fractions

if $\frac{16}{(x-2)(x+2)^2}$ or $\frac{2x+1}{(x-1)(x^2+1)}$ or $\frac{2x+3}{(x-3)(x+1)}$

or $\frac{x^3-2x^2-13x-12}{x^4-x^2-3x-10}$

(i) $\frac{1}{(x-a)(x-b)} = \frac{A}{(x-a)} + \frac{B}{(x-b)}$.

(ii) $\frac{1}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$.

(iii) $\frac{1}{(x-a)^2} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2}$

(iv) $\frac{1}{(x-a)^3} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3}$

(v) $\frac{1}{(x-a)(x-b)(x-c)^2} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)} + \frac{D}{(x-c)^2}$

(vi) $\frac{Px+q}{(x-a)(x+b)} = \frac{A}{(x-a)} + \frac{B}{(x+b)}$

(vii) $\frac{Px+2}{(x-a)^2(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)} + \frac{D}{(x-c)}$

i) $$\frac{1}{(x-a)(ax^2+bx+c)} = \frac{A}{(x-a)} + \frac{Bx+C}{(ax^2+bx+c)}$$

ii) $$\frac{Px^2+qx+r}{(x-a)(x-b)(ax^2+bx+c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{Cx+D}{(ax^2+bx+c)}.$$

iii) $$\frac{Px+q}{(x-a)(x-b)^2(x-c)(ax^2+bx+c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-b)^2}$$

$$+ \frac{D}{(x-c)} + \frac{Ex+F}{(ax^2+bx+c)}.$$

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i) $\int \frac{2x-3}{(x^2-1)(2x+3)}\,dx$ ii) $\int \frac{2x+1}{(x+2)(x-3)}\,dx$ iii) $\int \frac{1}{(x+1)(x+2)}\,dx$

iv) $\int \frac{3x-1}{(x-1)(x-2)(x-3)}\,dx$ v) $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}\,dx$ vi) $\int \frac{e^x}{(1+e^x)(2+e^x)}\,dx$

vii) $\int \frac{1}{x(x^2+1)}\,dx$ viii) $\int \frac{1}{x(x^5+1)}\,dx$ ix) $\int \frac{x}{(x+1)^2(x+2)}\,dx$

x) $\int \frac{2x}{(x^2+1)(x^2+3)}\,dx.$

Ex:- $I = \int \frac{2x-3}{(x^2-1)(2x+3)} dx$

let $\frac{2x-3}{(x^2-1)(2x+3)} = \frac{Ax+B}{(x^2-1)} + \frac{C}{(2x+3)}$

$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{(Ax+B)(2x+3) + C(x^2-1)}{(x^2-1)(2x+3)}$

$2x-3 = 2Ax^2 + 3Ax + 2Bx + 3B + Cx^2 - C$

$2x-3 = (2A+C)x^2 + (3A+2B)x + (3B-C)$

$2A + C = 0$, $3A + 2B = 2$, $3B - C = -3$

$2A = -C$, $2A + 3B = -3$ $3B + 2A = -3$

$3A + 2B = 2 \times 2$ $\Rightarrow$ $6A + 4B = 4$

$2A + 3B = -3 \times 3$ $\quad$ $\underline{6A \pm 9B = \mp 9}$

$-5B = +13$

$B = \frac{-13}{5}$

$\boxed{B = -13/5}$

$\boxed{A = 12/5}$

$\boxed{C = -24/5}$

$3A + 2B = 2$

$3A + 2\left(\frac{-13}{5}\right) = 2$

$2A = -C$

$3A = 2 + \frac{26}{5} = \frac{36}{5}$

$\frac{2 \times 12}{5} = -C$

$A = \frac{12}{5}$

$\boxed{C = \frac{-24}{5}}$

$$I = \int \frac{2x-3}{(x^2-1)(2x+3)}\,dx = \int\left(\frac{Ax+B}{x^2-1} + \frac{C}{2x+3}\right)dx$$

$$A\int \frac{x}{x^2-1}\,dx + B\int \frac{1}{x^2-1}\,dx + C\int \frac{dx}{2x+3} =$$

$$\frac{12}{5}\cdot\frac{1}{2}\log|x^2-1| - \frac{13}{5}\left(\frac{1}{2}\right)\log\left|\frac{x-1}{x+1}\right| + \left(\frac{-24}{5}\right)\log|2x+3|$$

Diploma wallah

# DEFINITE INTEGRALS

1. if $f(x) = \ldots\ldots$ be a given functions in the closed internal $x \in [a,b]$ then definite Integration is defined as:

$$I = \int_a^b f(x)dx = [f(x)]_a^b = f(b) - f(a).$$

Note:- $I = \int_a^b f(x)dx = [f(x)+c]_a^b$

$$= f(b)+\cancel{c} - f(a) - \cancel{c} = f(b) - f(a)$$

Ex: $\int_2^3 x^2 dx$  ii) $\int_{-1}^0 (x+1)dx$  iii) $\int_0^1 \frac{1}{1+x^2}dx$

iv) $\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$  v) $\int_1^2 x \log x\,dx$  vi) $\int_0^{-1} \frac{1}{|x|\sqrt{x^2-1}}dx$

vii) $\int_0^1 \tan^{-1}x\,dx$  viii) $\int_0^1 \log x\,dx$  ix) $\int_0^1 (x^2+x-3)dx$

2. Definite Integral by subsitution.

ex: $\int_0^2 e^{x/2}dx$  Put $\frac{x}{2} = t$

$\frac{dx}{dt} = 2$

lower limit if $x=0$, $t=0$

Upper limit if $x=2$, $t=1$

$$\int_0^1 e^t\, 2dt = 2\int_0^1 e^t dt = [e^t]_0^1 = 2[e^1 - e^0]$$

$$= 2(e-1)$$