civil Engineering

MATHEMATICS

Algebra

UNIT-3

I. complex Number.
II. partial fraction.
III. permutation and combination
IV. Binomial theorem
V. Matrices and Determinant.

I. Complex Number

Imaginary Numbers:- if the square of a given number is negative then such a number is called an Imaginary Number.

For example, $\sqrt{-1}$, $\sqrt{-2}$ etc are Img. No.

$\sqrt{-1}$ by greek letter iota 'i'

Thus,

$$\sqrt{-4} = 2i, \quad \sqrt{-9} = 3i, \quad \sqrt{-5} = i\sqrt{5} \text{ etc}$$

powers of i.

* $i^0 = 1$
* $i^1 = i$
* $i^2 = -1$
* $i^3 = -i$
* $i^4 = 1$

Ex!- $i^{23} = i^{(4\times5)+3} = (i^4)^5 \times i^3 = i^3 = -i$.

Ex!- $i^{998} = i^{4\times249+2} = (i^4)^{249} \times i^2 = -1$.

Ex!- $i^{-998} = \frac{1}{i^{998}} \times \frac{i^2}{i^2} = \frac{i^2}{i^{1000}} = \frac{-1}{1} = -1$.

Ex!- $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$.

$i^n(1 + i^1 + i^2 + i^3) = 0$

$i^n(1 + i - 1 - i) = 0$.

Ex!- $(1+i)^4 \times \left(1+\frac{1}{i}\right)^4 = 16$.

$(1+i)^4 \left(1+\frac{i}{i^2}\right)^4$

$(1+i)^4 (1-i)^4 = [(1^2-(i)^2)]^4$

$= [1+1]^4 = 2^4 = 16$.

Ex!- $\sqrt{-25} \times \sqrt{-49} \Rightarrow 5i \cdot 7i = 35i^2 = -35$.

Complex Numbers – The numbers of the form $(a+ib)$, where a and b are real numbers and $i = \sqrt{-1}$, are known as complex numbers.

It is denoted by C.

$\therefore\ C = \{(a+ib) : a, b \in R\}$.

Ex!- $(5+8i)$, $(-3+\sqrt{2}i)$, $\left(\frac{2}{3}-\frac{5}{7}i\right)$ .... is a complex.

* Conjugate of a complex Number.

Conjugate of a complex number $z = (a+ib)$ is defined as, $\bar{z} = (a-ib)$

(i) $\overline{(3+8i)} = (3-8i)$

(ii) $\overline{(-6-2i)} = (-6+2i)$.

(iii) $\overline{-3} = -3$.

* Modulus of a complex Number

$z = (a+ib)$ then, $|z| = \sqrt{a^2+b^2}$

Ex: (i) $z = 2+3i$; $|z| = \sqrt{2^2+3^2} = \sqrt{13}$

(ii) $z = -5-4i$; $|z| = \sqrt{(-5)^2+(-4)^2} = \sqrt{41}$.

* Equality of complex numbers.

$z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$.

Ex:- $2y + (3x-y)i = 5-2i$.

$2y = 5$; $3x - y = -2$

$\boxed{y = \frac{5}{2}}$

$3x - \frac{5}{2} = -2$

$3x = -2 + \frac{5}{2} = \frac{1}{2}$

$x = \frac{1}{2} \times \frac{1}{3}$

$\boxed{x = \frac{1}{6}}$

Sum and difference of complex numbers.

$z_1 = (a_1 + ib_1)$ and $z_2 = (a_2 + ib_2)$.

(i) $z_1 + z_2 = (a_1 + a_2) + i(b_1 + b_2)$.

(ii) $z_1 - z_2 = (a_1 - a_2) + i(b_1 - b_2)$.

(i) closure property.

$z_1 = (a_1 + ib_1)$ and $z_2 = (a_2 + ib_2)$.

$$z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2)$$
$$= (a_1 + a_2) + i(b_1 + b_2).$$

(ii) Commutative law $z_1 + z_2 = z_2 + z_1$

(iii) Associative law $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

(iv) Exitence of Additive Identity

$$z + 0 = 0 + z = z.$$

(v) Existence of Additive Inverse.

$$z + (-z) = (-z) + z = 0.$$

Multiplication of complex number.

$z_1 = (a_1 + ib_1)$ and $z_2 = (a_2 + ib_2)$.

$$z_1 z_2 = (a_1 + ib_1)(a_2 + ib_2)$$
$$= (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + b_1 a_2)$$

$\therefore z_1 z_2 = Re(z_1) \cdot Re(z_2) - Im(z_1) \cdot Im(z_2)$.

(i) $z_1 = (3+2i)$ & $z_2 = (5+4i)$.

$z_1 z_2 = (3+2i)(5+4i)$

$= (3\times5 + 3\times4i + 2i\times5 + 2i\times4i)$

$= (15 + 12i + 10i - 8)$

$= 7 + 22i$

I Closure property

$z_1 z_2 = (a_1+ib_1)(a_2+ib_2)$.

II Commutative law: $z_1 z_2 = z_2 z_1$

III Associative law: $(z_1 z_2) z_3 = z_1(z_2 z_3)$

IV Existence of multiplicative Identity

$z\times1 = 1\times z = z$.

V Existence of multiplicative Inverse.

$z = (a+ib)$

$z^{-1} = \frac{1}{z} = \frac{1}{(a+ib)}$

$\Rightarrow \frac{1}{(a+ib)} \cdot \frac{(a-ib)}{(a-ib)} = \frac{(a-ib)}{a^2+(b^2)}$

$z\times z^{-1} = z^{-1}\times z = 1$.

$$\boxed{z^{-1} = \frac{1}{z} = \frac{\bar{z}}{|z|^2} \qquad \therefore z\bar{z} = |z|^2}$$

Division of two complex numbers:-

let z_1 and z_2 be complex numbers such that $z_2 \neq 0$.

Then, $\frac{z_1}{z_2} = z_1 \cdot \frac{1}{z_2} = z_1 z_2^{-1}$.

Ex. $z_1 = 6+3i$ and $z_2 = 3-i$

$$\frac{z_1}{z_2} = z_1 \cdot z_2^{-1}$$

$$= (6+3i)\left(\frac{1}{3-i}\right)^{-1}$$

$$= (6+3i)\left(\frac{3+i}{10}\right)$$

$$= \frac{(18-3)+i(15)}{10}$$

$$= \frac{15+15i}{10}$$

$$= \frac{3+3i}{2} = \frac{3(1+i)}{2}.$$

$$z_2^{-1} = \frac{\overline{z_2}}{|z_2|^2}$$

$$= \frac{\overline{(3-i)}}{|3-i|^2}$$

$$= \frac{3+i}{3^2+(-1)^2}$$

$$= \frac{3+i}{10}.$$

polar Representation of complex No.

polar form of $z = x + iy$ is $r(\cos\theta + i\sin\theta)$

(i) $r = |z| = \sqrt{x^2 + y^2}$.

(ii) $\tan\alpha = \left|\frac{y}{x}\right| = \left|\frac{Im(z)}{Re(z)}\right|$

(iii) When $-\pi < \theta \le \pi$, then θ is the principal argument of z.

Quad. in which z lies	arg(z)
I	$\theta = \alpha$
II	$\theta = \pi - \alpha$
III	$\theta = -(\pi - \alpha)$
IV	$\theta = -\alpha$ or $(2\pi - \alpha)$

Example:— convert in polar form.

(i) 3.

$z = 3 + 0i$

Let it polar form be $z = r(\cos\theta + i\sin\theta)$

$\tan\alpha = \left|\frac{Im(z)}{Re(z)}\right| = \frac{0}{3} = 0.$

$\alpha = 0°$.

Now, $z = r(\cos 0° + i\sin 0°)$

clearly, $z = 3 + 0i$ is represented by the point $P(3,0)$, which lies on the positive side of the x-axis.

Example Argument of the complex No.

(i) $1+i$

$z = 1 + i$

$Re(z) = 1$ & $Im(z) = 1$

$\tan^{-1}\left|\frac{Im(z)}{Re(z)}\right| = \frac{1}{1} = 1$

$a = \frac{\pi}{4}$.

the point representing $z = 1+i$ is $P(1,1)$, which lies in the first quadrant.

Example!- convert $(1+i\sqrt{3})$ into polar form.

$z = 1+\sqrt{3}i$

$|z| = r = \sqrt{1^2+(\sqrt{3})^2}$

$= \sqrt{1+3} = \sqrt{4} = 2.$

let ~~it~~ ~~if~~ the polar form is $r(\cos\theta + i\sin\theta)$.

$z = 1+\sqrt{3}i$; $Re(z) = 1$ & $Im(z) = \sqrt{3}$

$\tan a = \frac{Im(z)}{Re(z)} = \frac{\sqrt{3}}{1} = \sqrt{3}$

$a = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$. $\boxed{\therefore a = \frac{\pi}{3}}$

clearly, it is the point is representing $P(1,\sqrt{3})$ in the first quadrant.

The required polar form of $z = 1+\sqrt{3}i$ is $r(\cos\theta + i\sin\theta)$

$2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ Ans

Convert $4(\cos 300° + i\sin 300°)$ into Cartesian form.

$4(\cos 300° + i\sin 300°)$

$4(\cos(360-60) + i\sin(360-60))$

$4[\cos 60° + i\sin 60°]$

$4\left[\frac{1}{2} + \frac{i\sqrt{3}}{2}\right] = 2 - 2i\sqrt{3}$.

Quadratic Equations (with complex roots)

A polynomial equation of degree n has at the most n roots.

Ex!- solve $x^2+3=0$

$x^2+3=0$

$x^2=-3$

$x=\sqrt{-3}=\pm\sqrt{3}i$

$x=+\sqrt{3}i, -\sqrt{3}i$.

$x^2+3x+9=0$.

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-3\pm\sqrt{(+3)^2-4\times1\times9}}{2\times1}$

$=-3\pm\sqrt{9-36}/2$

$=\frac{-3\pm\sqrt{-27}}{2}$

$=\frac{-3\pm3\sqrt{3}i}{2}, \frac{-3}{2}+\frac{3\sqrt{3}i}{2}, \frac{-3}{2}-\frac{3\sqrt{3}i}{2}$

Square roots of a complex No.

$\sqrt{a+ib}$

let $\sqrt{a+ib} = x+iy$

On squaring both sides -

$a+ib = (x+iy)^2$

$a+ib = x^2+(iy)^2+2x(iy)$

$a+ib = x^2-y^2+i2xy.$

$a = x^2-y^2$ & $b = 2xy.$

$\Rightarrow (x^2+y^2) = \sqrt{(x^2-y^2)^2+4x^2y^2} = \sqrt{a^2+b^2}$

Q1 - $\sqrt{6+8i}$

let $\sqrt{6+8i} = x+iy$

On squaring both sides of (i), we get

$6+8i = (x+iy)^2$

$6+8i = x^2-y^2+2xyi$

$6 = x^2-y^2$ & $8 = 2xy$

$6 = x^2-y^2$ & $xy = 4.$

$\Rightarrow (x^2+y^2) = \sqrt{(x^2-y^2)^2+4x^2y^2}$

$= \sqrt{6^2+4\times16}$

$= \sqrt{36+64}$

$= \sqrt{100} = 10.$

$x^2-y^2 = 6$ & $x^2+y^2 = 10$

$2x^2 = 16$ / $y^2 = 2.$

$x^2 = 8$

$x = \pm2\sqrt{2}$

$y = \pm\sqrt{2}$

II. partial fraction

if $f(x)$ and $g(x)$ are polynomials, then the quotient $\frac{f(x)}{g(x)}$ is termed as rational algebraic fraction.

$$\frac{3x}{x^2-16}, \quad \frac{x^2+5x}{x^2+2x-8}, \quad \frac{x^3-2x^2+1}{x^2-1} \text{ etc}$$

- proper fraction – In any fraction if the numerator is lower degree than its denominator, it is called proper fraction.

Ex! – $\frac{2x+5}{x^2+1}$, $\frac{3x+8}{x^2+4x+3}$.

- Improper fraction – In Improper fraction the degree of numerator is equal to or greater than the degree of its denominator.

Ex! – $\frac{x^2+2x}{x^2+3x-5}$ and $\frac{x^3-2x^2+1}{2x^2+2x-1}$, partial fraction - if we split up the given algebraic fraction into different fractions whose denominators are the factors of the denominator of the given fraction, these fractions are called partial fractions.

Ex:- $\frac{7x-1}{2x^2-x-1} = \frac{3}{2x+1} + \frac{2}{x-1}$.

Resolving into partial fractions -

When the factors of the denominator are linear and non-repeating.

When one or more linear factors of the denominator are repeating.

When the denominator has one or more quadratic non-repeating factors.

When the denominator has one or more quadratic repeating factors.

Case - I.
When Denominator has linear non repeated factors:.

$$\frac{1}{(x+4)(x+6)} = \frac{A}{x+4} + \frac{B}{x+6}.$$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}.$$

$$\frac{x^2}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}.$$

$$\frac{(x-1)(x-2)(x-3)}{(x-4)(x-5)(x-6)} = 1 + \frac{A}{x-4} + \frac{B}{x-5} + \frac{C}{x-6}.$$

$$\frac{2x^3+7x^2-2x-2}{2x^2+x-6} = \frac{A}{x+2} + \frac{B}{2x-3} + (x+D)$$

Case-II When the factors in the denominator are linear and repeated.

Ex!- $$\frac{9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

$$\frac{x^2+x}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}.$$

(iii) When the Denominator contains one or more factors which are non-repeated quadrate.

$$\frac{3x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\frac{42-19x}{(x^2+1)(x-4)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{x-4}.$$

$$\frac{x^4}{x^3+1} = x - \frac{x}{(x+1)(x^2-x+1)}$$

$$= x - \left(\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}\right).$$

$$\frac{x^2+6}{(x^2+3)(x^2+5)} = \frac{Ax+B}{x^2+3} + \frac{Cx+D}{x^2+5}.$$

(iv) When Denominator has repeated Quadratic factor

$$\frac{1}{(x+1)(x^2+1)^2} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

$$\frac{3x^2+5x+3}{(x^2+x+1)^2} = \frac{Ax+B}{x^2+x+1} + \frac{Cx+D}{(x^2+x+1)^2}$$

III. permutation and combination.

Factorial - The factorial of a positive Integer n is defined as the product of all positive Integers less than or equal to n.

It is denoted by $n!$.

$$n! = n(n-1)(n-2)(n-3) \cdots 3 \times 2 \times 1.$$

$$n! = n(n-1)!$$

$$\boxed{(n-1)! = \frac{n!}{n}}$$

put $n=1$ $\quad (1-1)! = \frac{1!}{1}$

*** $\boxed{0! = 1}$

put $n=0$ $\quad (0-1)! = \frac{0!}{0}$

$(-1)! = \frac{1}{0}$ undefined.

put $n=\frac{1}{2}$ $\quad \left(\frac{1}{2}-1\right)! = \frac{\frac{1}{2}!}{1/2}$

$\left(-\frac{1}{2}\right)! = \frac{1/2!}{1/2}$ undefined.

- Factorial is not defined for proper fraction or nega

Permutations— A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Ex!—

$${}^{10}P_6 = \frac{10!}{(10-6)!} = \frac{10!}{4!} = \frac{10\times9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{4!}} = 151200.$$

$${}^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

$${}^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1.$$

— : Combination : —

A Combination is an unordered collection of some or all of the objects in a set.

Ex!—

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$$

$${}^nC_r = \frac{{}^nP_r}{r!}$$

$$\Rightarrow \boxed{{}^nP_r = {}^nC_r \cdot r!}$$

* $$^{20}C_{18} = \frac{20!}{18!(20-18)!} = \frac{\cancel{20}^{10} \times 19 \times \cancel{18!}}{\cancel{18!} \times \cancel{2!}} = 190.$$

* $$^{n}P_4 = 30\,^{n}C_5 \Rightarrow \frac{\cancel{n!}}{(n-4)!} = 30\frac{\cancel{n!}}{5!(n-5)!}$$

$$\frac{1}{\cancel{(n-4)!}} = \frac{30}{5!\,(n-5)\cancel{(n-4)!}}$$

$$5!\,(n-5) = 30$$

$$\cancel{5 \times 4 \times 3 \times 2 \times 1 \neq (n-5) = 30}$$

$$4n - 20 = 0$$

$$4n = 20$$

$$\boxed{n = 5}$$

$$\frac{1}{(n-4)\cancel{(n-5)!}} = \frac{30}{5!\,\cancel{(n-5)!}}$$

$$\frac{\cancel{5} \times 4 \times 3 \cancel{\times 2} \times 1}{(n-4)} = \cancel{30}$$

$$n-4 = 4 \qquad \boxed{n = 8}.$$

$$^{n+1}C_3 = 2\,^{n}C_2$$

$$\frac{(n+1)!}{3!\,(n+1-3)!} = 2\frac{n!}{2!\,(n-2)!}$$

$$\frac{(n+1)\cancel{(n!)}}{3!\,\cancel{(n-2)!}} = 2\frac{\cancel{n!}}{2!\,\cancel{(n-2)!}}$$

$$\frac{n+1}{3 \times \cancel{2!}} = \frac{2}{\cancel{2!}}$$

$$n+1 = 6$$

$$\boxed{n = 5}$$

BINOMIAL THEOREM

$$(x+a)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1} a + {}^nC_2 x^{n-2} a^2 + \cdots + {}^nC_r x^{n-r} a^r + \cdots + {}^nC_n a^n.$$

$$(1+x)^n = 1 + {}^nC_1 x + {}^nC_2 x^2 + \cdots + {}^nC_r x^r + \cdots + {}^nC_n x^n.$$

$(a+b)$	$= a + b$	1 1
$(a+b)^2$	$= a^2 + 2ab + b^2$	1 2 1
$(a+b)^3$	$= a^3 + 3a^2b + 3ab^2 + b^3$	1 3 3 1
$(a+b)^4$	$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1 4 6 4 1
$(a+b)^5$	$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	1 5 10 10 5 1

Ex! — $(1-2x)^5$

$$1 + 5(1)^4(-2x) + 10(1)^3(-2x)^2 + 10(1)^2(-2x)^3 + 5(1)(-2x)^4 + (-2x)^5.$$

$$\Rightarrow 1 - 10x + 40x^2 - 80x^3 + 80x^4 + 32x^5.$$

Ex! — $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$

$$\Rightarrow (\sqrt{3})^4 + 4(\sqrt{3})^3(\sqrt{2}) + 6(\sqrt{3})^2(\sqrt{2})^2 + 4(\sqrt{3})(\sqrt{2})^3 + (\sqrt{2})^4 - \left[(\sqrt{3})^4 + 4(\sqrt{3})^3(-\sqrt{2}) + 6(\sqrt{3})^2(-\sqrt{2})^2 + 4(\sqrt{3})(-\sqrt{2})^3 + (\sqrt{2})^4\right]$$

$$\Rightarrow \cancel{(\sqrt{3})^4} + 4(\sqrt{3})^3(\sqrt{2}) + \cancel{6(\sqrt{3})^2(\sqrt{2})^2} + 4\sqrt{3}(\sqrt{2})^3 + \cancel{(\sqrt{2})^4} - \cancel{(\sqrt{3})^4} + 4(\sqrt{3})^3(\sqrt{2}) - \cancel{6(\sqrt{3})^2(\sqrt{2})^2} + 4(\sqrt{3})(\sqrt{2})^3 - \cancel{(\sqrt{2})^4}$$

$$12\sqrt{6} + 12\sqrt{6} + 8\sqrt{6} + 8\sqrt{6} = 24\sqrt{6} + 16\sqrt{6} \qquad \sqrt{6}(40) = 40\sqrt{6}.$$

General term —

$$(x+y)^n = \sum_{r=0}^{n} {}^nC_r \, x^{n-r} y^r.$$

$$= {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + \cdots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

1st term $= {}^nC_0 x^n y^0$

2nd term $= {}^nC_1 x^{n-1} y^1$

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n^{th} term $= {}^nC_{n-1} x^1 y^{n-1}$

$(n+1)^{th}$ term $= {}^nC_n x^0 y^n$.

So, the $(r+1)^{th}$ term (or general term) of the binomial expansion of $(x+y)^n$ ...

$$\boxed{T_{r+1} = {}^nC_r \, x^{n-r} y^r.}$$

* Middle term of the Binomial Expansio

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + \cdots + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0$$

→ The number of terms in the binomial expansion of $(x+y)^n = n+1$.

Case-I : n is even (i.e $n+1$ is odd)

middle term $= \left(\frac{n}{2}+1\right)^{th}$

Case-II : n is odd.

middle term $= \frac{1}{2}(n+1)^{th}$ term & $\left\{\frac{(n+1)}{2}+1\right\}^{th}$ term

p^{th} term from the end is $(a+b)^n$.

$= (n+1-p+1)$ th term from the begining

$= (n-p+2)$ th term from the begning.

Ex! → middle term in the Expansion of $\left(\frac{x}{3}+9y\right)^{10}$

Sol^n! → $n=10$, even

middle term $= \left(\frac{n}{2}+1\right)^{th}$

$= \left(\frac{10}{2}+1\right)^{th} = 6^{th}$ term

$T_{6th} = T_{r+1} = {}^nC_r\, x^{n-r} y^r$

$T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{x}{3}\right)^{10-5} (9y)^5$

$= \frac{10!}{5!(10-5)!} \left(\frac{x}{3}\right)^5 (9y)^5$

$= \frac{10!}{5!\,5!} \left(\frac{x^5}{3^5}\right)(9^5)(y^5)$

$= 61236\, x^5 y^5$

Ex:- 5^{th} term from the end $= \left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$.

Sol^n:- p^{th} term from the end $= (n-p+2)^{th}$ term from the begining.

5^{th} term from the end $= (9-5+2)^{th} = 6$

$$T_6 = T_{5+1} = {}^9C_5 \left(\frac{x^3}{2}\right)^{9-5} \left(\frac{-2}{x^2}\right)^5$$

$$= \frac{9!}{5!(9-5)!} \times \left(\frac{x^3}{2}\right)^4 \times \left(\frac{-2}{x^2}\right)^5$$

$$= \frac{9!}{5! \times 4!} \times \frac{\cancel{x^{12}}}{2^4} \times \frac{(-2)^5}{\cancel{x^{10}}}$$

$$= \frac{\cancel{9 \times 8 \times 7 \times 6 \times 5!}}{\cancel{5! \times 4 \times 3 \times 2}} \times \frac{x^2 (-2^5)^2}{\cancel{2^4}}$$

$$= -9 \times 7 \times 2 \times 2 \; x^2$$
$$= -252x^2.$$

Ex:- $\left(2x - \frac{1}{x}\right)^{10}$ is independent of x.

Sol^n:- Let T_{r+1} be independent of x.

$$T_{r+1} = (-1)^r \, {}^{10}C_r \, (2x)^{10-r} \left(\frac{1}{x}\right)^r$$

$(-1)^r \times {}^{10}C_r \times (2)^{10-r} \times x^{10-r} \cdot x^{-r}$

$(-1)^r \times {}^{10}C_r \times 2^{10-r} \therefore x^{10-2r}$

if binomial theorem of Expansion is independent of x.

$x^{10-2r} = x^0$

$10-2r = 0$

$2r = 10$

$\boxed{r = 5}$

$\Rightarrow (r+1) = (5+1) = 6$

$T_6 = T_{5+1} = (-1)^5 \times {}^{10}C_5 \times (2)^{10-5} \times x^0$

$= -1 \times \frac{10!}{5! \times 5!} \times 2^5$

$= -1 \times \frac{\cancel{10} \times \cancel{9} \times \cancel{8} \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times 2^5$

$= -3 \times 2 \times 7 \times 6 \times 2^5 = -8064.$

Ex:- $(99)^5$.

$(100-1)^5$

$\Rightarrow (100)^5 + 5(100)^4(-1) + 10(100)^3(-1)^2 + 10(100)^2(-1$

$+ 5(100)(-1)^4 + (-1)^5$.

$\Rightarrow (100)^5 - 5(100)^4 + 10(100)^3 - 10(100)^2 + 500 -$

$\Rightarrow 10000000000 - 500000000 + 10000000$

$100000 + 500 - 1$

$\Rightarrow 9509900499$.