civil engineering

UNIT-2 Co-ordinate Geometry MATHEMATICS

STRAIGHT LINES

1. Distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by —

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2. Area of ΔABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is given by —

$$\Delta = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right| \text{ sq units}$$

Note → The points A, B, C are collinear. then, Area of $\Delta ABC = 0$.

3. (i) if the point $P(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m:n$.

$$x = \frac{mx_2 + nx_1}{m+n} \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

(ii) if $P(x, y)$ is the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$.

$$x = \frac{1}{2}(x_1 + x_2) \quad \text{and} \quad y = \frac{1}{2}(y_1 + y_2).$$

1. Find the distance between the points (2,-3) and (-6,3)

$\therefore A(2,-3)$ and $B(-6,3)$

$$AB = \sqrt{(-6-2)^2 + (3-(-3))^2}$$

$$= \sqrt{(-8)^2 + (6)^2} = 10 \text{ unit}.$$

• find the area of the triangle whose vertices are A(4,4), B(3,-16) and C(3,-2).

$A\begin{pmatrix} x_1 = 4 \\ y_1 = 4 \end{pmatrix}$ $B\begin{pmatrix} x_2 = 3 \\ y_2 = -16 \end{pmatrix}$ $C\begin{pmatrix} x_3 = 3 \\ y_3 = -2 \end{pmatrix}$

$$\Delta = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$$

$$= \frac{1}{2}\left| 4(-16+2) + 3(-2-4) + 3[4-(-16)] \right|$$

$$= \frac{1}{2}\left| 4\times -14 + 3\times -6 + 3\times 20 \right|$$

$$= \frac{1}{2}(-56 - 18 + 60) = \frac{1}{2}|-74+60| = \frac{|-14|}{2} = 7 \text{ sq. u.}$$

Q. find the co-ordinates of the point which divides the line segment joining the points A(5,-2) and B(9,6) in the ratio 3:1.

$$x = \cancel{3\times 9}\ \frac{mx_2 + nx_1}{m+n} \quad ; \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{3\times 9 + 1\times 5}{3+1} \quad ; \quad y = \frac{3\times 6 + 1\times -2}{3+1}$$

$$= \frac{32}{4} = 8 \qquad y = \frac{16}{4} = 4.$$

Q. find the co-ordinates of the midpoint of the line segment joining the points A(-2,-5) and B(3,-1).

Sol^n:- $x = \frac{(-2+3)}{2} = \frac{1}{2}$; $y = \frac{-5+(-1)}{2} = -3$

the required point is $\left(+\frac{1}{2}, -3\right)$.

— Slope of a line —

$0° \leq \theta < 180°$.

- Horizontal line - Any line parallel to the x-axis or the axis itself is called a horizontal line.

- vertical line - Any line parallel to y-axis or the y-axis itself is called a vertical line.

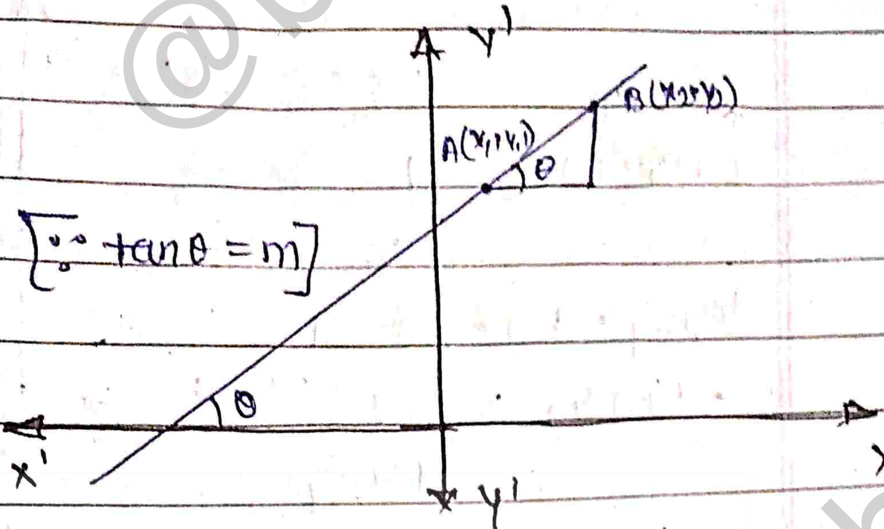
- oblique line - A line which is neither horizontal nor vertical is called oblique line.

slope or gradient of a line -

$\boxed{m = \tan\theta}$

- slope of a line passing through two given points.

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$



$[\because \tan\theta = m]$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Q. If the slope of line passing through the points (2,5) and (x,3) is 2.

$$\text{slope of } AB = \frac{3-5}{x-2}$$

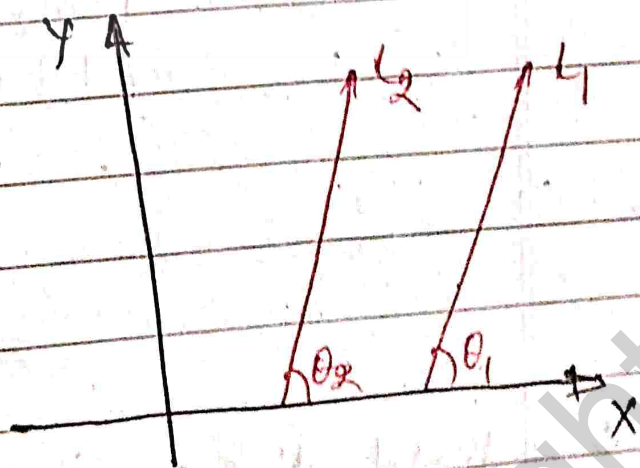
$$\cancel{2} = \frac{-\cancel{2}}{x-2} \Rightarrow x-2 = -1$$

$$x = -1+2$$

$$\boxed{x = 1}.$$

- slopes of parallel lines.

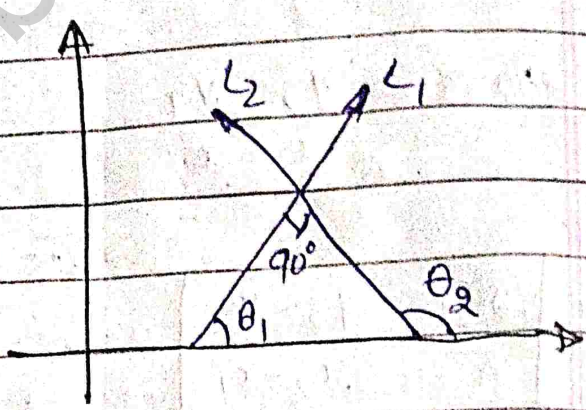
$$m_1 = m_2$$



l_1 and l_2 are parallel.

- Slopes of perpendicular lines–

$$\boxed{m_1 \cdot m_2 = -1}$$



$\tan\theta_1 \cdot \tan\theta_2 = -1$

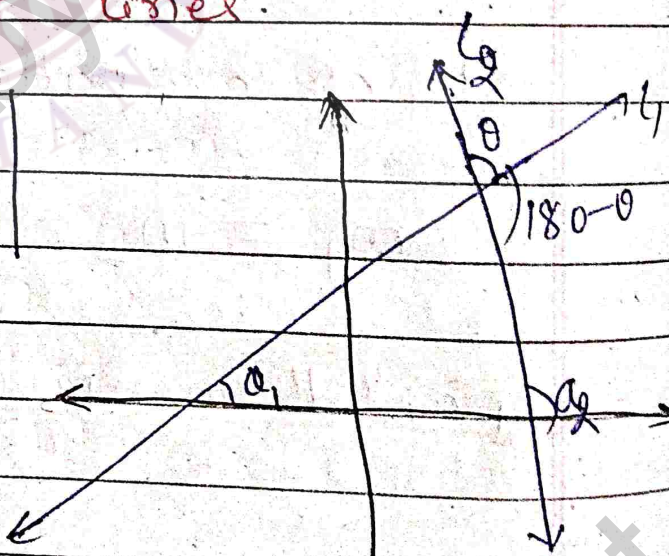
$\tan\theta_2 = \frac{-1}{\tan\theta_1}$

L_1 and L_2 are perpendicular to each other,

(i)	$L_1 \parallel L_2$	$\Leftrightarrow$	$m_1 = m_2$
(ii)	$L_1 \perp L_2$	$\Leftrightarrow$	$m_1 \cdot m_2 = -1$

- Angle between two Non-vertical and Non-perpendicular lines.

$$\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|$$



Q. find the angle b/w the lines whose slopes are $1/2$ and 3.

$m_1 = \frac{1}{2}, m_2 = 3$

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| \Rightarrow \tan\theta = \left|\frac{3 - 1/2}{1 + 3 \cdot \frac{1}{2}}\right|$

$\tan\theta = \left|\frac{5/2}{5/2}\right| \Rightarrow \tan\theta = 1$ or $\boxed{\theta = \pi/4}$.

Equation of a line –

The linear relation between two variables x and y, which is satisfied by the co-ordinates of each and every points on the line and not by those of any other point in the cartesian plane.

(i) EQUATION OF X-AXIS.

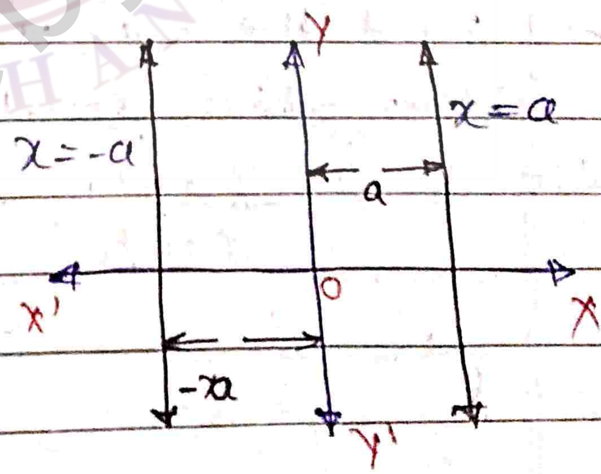
if P(x, y) is any point on the x-axis then $y = 0$.

(ii) EQUATON OF Y-AXIS.

if P(x, y) is any point on the y-axis then $x = 0$

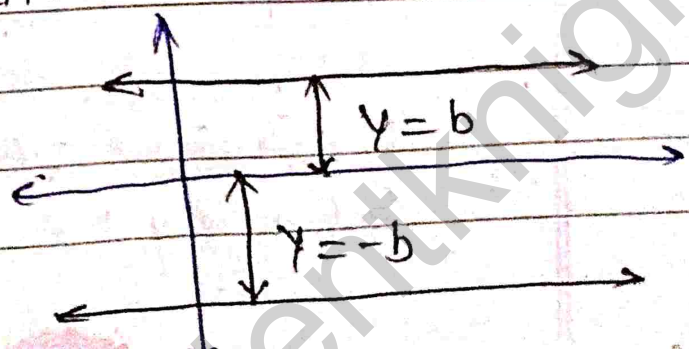
(iii) EQUATION OF A LINE PARALLEL TO Y-AXIS.

$x = a$, $x = -a$



(iv) EQUATION OF A LINE PARALLEL TO X-AXIS.

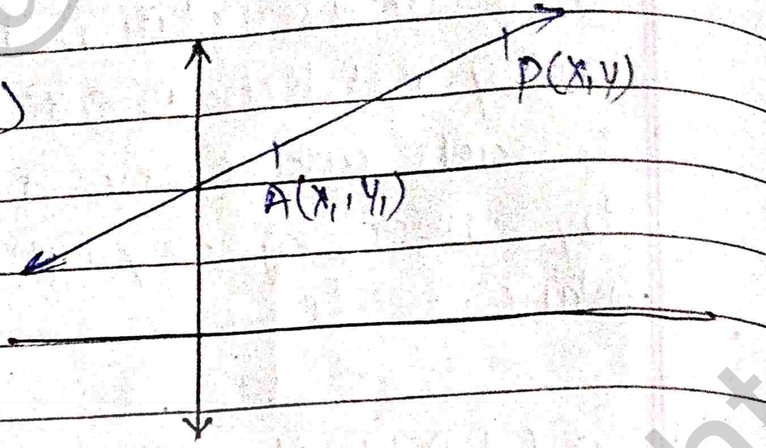
$y = b$, $y = -b$



Equation of a line in point-slope form

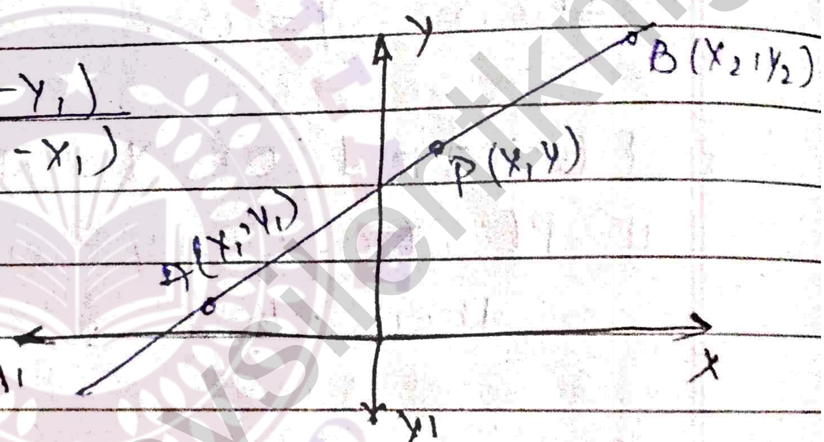
$$(y - y_1) = m(x - x_1)$$

$$\boxed{m = \frac{y - y_1}{x - x_1}}$$



Equation of a line in two point form

$$\frac{(y - y_1)}{(x - x_1)} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$



Q. Find the equation of a line passing through the points (−1,1) and (2,−4).

Soln:-

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

or, $$\frac{y - 1}{x + 1} = \frac{-4 + 1}{2 + 1}$$

$$\frac{y - 1}{x + 1} = \frac{-5}{3}$$

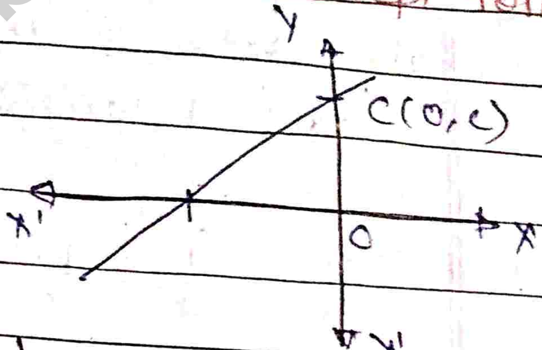
$$3y - 3 = -5x + 5$$

$$5x + 3y + 2 = 0.$$

• Equation of a line in slope-Intercept form.

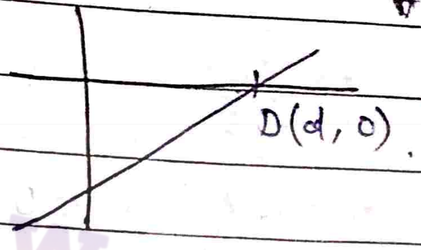
→ y-intercept c is —

$y = mx + c$.



→ x-intercept d is —

$y = m(x - d)$.



Q: find the equation of a line whose slope is $\frac{1}{2}$ and y-intercept equal to $-5/4$.

$y = mx + c$ $\qquad m = \frac{1}{2}$, $c = \frac{-5}{4}$

$y = \frac{1}{2}x + \left(\frac{-5}{4}\right)$

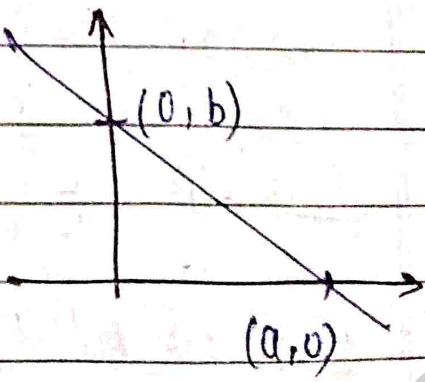
$y = \frac{1}{2}x - \frac{5}{4} \Rightarrow 4y = 2x - 5$

$2x - 4y - 5 = 0$.

→ Equation of a line in intercepts form —

• The line making intercept a and b on the x- and y-axis.

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$



Q. find the equation of line which make intercept 2 & -3 on the x-axis and y-axis respectively.

$\frac{x}{a} + \frac{y}{b} = 1 \Rightarrow \frac{x}{2} + \frac{y}{-3} = 1 \Rightarrow \frac{-3x + 2y}{-6} = 1$

$-3x + 2y = -6 \Rightarrow 3x - 2y - 6 = 0$

Q. find the equation of the line which makes intercepts 2 and -3 on the x-axis and y-axis respectively.

Sol^n- $\frac{x}{a} + \frac{y}{b} = 1$. $a = 2$, $b = -3$

$\frac{x}{2} + \frac{y}{-3} = 1$

$\frac{-3x + 2y}{-6} = 1$

$-3x + 2y = -6$

$3x - 2y - 6 = 0$.

Q- Find the equation of the line which passes through the point (3,4) and the sum of whose intercepts on the axes is 14.

Sol: let the intercepts made by the line on the x-axis & y-axis be a and $(14-a)$...

the required equation is $\frac{x}{a} + \frac{y}{14-a} = 1$.

since it passes through the point (3,4).

$\frac{3}{a} + \frac{4}{14-a} = 1 \Rightarrow \frac{3(14-a)+4a}{a(14-a)} = 1$

$42 - 3a + 4a = 14a - a^2$

$42 + a = 14a - a^2$

$a^2 - 14a + a + 42 = 0$

$a^2 - 13a + 42 = 0$

$a^2 - 6a - 7a + 42 = 0$

$a(a-6) - 7(a-6) = 0$

$a = 6$ or $a = 7$.

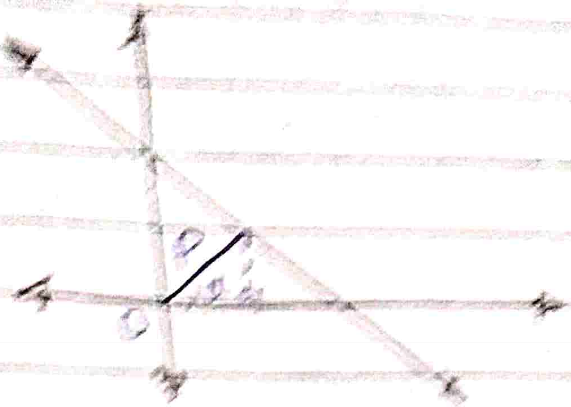
so, the required equation

$\frac{x}{6} + \frac{y}{8} = 1$

$x + y - 7 = 0$.

Equation of a line in normal form

$\boxed{x\cos\theta + y\sin\theta = p}$



* Find the equation of a line whose perpendicular distance from the origin is 5 units and the angle between the positive direction of the x-axis and the perpendicular is 30°.

Sol — $p = 5$ and $\theta = 30°$

So, the required solution is $x\cos\theta + y\sin\theta = p$

$x\cos 30° + y\sin 30° = 5$

$x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 5$

$\sqrt{3}x + y = 10$

$\sqrt{3}x + y - 10 = 0$

* General Equation of a line — $\boxed{y = mx + c}$

* REDUCTION OF GENERAL FORM TO STANDARD FORM —

• Given equation of a line $Ax + By + C = 0$

I Slope Intercept form –

$$Ax + By + C = 0 \Rightarrow By = -Ax - C$$

$$\Rightarrow y = \frac{-Ax - C}{B}$$

$y = mx + c$

$$\Rightarrow \boxed{y = \left(\frac{-A}{B}\right)x + \left(\frac{-C}{B}\right)}$$

Where,

$$m = \frac{-A}{B}, \quad c = \frac{-C}{B}$$

II Intercepts form

$$Ax + By + C = 0 \Rightarrow Ax + By = -C$$

$$\Rightarrow \left(\frac{A}{-C}\right)x + \left(\frac{B}{-C}\right)y = 1$$

$\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \boxed{\frac{x}{\left(\frac{-C}{A}\right)} + \frac{y}{\left(\frac{-C}{B}\right)} = 1}$$

Where,

$$a = \frac{-C}{A}, \quad b = \frac{-C}{B}.$$

III Normal form

$$Ax + By + C = 0 \Rightarrow \frac{A}{\sqrt{A^2+B^2}}x + \frac{B}{\sqrt{A^2+B^2}}y + \frac{C}{\sqrt{A^2+B^2}}$$

$x\cos\alpha + y\sin\alpha = p$

$$\boxed{\frac{-A}{\sqrt{A^2+B^2}}x + \frac{-B}{\sqrt{A^2+B^2}} = \frac{C}{\sqrt{A^2+B^2}}}$$

Where,

$$\cos\alpha = \frac{-A}{\sqrt{A^2+B^2}}; \quad \sin\alpha = \frac{-B}{\sqrt{A^2+B^2}}; \quad p = \frac{C}{\sqrt{A^2+B^2}}$$

Q. Reduce the equation $\sqrt{3}x + y + 2 = 0$.
① slope-intercepts form. find slope & y-intercept
② intercepts form and find the intercepts on the axes

Ans: $\sqrt{3}x + y + 2 = 0$

① $y = -\sqrt{3}x - 2$ $m = -\sqrt{3}$ & $c = -2$.

② $\sqrt{3}x + y + 2 = 0$

$\sqrt{3}x + y = -2$ x-intercept   y-intercept

$\frac{\sqrt{3}x}{-2} + \frac{y}{-2} = 1$ $a = \frac{-2}{\sqrt{3}}$, $b = -2$.

$\therefore \frac{x}{\left(\frac{-2}{\sqrt{3}}\right)} + \frac{y}{(-2)} = 1$

Q. Reduce the equation $x + \sqrt{3}y + 5 = 0$ to the normal form.

Ans: $x + \sqrt{3}y + 5 = 0$

$-x - \sqrt{3}y = 5$ [keeping constant +ve].

Now,

On dividing by $\sqrt{A^2 + B^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$.

again,

$-\frac{x}{2} - \frac{\sqrt{3}y}{2} = \frac{5}{2}$

$\cos a = -\frac{1}{2}$; $\sin a = \frac{-\sqrt{3}}{2}$; $p = \frac{5}{2}$

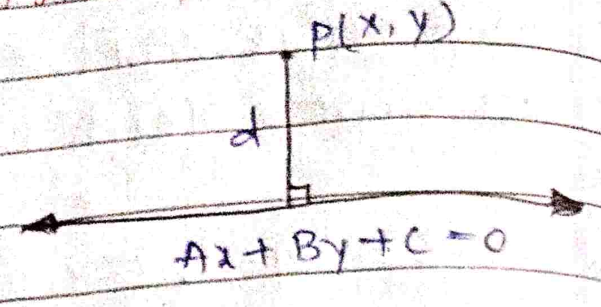
$a = 120°$ $\tan a = \frac{\sin a}{\cos a} = \frac{\frac{-\sqrt{3}}{2}}{\frac{-1}{2}} = \sqrt{3}$, $= 30°$

$\Downarrow$

$240°$

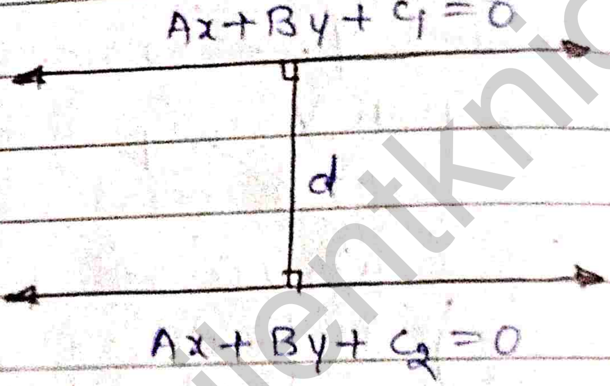
Distance of a point from a line.

$$\boxed{d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}}$$



Distance between two parallel lines

$$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{A^2 + B^2}}}$$



Distance between two parallel lines $y = mx + c_1$ and $y = mx + c_2$ is given by

$$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{1 + m^2}}}$$

Q. find the distance of the point $(4, 1)$ from the line $3x - 4y + 12 = 0$.

$$d = \frac{|3 \times 4 + (-4 \times 1) + 12|}{\sqrt{3^2 + (-4)^2}}$$

$$= \frac{|12 - 4 + 12|}{5} = \frac{20}{5} = 4 \text{ units.}$$

Q. find the distance between the parallel lines $15x + 8y - 34 = 0$ and $15x + 8y + 31 = 0$.

Ans. $15x + 8y - 34 = 0$

$y = -\frac{15}{8}x + \frac{17}{4}$ —— (I) $m = -\frac{15}{8}, c_1 = \frac{17}{4}$

$15x + 8y + 31 = 0$

$y = -\frac{15}{8}x - \frac{31}{8}$ —— (II) $m = -\frac{15}{8}, c_2 = -\frac{31}{8}$

distance $(d) = \frac{|c_2 - c_1|}{\sqrt{1+m^2}}$

$= \frac{\left|-\frac{31}{8} - \frac{17}{4}\right|}{\sqrt{1+\left(\frac{-15}{8}\right)^2}} = \frac{\left|\frac{-31-34}{\cancel{8}}\right|}{\frac{\sqrt{8^2+(-15)^2}}{\cancel{8}}}$

$d = \left|\frac{-65}{17}\right|$; $d = \frac{65}{17}$ units.