CHAPTER - 6

WORK, ENERGY AND POWER

~~Syllabus 2020-21~~

Work done :- work is said to be done when a force applied on the body displace the body through a certain distance in the direction of applied force.

- it is measured by the product of the force and the distance moved in the direction of the force i.e. $W = F - S$

- if an object undergoes a displacement 's' along a straight line while acted on a force F that makes an angle θ w.r.t S as shown.

$$W = FS\cos\theta = \vec{F}\cdot\vec{S}$$

- work done is a scalar quantity measured in newton metre [1 newton metre = 1 Joule]
- its dimensions is $[ML^2T^{-2}]$

→ following are some significant points about work done, derived from the definition given below

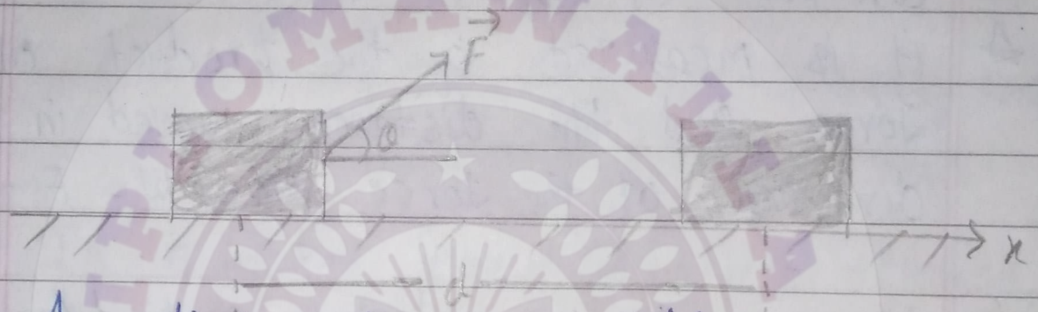
i) work done by a force is zero if displacement is perpendicular to the force ($\theta = 90°$)

ii) work done by the force is positive if angle between force and displacement is acute. ($\theta < 90°$)

iii) Work done by the force is negative if angle between force and displacement is obtuse ($\theta > 90°$)

→ if the applied force varies with time/position, the work done is given by:-

$$W = \int \vec{F} \cdot \vec{ds}$$



An object undergoes a displacement d under the influence of the force F.

* if we plot a graph between force applied and the displacement, then work done can be obtained by finding the area under the F-S graph.

* if a spring is stretched or compressed by a small distance from its unstreched configuration, the spring will exert a force on the block given by

$$F = -kx$$

where x is compression or elongation in spring, k is a constant called spring

Constant whose value depends inversely on unstreched length and the nature of material of spring.

→ The negative sign indicates that the direction of the spring force is opposite to x, the displacement of the free-end.

→ work done by spring when block is displaced by x_0 is given by

$$W = -\int_0^{x_0} kx\,dx = -\frac{1}{2}kx_0^2$$

work done by an agent in giving an elongation or Compression of x_0 is given as $\frac{1}{2}kx_0^2$.

Energy:— The energy of a body is its Capacity to do work.

- Anything which is able to do work is said to possess energy.
- Energy is measured in the same unit as that of work, namely Joule.

Mechanical energy of two types:—

i) **Kinetic energy:—** The energy possessed by a body by virtue of its motion is known as its kinetic energy.

→ For an object of mass m and having a velocity v, the kinetic energy is given by:—

$$K = \frac{1}{2}mv^2$$

Teacher's Sign.

ii) **Potential energy:-** The energy possessed by a body by virtue of its position or condition is known as its potential energy.

☆ There are two common forms of potential energy: gravitational and elastic.

→ gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

it is given by

(U) P.E. = mgh

where m = mass of the body.

g = acceleration due to gravity on the surface of earth.

h = height through which the body is raised.

→ when an elastic body is displaced from its equilibrium position, work is needed to be done against the restoring elastic force. The work done is stored up in the body in the form of its elastic potential energy.

☆ if an elastic spring is stretched by a distance y from its equilibrium position, then its elastic potential energy is given by

$$U = \frac{1}{2} kx^2$$

K → force Constant of given spring.

Work - Energy theorem:-

According to work energy theorem, the work done by a force on a body is equal to the change in kinetic energy of the body.

W = change in K.E. of a body = $\Delta(K.E.)$

$\Delta(K.E.)$ = The difference between the final and initial kinetic energies of the body.

☆ Energy and momentum are related by

$$E = \frac{p^2}{2m}$$ [m = mass of the body]

The Law of Conservation of Energy:-

According to the law of conservation of energy, the total energy of an isolated system does not change. Energy may be transformed from one form to another but the total energy of an isolated system remains Constant.

☆ Energy Can neither be created, nor destroyed

☆ Besides mechanical energy, the energy may manifest itself in many other forms. Some of these forms are: thermal energy, electrical energy, chemical energy, Visual light energy, nuclear energy etc.

Equivalence of mass and energy :—

According to Einstein, mass and energy are inter-convertible. That is, mass can be converted into energy and energy can be converted into mass.

→ The energy (E) equivalent to mass m is given by the relation

$$E = mc^2$$

where, $c = 3 \times 10^8$ m/s, velocity of vaccum in or air.

☆ when a body moves with a velocity v, comparable to the velocity of light c its mass m is given by

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}} \quad [m_0 = \text{rest mass}]$$

Power :— it is the rate of doing work i.e. The work done per unit time.

☆ $$P = \frac{dw}{dt} = \vec{F} \cdot \vec{v} = Fv \cos a$$

where a is the angle between the force F and the velocity v.

☆ Power is a scalar quantity.

☆ its S.I. unit is watt. [1 watt = 1 J/s].

☆ The dimensional formula of power is $[M^1 L^2 T^{-3}]$

✪ other Commonly used units of power are:—

1 kilowatt = 1 kW = 10^3 W

1 megawatt = 1 MW = 10^3 kW = 10^6 W

1 horse power = 746 watt = 0.746 kW.

Collision:— it is defined as an isolated event in which two or more Colliding bodies exert relatively short time.

✪ Collision between particles have been divided broadly into two types:—

i) **Elastic Collisions:**— A Collision between two particles or bodies is Said to be elastic if both the linear momentum and the kinetic energy of the System remain Conserved.

e.g. Collisions between atomic particles, atoms, marble balls and billiard balls.

ii) **Inelastic Collision:**— A collision b/w two particles or bodies is Said to be elastic if both the linear momentum and the kinetic energy of the system remain Conserved.

A Collision is Said to be inelastic if the linear momentum of the system remains Conserved but its Kinetic energy is not Conserved.

ex. when we drop a ball of wet putty on to the the floor then the Collision between ball and floor is an inelastic Collision.

Conservative force:- Force are Said to be Conserative in nature if work done against the forces gets Converged in the body in form of potential energy.
E.g. gravitational forces, Elastic forces & all the central forces.

Properties of Conservative forces:-
i) Work done against these forces is Conserved & gets stored in the body in the form of P.E.
ii) Work done against these force is never dissipated by being Converted into nonusable forms of energy like heat, light, Sound etc.
iii) Work done against Conservative forces is a state function & not path function
iv) Work done against Conservative forces is zero in a Complete cycle.

Non - Conservative forces:- These are the forces, work done against which does not get Conserved in the body in the form of potential energy.

Properties of Non-Conservative forces:-

i) Work done against these forces does not get conserved in the body in the form of P.E.

ii) Work done against these forces is always dissipated by being converted into non usable forms of energy like heat, light, sound etc.

iii) Work done against non-conservative force is a path function and not a state function.

iv) Work done against non-conservative force in a complete cycle is not zero.

Average power:- it is defined as the ratio of total work done by the body to total time taken.

$$P_{avg} = \frac{\text{Total work done}}{\text{Total time Taken}}$$

$$= \frac{\Delta W}{\Delta t}$$

Instantaneous power:- Average power evaluated for very short duration of time is known as instantaneous power.

$$P_{inst} = \lim_{\Delta t \to 0} P_{avg}$$

$$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$P_{inst} = \frac{dW}{dt}$$

$$P_{inst} = \frac{d\vec{F} \cdot \vec{S}}{dt}$$

$$P_{inst} = \vec{F} \cdot \frac{d\vec{S}}{dt}$$

$$\boxed{P_{inst} = \vec{F} \cdot \vec{V}}$$

Efficiency :- it is defined as the ratio of power/work/energy output to Power/work/Energy input.

$$n = \frac{P_{output}}{P_{input}} = \frac{E_{output}}{E_{input}} = \frac{W_{output}}{W_{input}}$$

Percentage efficiency :-

Percentage efficiency = efficiency x 100

Collision in one dimensions :-

→ Collision is said to be one dimensional, if the colliding particles, move along the same straight line path both before as well as after the collision.

→ in one dimesional elastic collision the relative velocity of approach before collision is equal to the relative velocity of seperation after collision.

→ if two particles of mass m_1 and m_2 moving with velocities $\bar{u}_1$ and $\bar{u}_2$ respectively collide head on such that $\bar{v}_1$ and $\bar{v}_2$ be their respective velocities after collision, then,

$$\bar{v}_1 = \frac{(m_1 - m_2)\bar{u}_1 + 2m_2\bar{u}_2}{(m_1 + m_2)}$$

$$\bar{v}_2 = \frac{2m_1\bar{u}_1 + (m_2 - m_1)\bar{u}_2}{(m_1 + m_2)}$$

Coefficient of Restitution or Coefficient of Resilience :—

it is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision.

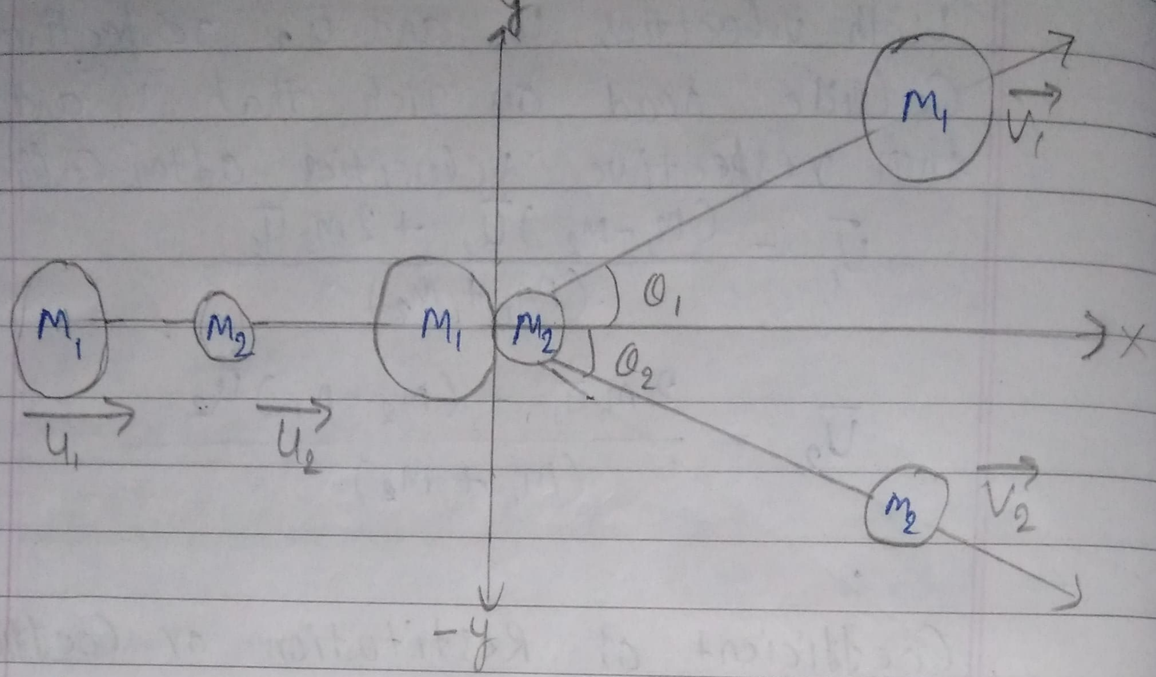
☆ it is represented by 'e'.

$$\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

Collision in two dimensions :—

Consider two bodies A and B of masses m_1 and m_2 moving along the same straight line along x - axis with velocities and u_2 respectively as shown in the figure.

Collision in two dimensions:-



→ By Conservation of momentum along x - axis

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

→ By Conservation of momentum along y-axis

$$m_1 u_{1y} = m_2 u_{2y} = m_1 u_{1y} + m_2 v_{2y}$$

$$m_1 \times 0 + m_2 \times 0 = m_1 v_1 \sin\theta_1 + (-m_2 v_2 \sin\theta_2$$

$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

→ By Conservation of kinetic energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

Physical Quantity	Symbol	Dimensions	Units	Remarks
work	W	$[ML^2T^{-2}]$	J	$W = F \times d$
Kinetic energy	K	$[ML^2T^{-2}]$	J	$K = \frac{1}{2}mv^2$
Potential energy	$V(x)$	$[ML^2T^{-2}]$	J	$F(x) = \frac{dV(x)}{dx}$
Mechanical energy	E	$[ML^2T^{-2}]$	J	$E = K + V$
spring Constant	k	$[MT^{-2}]$	N/m	$F = -kx$ $V(x) = \frac{1}{2}kx^2$
Power	P	$[ML^2T^{-3}]$	W	$P = F \times V$ $P = \frac{dW}{dt}$