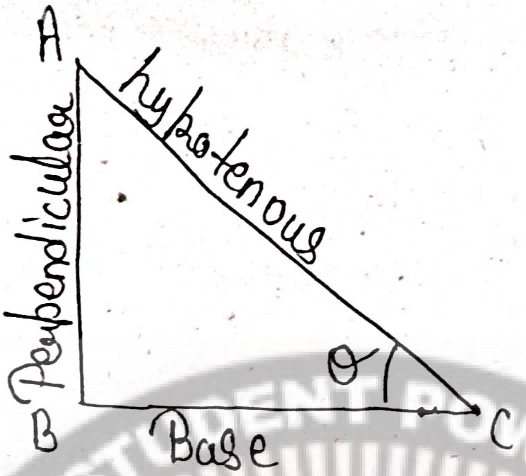
Trigonometry



Sinθ	Cosθ	tanθ
p	b	p
h	h	b
Cosecθ	Secθ	Cotθ

Sinθ	Cosθ	tanθ
L	A	L
K	K	A
Cosecθ	Secθ	Cotθ

* Formula

i) $\sin\theta = \frac{p}{h} = \frac{L}{K}$

ii) $\cos\theta = \frac{b}{h} = \frac{A}{K}$

iii) $\tan\theta = \frac{p}{b} = \frac{L}{A}$

iv) $\cot\theta = \frac{b}{p} = \frac{A}{L}$

v) $\sec\theta = \frac{h}{b} = \frac{K}{A}$

vi) $\operatorname{cosec}\theta = \frac{h}{p} = \frac{K}{L}$

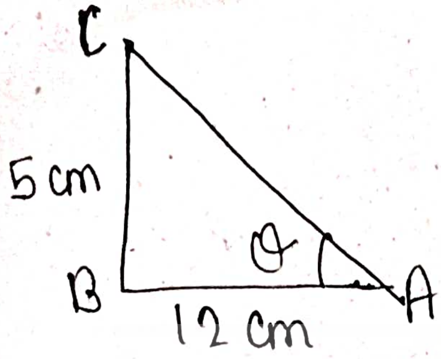
* Formula

i) $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$ / $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

ii) $\cos\theta = \frac{1}{\sec\theta}$ / $\sec\theta = \frac{1}{\cos\theta}$

iii) $\tan\theta = \frac{1}{\cot\theta}$ / $\cot\theta = \frac{1}{}$

iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$



All Cosine function = ?

$p = 5\text{ cm}$, $b = 12\text{ cm}$

$$h = \sqrt{p^2 + b^2}$$
$$= \sqrt{(5)^2 + (12)^2}$$
$$= \sqrt{25 + 144}$$
$$= \sqrt{169}$$
$$= 13\text{ cm}$$

$\sin\theta = \dfrac{p}{h} = \dfrac{5}{13}$, $\operatorname{cosec}\theta = \dfrac{h}{p} = \dfrac{13}{5}$

$\cos\theta = \dfrac{b}{h} = \dfrac{12}{13}$, $\sec\theta = \dfrac{h}{b} = \dfrac{13}{12}$

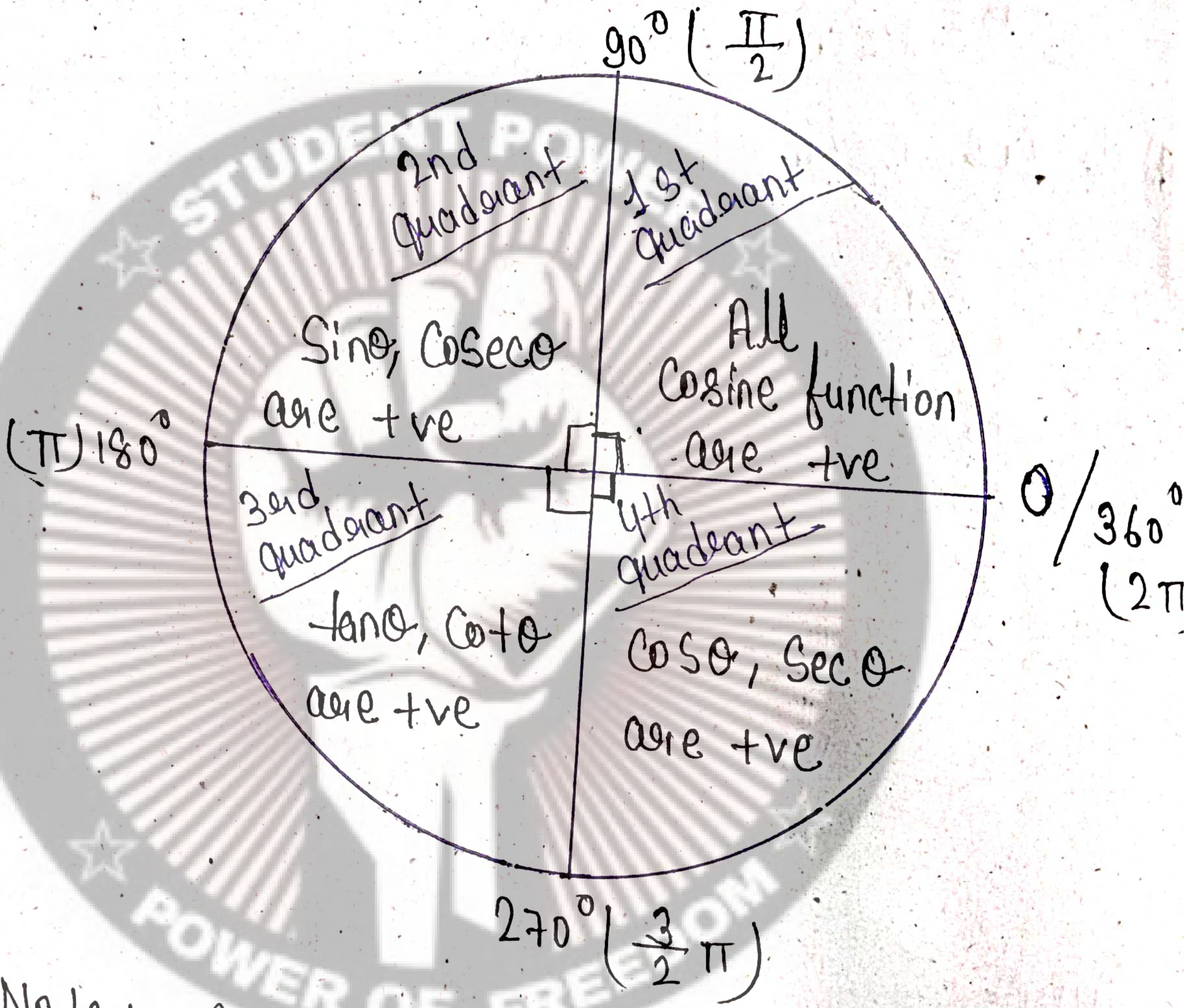
$\tan\theta = \dfrac{p}{b} = \dfrac{5}{12}$, $\cot\theta = \dfrac{b}{p} = \dfrac{12}{5}$

Formula

i) $\sin^2\theta + \cos^2\theta = 1$

ii) $\sec^2\theta - \tan^2\theta = 1$, $\sec^2\theta = 1 + \tan^2\theta$

iii) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$



Note :- All Student to College

All → All Cosine function

Student → Sin/cosec

to → tan/cot

College → cos/sec

* $1^0 = \frac{1}{360^0}$ whole circle

* $1^0 = \frac{1}{60^0} = 1'$ (1 minute)

* $1' = \frac{1}{60} = 1''$ (1 Second)

☆ 1st Quadrant

i) $\sin(90^0 - \theta) = \cos\theta$
ii) $\cos(90^0 - \theta) = \sin\theta$
iii) $\tan(90^0 - \theta) = \cot\theta$
iv) $\cot(90^0 - \theta) = \tan\theta$
v) $\operatorname{cosec}(90^0 - \theta) = \sec\theta$
vi) $\sec(90^0 - \theta) = \operatorname{cosec}\theta$

i) $\sin(360^0 + \theta) = \cos\theta$
ii) $\cos(360^0 + \theta) = \sin\theta$
iii) $\tan(360^0 + \theta) = \cot\theta$
iv) $\cot(360^0 + \theta) = \tan\theta$
v) $\sec(360^0 + \theta) = \operatorname{cosec}\theta$
vi) $\operatorname{cosec}(360^0 + \theta) = \sec\theta$

☆ 2nd quadrant

* i) $\sin(90^0 + \theta) = \cos\theta$
ii) $\cos(90^0 + \theta) = -\sin\theta$
iii) $\tan(90^0 + \theta) = -\cot\theta$
iv) $\cot(90^0 + \theta) = -\tan\theta$.

v) $\sec(90^\circ+\theta) = -\operatorname{cosec}\theta$

vi) $\operatorname{cosec}(90^\circ+\theta) = \sec\theta$

* i) $\sin(180^\circ-\theta) = \sin\theta$

ii) $\cos(180^\circ-\theta) = -\cos\theta$

iii) $\tan(180^\circ-\theta) = -\tan\theta$

iv) $\cot(180^\circ-\theta) = -\cot\theta$

v) $\sec(180^\circ-\theta) = -\operatorname{cosec}\theta$

vi) $\operatorname{cosec}(180^\circ-\theta) = \sec\theta$

☆ 3rd quadrant

* i) $\sin(180^\circ+\theta) = -\sin\theta$

ii) $\cos(180^\circ+\theta) = -\cos\theta$

iii) $\tan(180^\circ+\theta) = \tan\theta$

iv) $\cot(180^\circ+\theta) = \cot\theta$

v) $\sec(180^\circ+\theta) = -\operatorname{cosec}\theta$

vi) $\operatorname{cosec}(180^\circ+\theta) = -\sec\theta$

* i) $\sin(270^\circ-\theta) = -\cos\theta$

ii) $\cos(270^\circ-\theta) = -\sin\theta$

iii) $\tan(270^\circ-\theta) = \cot\theta$

iv) $\cot(270^\circ-\theta) = \tan\theta$

v) $\sec(270^\circ-\theta) = -\operatorname{cosec}\theta$

vi) $\operatorname{cosec}(270^\circ-\theta) = -\sec\theta$

☆ 4^{th} quadrant

i) $\sin(270^\circ+\theta) = -\cos\theta$

ii) $\cos(270^\circ+\theta) = \sin\theta$ 30°

iii) $\tan(270^\circ+\theta) = -\cot\theta$

iv) $\cot(270^\circ+\theta) = -\tan\theta$

v) $\sec(270^\circ+\theta) = \operatorname{cosec}\theta$

vi) $\operatorname{cosec}(270^\circ+\theta) = -\sec\theta$

i) $\sin(360^\circ-\theta) = -\sin\theta$

ii) $\cos(360^\circ-\theta) = \cos\theta$

iii) $\tan(360^\circ-\theta) = -\tan\theta$

iv) $\cot(360^\circ-\theta) = -\cot\theta$

v) $\operatorname{cosec}(360^\circ-\theta) = -\operatorname{cosec}\theta$ 45°

vi) $\sec(360^\circ-\theta) = \sec\theta$

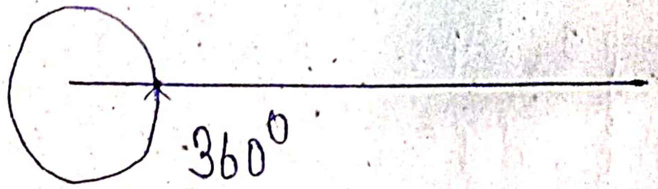
In 1st quadrant All Cosine function will be positive. 30°

In 2nd quadrant $\sin\theta$ and $\operatorname{cosec}\theta$ will be positive.

In 3rd quadrant $\tan\theta$ and $\cot\theta$ will be positive.

In 4^{th} quadrant $\cos\theta$ and $\sec\theta$ will be positive.

★ i) Sexagecimal System (Degree)



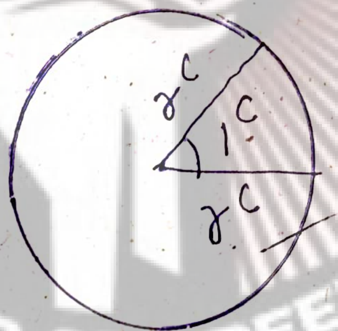
$$1^0 = \frac{1}{360^0}$$

$$1' = \frac{1^0}{60^0}$$

$$1'' = \frac{1'}{60^0}$$

ii) Circular System (radian)

1^c = If arc length equal to its radius.



$$\therefore 180^0 = \pi^c$$

$$\therefore 1^0 = \left(\frac{\pi}{180^0}\right)^c$$

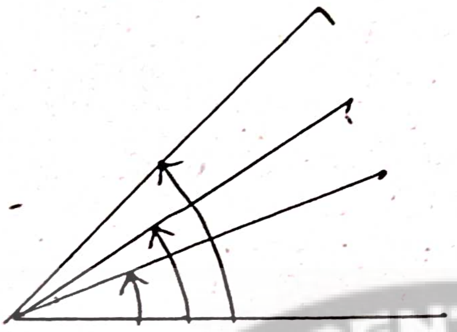
Ex – i) $90^0 = \cancel{90}^1 \times \left(\frac{\pi}{\cancel{180}_2}\right)^c = \frac{\pi}{2}^c$

iii) $270^0 = \cancel{270}^3 \times \frac{\pi}{\cancel{180}_2} = \left(\frac{3}{2}\pi\right)^c$

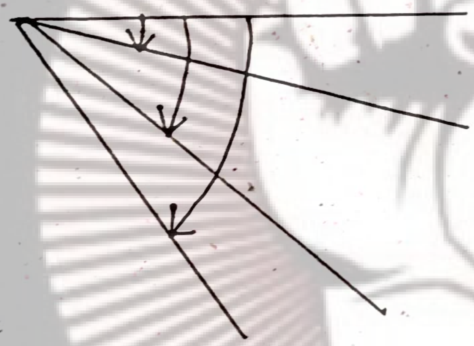
ii) $180^0 = (\pi)^c$

iv) $360^0 = 2\pi$

Positive angle :- If angle measured in Anti-clock-wise (ACW) direction then called positive angle.



Negative angle :- If any angle measure in C.W. direction then It called Negative angle.



degree to radian

30^0

We know that

$$1^0 = \left(\frac{\pi}{180^0}\right)^c$$

Hence

$$30^0 = \cancel{30^0} \times \frac{\pi}{\cancel{180^0}\,6}$$

$$= \left(\frac{\pi}{6}\right)^c$$

ii) 45^0

We know that

$$1^0 = \left(\frac{\pi}{180^0}\right)^c$$

Hence

$$45^0 = \cancel{45^0} \times \frac{\pi}{\cancel{180^0}\,4}$$

$$= \left(\frac{\pi}{4}\right)^c$$

30^0

45^0

30^0

iii) 60°

We know that

$$1^\circ = \left(\frac{\pi}{180^\circ}\right)^c$$

Hence

$$60^\circ = \cancel{60} \times \frac{\pi}{\cancel{180}_{3}} = \left(\frac{\pi}{3}\right)^c$$

iv) 75°

We know that

$$1^\circ = \left(\frac{\pi}{180^\circ}\right)^c$$

Hence

$$75^\circ = \overset{5}{\cancel{75}} \times \frac{\pi}{\underset{12^\circ}{\cancel{180^\circ}}} = \left(\frac{5\pi}{12}\right)^c$$

v) 90°

We know that

$$1^\circ = \left(\frac{\pi}{180^\circ}\right)^c$$

Hence

$$90^\circ = \cancel{90^\circ} \times \frac{\pi}{\cancel{180^\circ}_{2}} = \left(\frac{\pi}{2}\right)^c$$

vi) 105°

We know that

$$1^\circ = \left(\frac{\pi}{180^\circ}\right)^c$$

Hence

$$105^\circ = \overset{7}{\cancel{105}} \times \frac{\pi}{\cancel{180}_{12}} = \left(\frac{7\pi}{12}\right)^c$$

vii) 135°

We know

$$1^\circ = \left(\frac{\pi}{180^\circ}\right)^c$$

Hence

$$135^\circ = \overset{\cancel{9}\,3}{\cancel{135^\circ}} \times$$

$$= \left(\frac{3\pi}{4}\right)^c$$

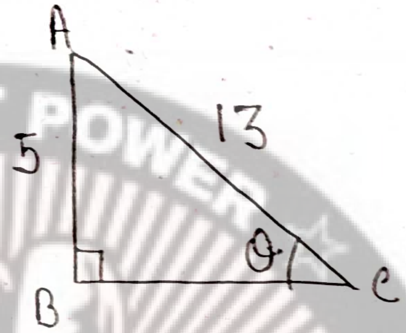
☆ Formula

Angle	0°	30°	45°	60°	90°
	0°	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

Exercise

1) Do as directed :-

a) From the Fig. 6.19, find ~~th~~ tan θ, Cosec θ, Sec θ, and Cot θ.



$$\sin\theta = \frac{p}{h} = \frac{5}{13}$$

$$h^2 = p^2 + b^2$$

$$b = \cancel{\sqrt{p^2+b^2}}\ \sqrt{h^2-p^2}$$

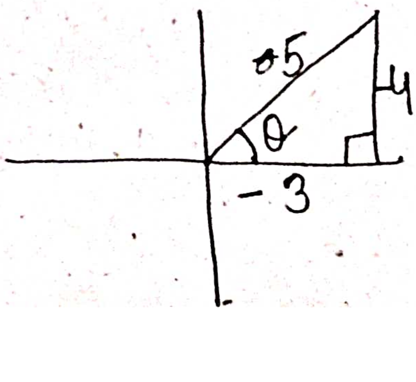
$$= \cancel{\sqrt{5^2+13^2}}\ \sqrt{13^2-5^2}$$

$$= \sqrt{169-25} = \sqrt{144} = 12$$

$$\tan\theta = \frac{p}{b} = \frac{5}{12}, \quad \cot\theta = \frac{b}{p} = \frac{12}{5}$$

$$\operatorname{cosec}\theta = \frac{h}{p} = \frac{13}{5}, \quad \sec\theta = \frac{h}{b} = \frac{13}{12}$$

b) Find out ~~of~~ the values of trigonometric ratios of the angle, if the Co-ordinates of a point on its terminal arm are (-3, -4)



$$\sin\theta = \frac{p}{h} = -\frac{4}{5}$$

$$\cos\theta = \frac{b}{h} = -\frac{3}{5}$$

$$\tan\theta = \frac{p}{b} = \frac{4}{3}$$

$$\cot\theta = \frac{b}{p} = \frac{3}{4}$$

$$\sec\theta = \frac{h}{b} = -\frac{5}{3}$$

$$\operatorname{cosec}\theta = \frac{h}{p} = -\frac{5}{4}$$

c) Evaluate :- i) $\sin^2 60^\circ + \tan^2 45^\circ - \operatorname{cosec}^2 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 - \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= \frac{3}{4} + \frac{1}{1} - \frac{4}{3}$$

$$= \cancel{\frac{9+12-16}{24}} = \frac{3+4-16}{4} = -\frac{9}{4}$$

ii) $\tan^2 45^\circ - \operatorname{cosec}^2 30^\circ + 2\sin 30^\circ$

$$(1)^2 - (2)^2 + \cancel{2} \times \frac{1}{\cancel{2}}$$

$$\Rightarrow 1 - 4 + 1$$

$$\Rightarrow -2 \text{ Ans}$$

$\sin(-45^\circ)\cdot\cos(-45^\circ) + \sin^2 30^\circ\cdot\cos^2(-30^\circ) - \sin(-60^\circ)\cos 30^\circ$

$-\sin 45^\circ\cdot\cos 45^\circ + \sin^2 30^\circ\cdot\cos^2 30^\circ + \sin 60^\circ\cos 30^\circ$

$-\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + \left(\frac{1}{2}\right)^2\cdot\left(\frac{\sqrt{3}}{2}\right)^2 + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$

$-\frac{1}{2} + \frac{1}{4}\cdot\frac{3}{4} + \frac{3}{4}$

$\Rightarrow -\frac{1}{2} + \frac{3}{16} + \frac{3}{4}$

$\Rightarrow \frac{-8+3+12}{16} = \frac{7}{16}$ Ans

Find 'x', if $\sin^2 60^\circ + \tan^2 45^\circ - \operatorname{cosec}^2 30^\circ = x\cdot\cot^3 45^\circ - \sec^2 30^\circ$

$\sin^2 60^\circ + \tan^2 45^\circ - \operatorname{cosec}^2 30^\circ = x\cdot\cot^3 45^\circ - \sec^2 30^\circ$

$\left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 - (2)^2 = x\cdot(1)^3 - \left(\frac{2}{\sqrt{3}}\right)^2$

$\frac{3}{4} + \frac{1}{1} - \frac{4}{1} = x - \frac{4}{3}$

$\frac{3}{4} + \frac{1}{1} - \frac{4}{1} + \frac{4}{3} = x$

$\frac{9+12-48+16}{12} = x$

$$\frac{-11}{12} = x$$

$$\boxed{x = \frac{-11}{12}}$$ Ans

e) If $\sin A = \frac{1}{2}$ and $\frac{\pi}{2} < A < \pi$, find $\cos A$, $\tan A$

$$\sin A = \frac{p}{h} = \frac{1}{2} \qquad 90^\circ < A < 180^\circ$$

$$b = \sqrt{h^2 - p^2}$$
$$= \sqrt{2^2 - 1^2}$$
$$= \sqrt{4-1} = \sqrt{3}$$

$$\boxed{\cos A \Rightarrow \frac{b}{h} = \frac{-\sqrt{3}}{2}}, \quad \boxed{\tan A \Rightarrow \frac{p}{b} = -\frac{1}{\sqrt{3}}}$$ Ans

f) If $\tan A = \frac{1}{\sqrt{3}}$, $\pi < A < \frac{3\pi}{2}$, find $\sin A + \cos A$

$$180^\circ < A < 270^\circ$$

$$\tan A = \frac{1}{\sqrt{3}} = \frac{p}{b}$$

$$h = \sqrt{p^2 + b^2}$$
$$= \sqrt{(1)^2 + (\sqrt{3})^2}$$
$$= \sqrt{4}$$
$$= 2$$

$\sin A + \cos A$

$$\Rightarrow -\frac{1}{2} + \left(-\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow -\frac{1}{2} - \frac{\sqrt{3}}{2}$$

$$= \frac{-1-\sqrt{3}}{2}$$

$$= -\frac{(1+\sqrt{3})}{2} \text{ Ans}$$

If $\cos A = \sin B = -\frac{1}{3}$, $\frac{\pi}{2} < A < \pi$, $\pi < B < \frac{3\pi}{2}$,

find ~~tanA~~ $\frac{\tan A - \tan B}{\tan A + \tan B}$

$\cos A = \frac{b}{h} = \frac{-1}{3}$, $\sin B = \frac{p}{h} = \frac{-1}{3}$

$p = \sqrt{h^2 - b^2}$ $\qquad$ $b = \sqrt{h^2 - p^2}$

$p = \sqrt{3^2 - (-1)^2}$ $\qquad$ $= \sqrt{(3)^2 - (-1)^2}$

$= \sqrt{9-1} = \sqrt{8}$ $\qquad$ $= \sqrt{9-1} = \sqrt{8}$

$\tan A = \frac{p}{b} = \frac{\sqrt{8}}{-1} = -\sqrt{8}$, $\tan B = \frac{p}{b} = \frac{-1}{\sqrt{8}} = +\frac{1}{\sqrt{8}}$

$$\frac{\tan A - \tan B}{\tan A + \tan B} = \frac{-\sqrt{8} - \frac{1}{\sqrt{8}}}{-\sqrt{8} + \frac{1}{\sqrt{8}}}$$

$$= \frac{\frac{-8-1}{\sqrt{8}}}{\frac{-8+1}{\sqrt{8}}} = \frac{-9}{-7} = \frac{9}{7} \text{ Ans}$$

$$\pi^c = 180^\circ$$

$$1^c = \frac{180}{\frac{22}{7}}$$

$$2^c = \frac{180}{\pi} \times 2$$

$$2^c = \frac{360}{\pi}$$

Let angles are $2x$, $3x$ and $5x$

$$\Rightarrow 2x + 3x + 5x = 180^\circ$$

$$10x = 180^\circ$$

$$x = 18^\circ$$

$$2x = 36^\circ$$

$$3x = 54^\circ$$

$$5x = 90^\circ$$

in radian

$$2x = \cancel{36} \times \frac{\pi}{\cancel{180}_{5}} = \frac{\pi}{5}$$

$$3x = \cancel{54}^{3} \times \frac{\pi}{\cancel{180}_{10}} = \frac{3\pi}{10}$$

$$5x = \cancel{90} \times \frac{\pi}{\cancel{180}_{2}} = \frac{\pi}{2}$$

1) Find $\sin \alpha$, if $\tan\left(\frac{\alpha}{2}\right) = 1$

~~We Know~~ that
Given

$$\tan\left(\frac{\alpha}{2}\right) = 1$$

We, Know,

$$\tan \alpha\left(\frac{\alpha}{2}\right) = 45^0$$

$$\cancel{\tan}\ \alpha = 90^0$$

$$\sin \alpha = 90^0$$

$$\boxed{\sin \alpha = 1}$$

2) Prove that

a) $\sec^2\theta - \sin^2\theta \cdot \sec^2\theta = 1$

$$\sec^2\theta \left(1 - \sin^2\theta\right) = 1$$

$$\sec^2\theta \cdot \cos^2\theta = 1$$

$$\frac{1}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} = 1$$

$$\boxed{1 = 1}$$

b) $\cot^2\theta - \cos^2\theta = \cot^2\theta \cdot \cos^2\theta$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$$

$$= \cos^2\theta \cdot \left(\frac{1}{\sin^2\theta} - \cancel{\cos^2}\,1\right)$$

$$= \cos^2\theta \cdot \left(\frac{1 - \sin^2\theta}{\sin^2\theta}\right)$$

$$= \cos^2\theta \cdot \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \boxed{\cos^2\theta \cdot \cot^2\theta} = R.H.S$$

$$\frac{1+2\sin^2\theta}{1+3\tan^2\theta}$$

$$= \frac{1+2\sin^2\theta}{1+3\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1+2\sin^2\theta}{\frac{\cos^2\theta+3\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta(1+2\sin^2\theta)}{\cos^2\theta+\sin^2\theta+2\sin^2\theta}$$

$$= \frac{\cos^2\theta\,\cancel{(1+2\sin^2\theta)}}{\cancel{1+2\sin^2\theta}}$$

$$= \cos^2\theta = R.H.S$$

d) $$\left[\frac{1}{\sin\theta}+\frac{1}{\tan\theta}\right]^2 = \frac{1+\cos\theta}{1-\cos\theta}$$

$$= \left[\operatorname{cosec}\theta + \cot^2\theta\right]^2$$

$$= \left[\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}\right]^2$$

$$= \left[\frac{1+\cos\theta}{\sin\theta}\right]^2$$

$$= \frac{(1+\cos\theta)^2}{\cancel{1-\cos}\sin^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{1^2-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)\cancel{(1+\cos\theta)}}{(1-\cos\theta)\cancel{(1+\cos\theta)}}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = R.H.S$$

e) $\tan^2 A - \sin^2 A = \sin^4 A \cdot \sec^2 A$

$$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$$

$$= \sin^2 A\left(\frac{1}{\cos^2 A} - 1\right)$$

$$= \sin^2 A\left(\frac{1-\cos^2 A}{\cos^2 A}\right)$$

$$= \sin^2 A\left(\frac{\sin^2 A}{\cos^2 A}\right)$$

$$= \frac{\sin^2 A \cdot \sin^2 A}{\cos^2 A}$$

$$= \frac{\sin^4 A}{\cos^2 A}$$

$$= \sin^4 A \cdot \frac{1}{\cos^2 A}$$

$$\Rightarrow \sin^4 A \cdot \sec^2 A = R.H.S$$

f) $(\operatorname{cosec} A - \cot A)^2 = \frac{1-\cos A}{1+\cos A}$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$

$$= \left(\frac{1-\cos A}{\sin A}\right)^2$$

$$= \frac{(1-\cos A)^2}{\sin^2 A}$$

$$= \frac{(1-\cos A)(1-\cos A)}{1-\cos^2 A}$$

$$= \frac{\cancel{(1-\cos A)}(1-\cos A)}{\cancel{(1-\cos A)}(1+\cos A)}$$

$$= \frac{(1-\cos A)}{(1+\cos A)} = R.H.S$$

g) $\frac{\sin A}{1+\cos A} + \frac{1-\cos A}{\sin A} = 2(\operatorname{cosec} A - \cot A)$

$$= \frac{\sin^2 A + (1-\cos A)(1+\cos A)}{(1+\cos A)(\sin A)}$$

$$\frac{\sin^2 A + (1-\cos^2 A)}{(1+\cos A)(\sin A)}$$

$$= \frac{\sin^2 A + \sin^2 A}{(1+\cos A)(\sin A)}$$

$$= \frac{2\sin^2 A}{(1+\cos A)(\sin A}$$

$$= \frac{2 \times 1-\cos^2 A}{\sin A(1+\cos A)}$$

$$= \frac{2 \times (1-\cos A)\cancel{(1+\cos A)}}{\sin A \cancel{(1+\cos A)}}$$

$$= 2 \cdot \frac{1-\cos A}{\sin A}$$

$$= 2 \cdot \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$

$$= 2 \cdot (\operatorname{cosec} A - \cot A) = R.H.S$$

$$\frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A} = 2\operatorname{cosec} A$$

$$= \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)(\sin A)}$$

$$= \cancel{\sin^2 A + (1+\cos A)(1+\cos A)}$$

$$= \frac{\sin^2 A + 1^2 + 2\cos A + \cos^2 A}{\sin A(1+\cos A)}$$

$$= \frac{1 + 1 + 2\cos A}{\sin A(1+\cos A)}$$

$$= \frac{2 + 2\cos A}{(1+\cos A)\sin A}$$

$$= \frac{2\,\cancel{(1+\cos A)}}{\cancel{(1+\cos A)}\sin A}$$

$$= 2 \cdot \frac{1}{\sin A}$$

$$\Rightarrow 2\ \text{cosec} A = R.H.S$$

i) $$\frac{\text{Cosec} A}{\text{Cosec} A - 1} + \frac{\text{Cosec} A}{\text{Cosec} A + 1} = 2\sec^2 A$$

$$\frac{\text{Cosec}^2 A + \cancel{1} + \text{Cosec}^2 A - \cancel{1}}{(\text{Cosec} A - 1)(\text{Cosec} A + 1)}$$

$$\frac{2\ \text{Cosec}^2 A}{\text{Cosec}^2 A - 1^2} = \frac{2\ \text{Cosec}^2 A}{\text{Cot}^2 A}$$

$$= 2 \cdot \frac{\frac{1}{\cancel{\sin^2 A}}}{\frac{\cos^2 A}{\cancel{\sin^2 A}}}$$

$$\Rightarrow 2 \cdot \frac{1}{\cos^2 A} = 2 \sec^2 A = R.H.S$$

Proved

$$\frac{\sec\theta - 1}{\tan\theta} = \frac{\tan\theta}{\sec\theta + 1}$$

$$L.H.S = \frac{\sec\theta - 1}{\tan\theta}$$

$$= \frac{\frac{1}{\cos\theta} - \frac{1}{1}}{\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{1-\cos\theta}{\cancel{\cos\theta}}}{\frac{\sin\theta}{\cancel{\cos\theta}}}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta - \cot\theta$$

$$R.H.S = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta} + 1}$$

$$= \frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1+\cos\theta}{\cancel{\cos\theta}}}$$

$$= \frac{\sin\theta}{1+\cos\theta} \times \frac{1-\cos\theta}{1-\cos\theta}$$

$$= \frac{\sin\theta(1-\cos\theta)}{1^2 - \cos^2\theta}$$

$$= \frac{\cancel{\sin\theta}(1-\cos\theta)}{\sin^{\cancel{2}}\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta - \cot\theta$$

K) $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$

L.H.S

$\sqrt{\sec^2 A + \operatorname{cosec}^2 A}$

$= \sqrt{\dfrac{1}{\cos^2 A} + \dfrac{1}{\sin^2 A}}$

$= \sqrt{\dfrac{(\sin^2 A + \cos^2 A)}{\cos^2 A \cdot \sin^2 A}}$

$= \sqrt{\dfrac{1}{\cos^2 A \cdot \sin^2 A}}$

$= \sqrt{\sec^2 A \cdot \operatorname{cosec}^2 A}$

$= \sec A \cdot \operatorname{cosec} A$

$\tan A + \cot A$

$= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$

$= \dfrac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}$

$= \dfrac{1}{\cos A \cdot \sin A}$

$= \sec A \cdot \operatorname{cosec} A$

$\therefore$ L.H.S = R.H.S

2nd

$\sqrt{(1+\tan^2 A) + (1+\cot^2 a)}$

$= \sqrt{1 + \tan^2 a + 1 + \cot^2 a}$

$= \sqrt{2 + \tan^2 a + \cot^2 a}$

$= \sqrt{2 + (\tan^2 a + \cot^2 a) - 2 \cancel{\cos \tan\theta} + 2 \cancel{\cos \tan}}$

$= \sqrt{2 + (\tan a + \cot^2 a)^2 - 2 \sin}$

$\tan a + \cot a$

$\cot^4 A + \cot^2 A = \operatorname{cosec}^4 A - \operatorname{cosec}^2 A$

L.H.S

$\cot^4 A + \cot^2 A$

$= \cot^2 A (\cot^2 A + 1)$

$= \cot^2 A \cdot \operatorname{cosec}^2 A$

$= (\operatorname{cosec}^2 A - 1) \cdot \operatorname{cosec}^2 A$

$= \operatorname{cosec}^4 A - \operatorname{cosec}^2 A$

3) Proof that

a) $\sqrt{\dfrac{1+\cos A}{1-\cos A}} = \operatorname{cosec} A + \cot A$

L.H.S $= \sqrt{\dfrac{1+\cos A}{1-\cos A} \times \dfrac{(1+\cos A)}{(1+\cos A)}}$

$= \sqrt{\dfrac{(1+\cos A)^2}{1^2 - \cos^2 A}}$

$= \sqrt{\dfrac{(1+\cos A)^2}{\sin^2 A}}$

$= \dfrac{1+\cos A}{\sin A}$

$$\frac{1}{\sin A} + \frac{\cos A}{\sin A}$$

$$= \operatorname{cosec} A + \cot A = R.H.S$$

b) $$\frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = 2 \operatorname{cosec} A$$

$$= \tan \left\{ \frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} \right\}$$

$$= \tan A \left\{ \frac{\sec A \cancel{+1} + \sec A \cancel{-1}}{\sec^2 - 1} \right\}$$

$$= \tan A \left\{ \frac{2 \sec A}{\tan^2 A} \right\}$$

$$= \frac{\cancel{\tan A} \cdot 2 \sec A}{\tan^{\cancel{2}} A}$$

$$= 2 \cdot \frac{\frac{1}{\cancel{\cos A}}}{\frac{\sin A}{\cancel{\cos A}}} = 2 \cdot \frac{1}{\sin A}$$

$$= 2 \operatorname{cosec} A \text{ Ps}$$

$$\frac{\sin A}{1-\cot A} + \frac{\cos A}{1-\tan A} = \sin A + \cos A$$

$$\frac{\sin A}{\frac{1}{1} - \frac{\cos A}{\sin A}} + \frac{\cos A}{\frac{1}{1} - \frac{\sin A}{\cos A}}$$

$$\frac{\sin A}{\frac{\sin A - \cos A}{\sin A}} + \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}}$$

$$\frac{\sin^2 A}{\sin A - \cos A} + \frac{\cos^2 A}{\cos A - \sin A}$$

$$\frac{\sin^2 A}{(\sin A - \cos A)} - \frac{\cos^2 A}{(\sin A - \cos A)}$$

$$\frac{\sin^2 A - \cos^2 A}{(\sin A - \cos A)}$$

$$\frac{(\sin A + \cos A)\cancel{(\sin A - \cos A)}}{\cancel{(\sin A - \cos A)}}$$

$$\sin A + \cos A = R.H.S$$

d) $\left(1+\cot A-\operatorname{cosec} A\right)\left(1+\tan A+\sec A\right)=2$

$$=\left(\frac{1}{1}+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(\frac{1}{1}+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)=2$$

$$\left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$

$$=\frac{\{(\sin A+\cos A)-1\}\{(\sin A+\cos A)+1\}}{\sin A\cdot\cos A}$$

$$=\frac{(\sin+\cos A)^2-(1)^2}{\sin A\cdot\cos A}$$

$$=\frac{\sin^2A+\cos^2A+2\cdot\sin A\cdot\cos A-1}{\sin A\cdot\cos A}$$

$$=\frac{\cancel{1}+2\cancel{\sin A\cdot\cos A}-\cancel{1}}{\cancel{\sin A\cdot\cos A}}$$

$$=2=R.H.S$$

e) $2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+1=0$

$$L.H.S=2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+1$$

$$=2\left\{(\sin^2\theta)^3+(\cos^2\theta)^3\right\}-3\left(\sin^4\theta+\cos^4\theta\right)+1$$

$$=2\left\{(\sin^2\theta+\cos^2\theta)\cdot(\sin^4\theta-\sin^2\theta\cdot\cos^2\theta+\cos^4\right.$$

$-3(\sin^4\theta + \cos^4\theta) + 1$

$2(\sin^4\theta + \cos^4\theta - \sin^2\theta \cdot \cos^2\theta) - 3(\sin^4\theta + \cos^4\theta)$

$= 2(\sin^4\theta + \cos^4\theta \cancel{)} - \sin^2\theta \cdot \cos^2\theta) - 3(\sin^4\theta + \cos^4\theta)$

$+1$

$= 2\sin^4\theta + 2\cos^4\theta - 2\sin^2\theta \cdot \cos^2\theta - 3\sin^4\theta - 3\cos^4\theta$

$+1$

$= -\sin^4\theta - \cos^4\theta - 2\sin^2\theta \cdot \cos^2\theta + 1$

$= -\left[(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2 \cdot \sin^2\theta \cdot \cos^2\theta\right] + 1$

$= -\left[(\sin^2\theta + \cos^2\theta)^2\right] + 1$

$= -1 + 1$

$= 0 = R.H.S$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$(A - B$

B

$\sin^2\theta$

f) $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \sec A + \tan A$

Dividing Num & Deno. by Cos A

$$L.H.S = \dfrac{\dfrac{\sin A - \cos A + 1}{\cos A}}{\dfrac{\sin A + \cos A - 1}{\cos A}}$$

$1 + \tan^2 A = \sec^2 A$

$1 = \sec^2 A - \tan^2 A$

$1 = (\sec A + \tan A)(\sec A - \tan A)$

$$= \dfrac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$$

$$= \dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \dfrac{(\tan A + \sec A) - (\sec A + \tan A)\cdot(\sec A - \tan A)}{\tan A - \sec A + 1}$$

$$= \dfrac{(\tan A + \sec A)\{1 - (\sec A - \tan A)\}}{\tan A - \sec A + 1}$$

$$= \dfrac{(\tan A + \sec A)\cancel{(1 - \sec A + \tan A)}}{\cancel{\tan A - \sec A + 1}}$$

$= \tan A + \sec A$

$= \sec A + \tan A = R.H.S$

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

$$L.H.S = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\tan\theta + \sec\theta - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan\theta + \sec\theta)\{1 - (\sec\theta - \tan\theta)\}}{\tan\theta - \sec\theta + 1}$$

$$= \frac{(\tan A + \sec A)\cancel{(1 - \sec A + \tan A)}}{\cancel{\tan A - \sec A + 1}}$$

$$= \tan A + \sec A$$

$$= \sec A + \tan A = R.H.S$$

$$\frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta + \tan\theta$$

h) $$\frac{1+\sin\theta}{1+\cos\theta} + \frac{1-\sin\theta}{1-\cos\theta} = 2(\csc^2\theta - \cot\theta)$$

$$L.H.S = \frac{1+\sin\theta}{1+\cos\theta} + \frac{1-\sin\theta}{1-\cos\theta}$$

$$= \frac{(1+\sin\theta)\cdot(1-\cos\theta) + (1-\sin\theta)\cdot(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cancel{\cos\theta}+\cancel{\sin\theta}-\sin\theta\cdot\cos\theta+1+\cancel{\cos\theta}-\cancel{\sin\theta}-\sin\theta\cdot\cos\theta}{1-\cos^2\theta}$$

$$= \frac{2-2(\sin\theta\cdot\cos\theta)}{\sin^2\theta}$$

$$= \frac{2}{\sin^2\theta} - \frac{2(\cancel{\sin\theta}\cdot\cos\theta)}{\sin^{\cancel{2}}\theta}$$

$$= 2\operatorname{cosec}^2\theta - 2\cot\theta$$

$$= 2(\operatorname{cosec}^2\theta - \cot\theta) = R.H.S$$

(i) $2+\frac{\sin^4\theta+\cos^4\theta}{\sin^2\theta\cdot\cos^2\theta} = \sec^2\theta\cdot\operatorname{cosec}^2\theta$

$$\Rightarrow 2+\frac{\cancel{\sin^2\theta}\cdot\sin^2\theta}{\cancel{\sin^2\theta}\cdot\cos^2\theta}+\frac{\cancel{\cos^2\theta}\cdot\cos^2\theta}{\sin^2\theta\cdot\cancel{\cos^2\theta}}$$

$$\Rightarrow 2+\tan^2\theta+\cot^2\theta$$

$$\Rightarrow 2+\sec^2\theta-1+\operatorname{cosec}^2\theta-1$$

$$\sec^2\theta+\operatorname{cosec}^2\theta+\cancel{2}-\cancel{2}$$

$$= \sec^2\theta+\operatorname{cosec}^2\theta$$

2) $\tan^4$

i) If $\tan A + \cot A = 5$, show that $\tan^4 A + \cot^4 A = 527$

$\tan A + \cot A = 5$

squaring both side

$2\tan A + \cot A = \tan^2 + \cot^2 A = 25$

$\Rightarrow 2 + \tan^2 A + \cot^2 A = 25$

$\tan^2 A + \cot^2 A = 25 - 2$

$(\tan^2 A + \cot^2 A)^2 = (23)^2$

$\tan^4 + A\cot^4 A + 2\tan^2 A \cdot \cos^2 A = 529$

$\tan^4 A + \cos^4 A = 529 - 2$

$\tan^4 A + \cot^4 A = 527$ Ans

k) If $a^2 \sec^2\theta - b^2 \tan^2\theta = c^2$) than show that $\operatorname{cosec}^2\theta = \dfrac{c^2 - b^2}{c^2 - a^2}$

$$\Rightarrow \frac{c^2 - b^2}{c^2 - a^2} = \operatorname{cosec}^2\theta$$

$$\Rightarrow \frac{a^2\sec^2\theta - b^2 + \tan^2\theta - b^2}{a^2\sec^2\theta - b^2\tan^2\theta - a^2}$$

$$\Rightarrow \frac{a^2\sec^2\theta - b^2(\tan^2 a + 1)}{+a^2(-1+\sec^2\theta) - b^2\tan^2 a.}$$

$$\Rightarrow \frac{a^2\sec^2\theta - b^2(\sec^2\theta)}{+a^2(\tan^2\theta) - b^2\tan^2\theta}$$

$$\Rightarrow \frac{\sec^2\theta\,\cancel{(a^2-b^2)}}{\cancel{+a^2 - b^2}\,(\tan^2\theta)}$$

$$\Rightarrow \frac{1}{\cancel{\cos^2\theta}} \quad \frac{\cancel{\cos^2\theta}}{\sin^2\theta}$$

$$\Rightarrow \operatorname{cosec}^2\theta \quad \text{Ans}$$

Exercise

1) Without using Calculator, find the value of :-

a) $\cos 15^\circ$

$= \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ As

b) $\sin 75^\circ$

$= \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ As

c) $\cos 105^\circ$

$= \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ As

d) $\tan 105^\circ$

$= \frac{\sin 105^\circ}{\cos 105^\circ}$

$= \frac{\sin(60^\circ + 45^\circ)}{\cos(60^\circ + 45^\circ)}$

$= \frac{\sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ}{\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ}$

$= \frac{\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}}{\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}}$

$= \frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{1-\sqrt{3}}{2\sqrt{2}}}$

$= \sqrt{3}+1$ As

Exercise

1) Without using Calculator, find the value of :-

a) $\cos 15^\circ$

$= \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cdot \cos 30^\circ + \sin 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

b) $\sin 75^\circ$

$= \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ Ans

c) $\cos 105^\circ$

$= \cos(60^\circ + 45^\circ)$

$= \cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$ Ans

d) $\tan 105^\circ$

$= \frac{\sin 105^\circ}{\cos 105^\circ}$

$= \frac{\sin(60^\circ + 45^\circ)}{\cos(60^\circ + 45^\circ)}$

$= \frac{\sin 60^\circ \cdot \cos 45^\circ + \cos 60^\circ \cdot \sin 45^\circ}{\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \cdot \sin 45^\circ}$

$= \frac{\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2} \cdot \frac{1}{\sqrt{2}}}{\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}}}$

$= \frac{\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{1-\sqrt{3}}{2\sqrt{2}}}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}}$ Ans

Without using Calculator, find the value of :-

a) $\sin 120^0$

$= \sin(90^0 + 30^0)$

$= \cos 30^0$

$= \frac{\sqrt{3}}{2}$

b) $\cos(-120^0)$

$= \cos 120^0$

$= \cos(90^0+30^0)$

$= -\sin 30^0$

$= -\frac{1}{2}$

c) $\sec(-660^0)$

$= \sec 660^0$

$= \sec(720^0 - 60^0)$

$= \sec 60^0$

$= 2$

d) $\sec^2(-765^0)$

$= \sec^2(765^0)$

$= \sec^2(720^0 + 45^0)$

$= \sec^2 45^0$

$= (\sqrt{2})^2$

$= 2$ Ans.

e) $\sin 4620^0$

$= \sin(4680^0 - 60^0)$ $(360^0 \times 13 =$

$= \sin 60^0$

$= -\frac{\sqrt{3}}{2}$

3) Find the value of :-

a) $\sin 420^0 \cdot \cos 390^0 + \cos(-300^0) \cdot \sin(330^0)$

$= \sin 420^0 = \sin(360^0 + 60^0) = \sin 60^0 = \frac{\sqrt{3}}{2}$

$= \cos 390^0 = \cos(360^0 + 30^0) = \cos 30^0 = \frac{\sqrt{3}}{2}$

$= \cos(-300^0) = \cos(360^0 - 60^0) = \cos 60^0 = \frac{1}{2}$

$= \sin(330^0) = \sin(360^0 - 30^0) = -\sin 30^0 = -\frac{1}{2}$

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \left(-\frac{1}{2}\right)$

$= \frac{3}{4} - \frac{1}{4}$

$= \frac{\cancel{2}\,1}{\cancel{4}\,2}$ Ans

b) $\cos 30^\circ + \sin 240^\circ + \tan 60^\circ + \tan 120^\circ$

$\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2}\right) + \sqrt{3} + (-\sqrt{3})$

$\cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{\sqrt{3}}{2}} + \cancel{\sqrt{3}} - \cancel{\sqrt{3}}$

$= 0$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\cos 6$

$\sin 240^\circ =$

$\sin(180^\circ + 60^\circ) = -\sin 60^\circ$

$= -\frac{\sqrt{3}}{2}$

$\tan 60^\circ = \frac{1}{\sqrt{3}}$

$\tan(90^\circ + 30^\circ)$

$\cot 30^\circ = -\sqrt{3}$

c) $\sin(-690^\circ) \cdot \cos(-330^\circ) + \cos(-750^\circ) \cdot \sin(-240^\circ)$

$-\sin 690^\circ \cdot \cos 330^\circ + \cos 750^\circ \cdot - \sin 240^\circ$

$\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \left(-\frac{\sqrt{3}}{2}\right)$

$\sqrt{3}$

$\sin(720^\circ - 30^\circ) = \sin 30^\circ$

$= \frac{-1}{2}$

$\cos(360^\circ - 30^\circ) = \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

$\cos(720^\circ + 30^\circ) = \cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

$\sin 240^\circ$

$\sin(180^\circ + 60^\circ) =$

$\sin 60^\circ = -\frac{\sqrt{3}}{2}$

$\tan(-495^\circ) \cdot \cot(-405^\circ) + \cot(495^\circ) \cdot \tan(-585^\circ)$

$-1 \cdot 1 + (-1) \cdot (-1)$

$-1 + 1$

$= 0$ Ans

$\tan(-495^\circ)$

$-\tan(450^\circ + 45^\circ)$

$-\cot 45^\circ = -1$

$\cot(-405^\circ)$

$-\cot(360^\circ + 45^\circ)$

$\cot 45^\circ = 1$

$\cot(495^\circ)$

$\cot(450^\circ + 45^\circ)$

$-\tan 45^\circ = -1$

$\tan(-585^\circ)$

$-\tan(540^\circ + 45^\circ)$

$-\cot 45^\circ = -1$

c) $\sin(-690^\circ) \cdot \cos$

$= -\sin 690^\circ \cdot \cos 330^\circ - \cos 750^\circ \cdot \sin 240$

$-\sin(720^\circ - 30^\circ) \cdot \cos(360 - 30) - \cos(720^\circ + 30^\circ) \cdot \sin(360 - 120^\circ)$

$= -(-\sin 30^\circ) \cdot \cos 30^\circ - \cos 30 (-\sin 120^\circ)$

$\sin 30 \cdot \cos 30 + \cos 30^\circ \cos 30^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{4} + \frac{3}{4} = \frac{3 + \sqrt{3}}{4}$

4) Prove that :-

a) $\sin(2A+3B)\cdot\cos(2A-3B) + \cos(2A+3B)\cdot\sin(2A-3B) = \sin$

$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$

$\sin\{(2A+3B) + (2A-3B)\}$

$\sin(2A+\cancel{3B}+2A-\cancel{3B})$

$\sin 4A$ //

b) $\sin(45^\circ+A)\cdot\cos(45^\circ-B) + \cos(45^\circ+A)\cdot\sin(45^\circ-B) =$ $\cos(A-B$

$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$

$\sin\{(45^\circ+A) + (45^\circ-B)\}$

$\sin(45^\circ+A+45^\circ-B)$

$\sin\{90^\circ+(A-B)\}$

$\cos(A-B)$ //

$$c) \quad \frac{\cos 21^\circ - \sin 21^\circ}{\cos 21^\circ + \sin 21^\circ} = \tan 24^\circ$$

L.H.S :-
$$= \frac{\cancel{\cos 21^\circ}\left(1 - \frac{\sin 21^\circ}{\cos 21^\circ}\right)}{\cancel{\cos 21^\circ}\left(1 + \frac{\sin 21^\circ}{\cos 21^\circ}\right)}$$

$$= \frac{1 - \tan 21^\circ}{1 + \tan 21^\circ}$$

$$= \frac{\tan 45^\circ - \tan 21^\circ}{1 + \tan 45^\circ \cdot \tan 21^\circ}$$

$$= \tan(45^\circ - 21^\circ)$$

$$\Rightarrow \tan 24^\circ = R.H.S$$

$$d) \quad \frac{\cos 35^\circ - \sin 35^\circ}{\cos 35^\circ + \sin 35^\circ} = \tan 10^\circ$$

L.H.S :-
$$= \frac{\cancel{\cos 35^\circ}\left(1 - \frac{\sin 35^\circ}{\cos 35^\circ}\right)}{\cancel{\cos 35^\circ}\left(1 + \frac{\sin 35^\circ}{\cos 35^\circ}\right)}$$

$$= \frac{1 - \tan 35^\circ}{1 + \tan 35^\circ}$$

$$= \frac{\tan 45^\circ - \tan 35^\circ}{1 + \tan 45^\circ \cdot \tan 35^\circ}$$

$$= \tan(45^\circ - 35^\circ)$$

$$= \tan 10^\circ \text{ proved}$$

$$\frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta} = \sec\theta$$

$$L.H.S = \frac{\sin 2\theta}{\sin\theta} - \frac{\cos 2\theta}{\cos\theta}$$

$$= \frac{2\cancel{\sin\theta}\cdot\cos\theta}{\cancel{\sin\theta}} - \frac{(\cos^2\theta - \sin^2\theta)}{\cos\theta}$$

$$= \frac{2\cos\theta}{1} - \frac{(\cos^2\theta - \sin^2\theta)}{\cos\theta}$$

$$\frac{2\cos^2\theta - (\cos^2\theta - \sin^2\theta)}{\cos\theta}$$

$$= \frac{2\cos^2\theta - \cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$= \sec\theta = R.H.S$$

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$= \frac{2\tan\theta}{1-\tan^2\theta}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

(f) $1 + \cot\theta \cdot \cot 2\theta = \cot\theta \cdot \operatorname{cosec} 2\theta$

$1 + \cot\theta \cdot \cot 2\theta$

$\frac{1}{1} + \frac{\cos\theta}{\sin\theta} \cdot \frac{\cos 2\theta}{\sin 2\theta}$

$$= \frac{\sin\theta \cdot \sin 2\theta + \cos\theta \cdot \cos 2\theta}{\sin\theta \cdot \sin 2\theta}$$

$$= \frac{\cos(\theta - 2\theta)}{\sin\theta \cdot \sin 2\theta}$$

$$= \frac{\cos(-\theta)}{\sin\theta \cdot \sin 2\theta} = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin 2\theta}$$

$$= \cot\theta \cdot \operatorname{cosec} 2\theta$$

g) $\sin^2(A+B) - \sin^2(A-B) = \sin 2A \cdot \sin 2B$

L.H.S

$\left[\sin(A+B) - \sin(A-B)\right]\left[\sin(A+B) + \sin(A-B)\right]$

$\left[\cancel{\sin A \cdot \cos B} + \cos A \cdot \sin B - \left(\cancel{\sin A \cdot \cos B} - \cos A \cdot \sin B\right)\right]$

$\left[\sin A \cdot \cos B + \cancel{\cos A \cdot \sin B} + \sin A \cos B - \cancel{\cos A \cdot \sin B}\right]$

$(2\cos A \cdot \sin B)(2\sin A \cdot \cos B)$

$(2\sin A \cdot \cos A)(2\sin B \cdot \cos B)$

$\Rightarrow \sin 2A \cdot \sin 2B = R \cdot H \cdot S$

$\sin 75^\circ + \cos 75^\circ = \sqrt{3} \cos 45^\circ$

$\sin(45^\circ + 30^\circ) + \cos(45^\circ + 30^\circ)$

$\sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ + \cos 45^\circ \cdot \sin 30^\circ - \sin 45^\circ \cdot \cos 30^\circ$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$\frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{1-\sqrt{3}}{2\sqrt{2}} \quad \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\cancel{1} + \sqrt{3} + \sqrt{3} - \cancel{1}}{2\sqrt{2}}$

$$\frac{\cancel{2}\sqrt{3}}{\cancel{2}\sqrt{2}}$$

$$= \sqrt{3}\cdot\left(\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{3}\cos 45^\circ$$

(9) $\tan 40^\circ + 2\tan 10^\circ = \tan 50^\circ$

$\tan 50^\circ = \tan(40^\circ + 10^\circ)$

i) $$\frac{\tan 32^\circ + \tan 88^\circ}{1 - \tan 32^\circ \cdot \tan 88^\circ} = -\sqrt{3}$$

$\tan(32^\circ + 88^\circ)$

$\tan 120^\circ$

$\tan(90^\circ + 30^\circ)$

$-\cot 30^\circ$

$-\sqrt{3} = R.H.S$

j) $\tan 20^\circ + 2\tan 50^\circ = \tan 70^\circ$

Since $70^\circ - 50^\circ = 20^\circ$

$= \tan(70^\circ - 50^\circ) = \tan 20^\circ$

$$= \frac{\tan 70^\circ - \tan 50^\circ}{1 + \tan 70^\circ \cdot \tan 50^\circ} = \tan 20^\circ$$

$= \tan 70^\circ - \tan 50^\circ = \tan 20^\circ (1 + \tan 70^\circ \cdot \tan 50^\circ)$

$=$

$\tan 20^\circ + 2\tan 50^\circ = \tan 70^\circ$

$\tan 70^\circ = \tan(50^\circ + 20^\circ)$

$\dfrac{\tan 70^\circ}{1} = \dfrac{\tan 50^\circ + \tan 20^\circ}{1 - \tan 50^\circ \cdot \tan 20^\circ}$

$\tan 20^\circ = \tan(90^\circ - 70^\circ)$
$= \cot 70^\circ$

$\tan\theta \cdot \cot\theta = 1$

$(A-B)^2$ B $\sin^2\theta$

$\tan 50^\circ + \tan 20^\circ = \tan 70^\circ - \tan 70^\circ \cdot \tan 50^\circ \cdot \tan 20^\circ$

$\tan 50^\circ + \tan 20^\circ = \tan 70^\circ - \underbrace{\tan 70^\circ \cdot \tan 50^\circ \cdot \cot 70^\circ}$

$\tan 50^\circ + \tan 20^\circ = \tan 70^\circ - \tan 50^\circ$

$\tan 50^\circ + \tan 20^\circ + \tan 50^\circ = \tan 70^\circ$

Q $\boxed{\tan 20^\circ + 2\tan 50^\circ = \tan 70^\circ}$ Proved

If $A + B = 45^\circ$, Prove that :- $\tan A + \tan B + \tan A \cdot \tan B = 1$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\tan 45^\circ = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\dfrac{1}{1} = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$\tan A + \tan B = 1 - \tan A \cdot \tan B$

$\tan A + \tan B + \tan A \cdot \tan B = 1$

5) Prove that :-

a) $\cos A \cdot \sin(B-C) + \cos B \cdot \sin(C-A) + \cos C \cdot \sin(A-B) = 0$

L.H.S

$\cos A \cdot (\sin B \cdot \cos C - \cos B \cdot \sin C) + \cos B \cdot (\sin C \cdot \cos A - \cos C \cdot \sin A)$
$+ \cos C (\sin A \cdot \cos B - \cos A \cdot \sin B)$

$\Rightarrow \cancel{\cos A \cdot \sin B \cdot \cos C} - \cancel{\cos A \cdot \cos B \cdot \sin C} + \cancel{\cos A \cdot \cos B \cdot \sin C} - \cancel{\cos C \cdot \sin A \cdot \cos B} + \cancel{\cos C \cdot \sin A \cdot \cos B} - \cancel{\cos A \cdot \sin B \cdot \cos C}$

$= 0 = R.H.S$

Proved

b) $\tan\left(\frac{\pi}{4} + A\right) \cdot \tan\left(\frac{\pi}{4} - A\right) = 1$

L.H.S :- $\left\{\tan\left(\frac{\pi}{4} + A\right)\right\} \cdot \left\{\tan\left(\frac{\pi}{4} - A\right)\right\}$

$= \tan\frac{\pi}{4} + \tan A \cdot \tan\frac{\pi}{4} - \tan A$

$= 1 + \cancel{\tan A} \cdot 1 - \cancel{\tan A}$

$= 1 = R.H.S$

Proved

$$\frac{\sin(A-B)}{\sin A\cdot\sin B}+\frac{\sin(B-C)}{\sin B\cdot\sin C}+\frac{\sin(C-A)}{\sin C\cdot\sin A}=0$$

$$L.H.S=\frac{\sin(A-B)}{\sin A\cdot\sin B}+\frac{\sin(B-C)}{\sin B\cdot\sin C}+\frac{\sin(C-A)}{\sin C\cdot\sin A}$$

$$\frac{\sin A\cdot\cos B-\cos A\cdot\sin B}{\sin A\cdot\sin B}+\frac{\sin B\cdot\cos C-\cos B\cdot\sin C}{\sin B\cdot\sin C}+\frac{\sin C\cdot\cos A-\cos C\cdot\sin A}{\sin C\cdot\sin A}$$

$$\frac{\cancel{\sin A}\cdot\cos B}{\cancel{\sin A}\cdot\sin B}-\frac{\cos A\cdot\cancel{\sin B}}{\sin A\cdot\cancel{\sin B}}+\frac{\cancel{\sin B}\cdot\cos C}{\cancel{\sin B}\cdot\sin C}-\frac{\cos B\cdot\cancel{\sin C}}{\sin B\cdot\cancel{\sin C}}+\frac{\cancel{\sin C}\cdot\cos A}{\cancel{\sin C}\cdot\sin A}$$

$$-\frac{\cos C\cdot\cancel{\sin A}}{\sin C\cdot\cancel{\sin A}}$$

$$=\cancel{\cot B}-\cancel{\cot A}+\cancel{\cot C}-\cancel{\cot B}+\cancel{\cot A}-\cancel{\cot C}$$

$=0$ ll

$$\tan 3A-\tan 2A-\tan A=\tan A\cdot\tan 2A\cdot\tan 3A$$

$L.H.S \Rightarrow \tan 3A-\tan 2A-\tan A$

Sol:- $\tan 3A=\tan 2A+\tan A$

$$\tan 3A=\tan(2A+A)$$

$$\tan 3A=\frac{\tan 2A+\tan A}{1\mp\tan 2A\cdot\tan A}$$

$$\tan 3A(1\mp\tan 2A\cdot\tan A)=\tan 2A+\tan A$$

$$\tan 3A\mp\tan 3A\cdot\tan 2A\cdot\tan A=\tan 2A+\tan A$$

$(A-B$ ... ^2B ... $\sin^2\theta$

$$\tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$$

Proved

5. (e) $$\frac{\cot\theta - \cot 2\theta}{\cot\theta + \cot 2\theta} = \frac{\sin\theta}{\sin 3\theta}$$

L.H.S. $$\frac{\dfrac{\cos\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\sin 2\theta}}{\dfrac{\cos\theta}{\sin\theta} + \dfrac{\cos 2\theta}{\sin 2\theta}}$$

$$= \frac{\dfrac{\sin 2\theta \cdot \cos\theta - \cos 2\theta \cdot \sin\theta}{\cancel{\sin\theta \cdot \sin 2\theta}}}{\dfrac{\sin 2\theta \cdot \cos\theta + \cos 2\theta \cdot \sin\theta}{\cancel{\sin\theta \cdot \sin 2\theta}}}$$

$$= \frac{\sin(2\theta - \theta)}{\sin(2\theta + \theta)}$$

$$= \frac{\sin\theta}{\sin 3\theta} = \text{R.H.S}$$

Proved

f) $$\frac{1}{\tan 3\theta - \tan\theta} - \frac{1}{\cot 3\theta - \cot\theta} = \cot 2\theta$$

R.H.S :- $$\frac{1}{\dfrac{\sin 3\theta}{\cos 3\theta} - \dfrac{\sin\theta}{\cos\theta}} - \frac{1}{\dfrac{\cos 3\theta}{\sin 3\theta} - \dfrac{\cos\theta}{\sin\theta}}$$

$$\frac{1}{\dfrac{\sin 3\theta\cdot\cos\theta-\cos 3\theta\cdot\sin\theta}{\cos 3\theta\cdot\cos\theta}}-\frac{1}{\dfrac{\sin\theta\cdot\cos 3\theta-\cos\theta\cdot\sin 3\theta}{\sin 3\theta\cdot\sin\theta}}$$

$$\frac{\cos 3\theta\cdot\cos\theta}{\sin(3\theta-\theta)}-\frac{\sin 3\theta\cdot\sin\theta}{\sin(\theta-3\theta)}$$

$$=\frac{\cos 3\theta\cdot\cos\theta}{\sin 2\theta}-\frac{\sin 3\theta\cdot\sin\theta}{\sin(-2\theta)}$$

$$=\frac{\cos 3\theta\cdot\cos\theta}{\sin 2\theta}+\frac{\sin 3\theta\cdot\sin\theta}{\sin 2\theta}$$

$$=\frac{\cos 3\theta\cdot\cos\theta+\sin 3\theta\cdot\sin\theta}{\sin 2\theta}$$

$$=\frac{\cos(3\theta-\theta)}{\sin 2\theta}$$

$$=\frac{\cos 2\theta}{\sin 2\theta}=\cot 2\theta=R.H.S$$

$$\cos^2 x+\cos^2(60-x)+\cos^2(60+x)=\frac{3}{2}$$

$$R.H.S:-\cos^2 x+\{\cos(60-x)\}^2+\{\cos(60+x)\}^2$$

$$=\cos^2 x+\{\cos 60\cdot\cos x+\sin 60\cdot\sin x\}^2+\{\cos 60\cdot\cos x-\sin 60\cdot\sin x\}^2$$

$$\cos^2 x + \left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)^2 + \left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right)^2$$

$$= \cos^2 x + 2\cdot\left\{\left(\frac{1}{2}\cos x\right)^2 + \left(\frac{\sqrt{3}}{2}\sin x\right)^2\right\}$$

$$= \cos^2 x + 2\left\{\frac{1}{4}\cos^2 x + \frac{\sqrt{3}}{4}\sin^2 x\right\}$$

$$= \cos^2 x + \cancel{2}\times\frac{1}{\cancel{4}\,2}\left\{\cos^2 x + \sqrt{3}\sin^2 x\right\}$$

$$= \cos^2 x + \frac{1}{2}\left(\cos^2 x + \sin^2 x + 2\sin^2 x\right)$$

$$= \cos^2 x + \frac{1}{2}\left\{1 + 2\left(1 - \cos^2 x\right)\right\}$$

$$= \cos^2 x + \frac{1}{2} + \frac{1}{\cancel{2}}\times\cancel{2}\left(1 - \cos^2 x\right)$$

$$= \cancel{\cos^2 x} + \frac{1}{2} + 1 - \cancel{\cos^2 x}$$

$$= \frac{3}{2} = R.H.S$$

$$(a+b)^2 + (a-b)^2$$

$$= a^2 + b^2 + \cancel{2ab} + a^2 + b^2 - \cancel{2ab}$$

$$= 2\left(a^2 + b^2\right)$$

Evaluate :-

$\cancel{\sin^2 37\frac{1}{2}^\circ - \sin^2 7\frac{1}{2}^\circ}$

$\sin^2 75^\circ - \sin^2 15^\circ$

$\sin^2(90^\circ - 15^\circ) - \sin^2 15^\circ$

$\cos^2 15^\circ - \sin^2 15^\circ$

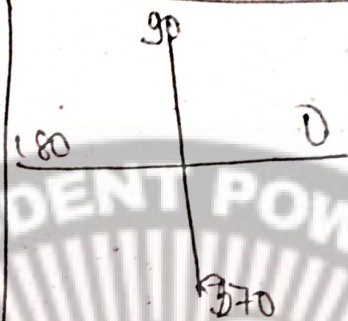
$\cos 2\times 15^\circ$

$\cos 30^\circ$

$= \frac{\sqrt{3}}{2}$

$* \ \sin^2 A - \sin^2 B = \sin(A+B)\cdot\sin(A-B)$

$* \ \cos^2 A - \sin^2 B = \cos(A+B)\cdot\cos(A-B)$



$\sin 90^\circ = 1$

$\sin 270^\circ = -1$

$\cos 0^\circ = 1$

$\cos 180^\circ = -1$

$s(A-B$

^2B

$\sin^2\theta$

$\sin^2 37\frac{1}{2} - \sin^2 7\frac{1}{2}$

$\sin\left(\frac{75}{2}+\frac{15}{2}\right)\cdot\sin\left(\frac{75}{2}-\frac{15}{2}\right)$

$\sin 45^\circ \cdot \sin 30^\circ$

$\frac{1}{\sqrt{2}}\cdot\frac{1}{2}$

$= \frac{1}{2\sqrt{2}}$

b) $\cos^2 7\frac{1}{2} + \cos^2 37\frac{1}{2}^\circ$

$\cos^2\frac{15}{2} + \cos^2\frac{75}{2}$

$\cos^2\frac{15}{2} + 1 - \sin^2\frac{75}{2}$

$= \cos^2\frac{15}{2} - \sin^2\frac{75}{2} + 1$

$= \cos\left(\frac{15}{2}+\frac{75}{2}\right)\cdot\cos\left(\frac{15}{2}-\frac{75}{2}\right) + 1$

$= \cos 45^\circ \cdot \cos 30^\circ + 1$

$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + 1$

$\frac{\cancel{2}+\sqrt{6}}{\cancel{2}\sqrt{2}} = \cancel{\frac{1+\sqrt{6}}{\sqrt{}}}$

$\frac{\sqrt{3}\times\sqrt{2}}{2\sqrt{2}\times\sqrt{2}} + 1$

$\frac{\sqrt{6}}{} + 1$

$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{} = \frac{\sqrt{3}+2\sqrt{2}}{2\sqrt{2}}$

c) $\dfrac{\tan 420^\circ + \tan 300^\circ}{1 - \tan 420^\circ \cdot \tan 300^\circ}$

$\tan(420^\circ + 300^\circ)$.

$\tan 420^\circ + \tan 300^\circ$

$\tan(360^\circ + 60^\circ) + \tan(360^\circ - 60^\circ)$

$\tan 60^\circ + (-\tan 60^\circ)$

$\tan 60^\circ - \tan 60^\circ$

$\sqrt{3} - \sqrt{3}$

$= 0$ Ans

d) $\dfrac{\tan 66^\circ + \tan 69^\circ}{1 - \tan 66^\circ \cdot \tan 69^\circ}$

$\tan(66^\circ + 69^\circ)$

$= \tan 135$

$= \tan(90^\circ + 45^\circ)$

$= -\cot 45^\circ$

$\Rightarrow -1$ Ans

e) $\dfrac{\sec 220^\circ}{\cos 580^\circ} - \dfrac{\tan 940^\circ}{\cot 220^\circ}$

~~Sec (270°~~

$\cos 580^\circ = \cos(1 \cdot 360 + 220)$ $\boxed{\cos(2\pi + \theta) = \cos\theta}$

$= \cos 220^\circ$

$\tan 940^\circ = \tan(2 \times 360^\circ + 220)$ $\boxed{\tan(2\pi + \theta) = \tan\theta}$

$= \tan 220^\circ$

formula	$\sec^2\theta - \tan^2\theta = 1$

$$\frac{\sec 220^\circ}{\cos 580} - \frac{\tan 940}{\cot 220}$$

$$= \frac{\sec 220}{\cos 220} - \frac{\tan 220}{\cot 220}$$

$$= \sec 220 \cdot \sec 220 - \tan 220 \cdot \tan 220$$

$$= \sec^2 220 - \tan^2 220$$

$$= 1 \text{ Ans}$$

$S(A-B$

^2B

$\sin^2\theta$

Do as Directed :-

If $\tan(A+B) = 3$, $\tan(A-B) = 5$, find $\tan 2A$, $\tan 2B$.

$\tan(A+B) = 3$, $\tan(A-B) = 5$

let, $A+B = x$ —(i) , $A-B = y$.

$\tan x = 3$, $\tan y = 5$

Adding (i) and (ii)

$2A = x+y$

Taking tan both sides

$\tan 2A = \tan(x+y)$

$$= \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\frac{3+5}{1-15} = \frac{\cancel{8}\,4}{-\cancel{14}\,7} = -\frac{4}{7} \text{ Ans}$$

Substract (i) and (ii)

$2B = (x-y)$

$\tan B = \tan(x-y)$

$$= \frac{\tan x - \tan y}{1 + \tan x \cdot \tan y}$$

$$= \frac{3-5}{1+3\cdot 5} = \frac{-\cancel{2}\,1}{\cancel{16}\,8} = -\frac{1}{8} \text{ Ans}$$

b) If $\tan(x+y) = \frac{1}{2}$ and $\tan(x-y) = \frac{1}{3}$, evaluate $\tan 2x$ and $\tan 2y$.

$$\tan(x+y) = \frac{1}{2}, \quad \tan(x-y) = \frac{1}{3}$$

$$x+y = A, \quad x-y = B$$

$$\tan A = \frac{1}{2}, \quad \tan B = \frac{1}{3}$$

Adding (i) and (ii)

$$2x = A+B$$

taking tan on both side

$$\tan 2x = \tan(A+B)$$

$$\tan 2x = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{1} - \frac{1}{2}\cdot\frac{1}{3}}$$

$$= \frac{\frac{3+2}{6}}{\frac{6-1}{6}}$$

$$= \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{5}}$$

$$= 1$$

Substract (i) and (ii)

$$2y = A-B$$

taking tan on both side

$$\tan 2y = \tan(A-B)$$

$$\tan 2y = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$= \frac{\frac{1}{2} - \frac{1}{3}}{\frac{1}{1} + \frac{1}{2}\cdot\frac{1}{3}}$$

$$= \frac{\frac{3-2}{6}}{\frac{6+1}{6}}$$

$$= \frac{1}{\cancel{6}} \times \frac{\cancel{6}}{7}$$

$$= \frac{1}{7}$$

If $\sin\alpha = \frac{12}{13}$, $\cos\beta = \frac{3}{5}$, $\frac{\pi}{2} < \alpha < \pi$, $0 < \beta < \frac{\pi}{2}$, find $\cos(\alpha+\beta)$.

$\sin\alpha = \frac{p}{h} = \frac{12}{13}$

$b = 5$

$\cos\alpha = \frac{b}{h} = -\frac{5}{13}$

~~$p = 4$~~

~~$\sin\beta = \frac{p}{}$~~ $\cos\beta = \frac{3}{5} = \frac{b}{h}$

$p = 4$

$\sin\beta = \frac{p}{h} = \frac{4}{5}$

$$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$= -\frac{\cancel{5}}{13} \cdot \frac{3}{\cancel{5}} - \frac{12}{13} \cdot \frac{4}{5}$$

$$= -\frac{3}{13} - \frac{48}{65}$$

$$= \frac{-15-48}{65} = -\frac{63}{65}$$

If $\sin\alpha = -\frac{5}{13}$ and $\cos\beta = -\frac{7}{25}$, α and β lies in the third quadrant, Find $\sin(\alpha-\beta)$

$\sin\alpha = \frac{-5}{13} = \frac{p}{h}$

$b = 12$

$\cos\alpha = -\frac{12}{13}$

$\cos\beta = -\frac{7}{25} = \frac{p}{h}$

$b = 24$

$\sin\beta = -\frac{24}{25}$

e) If $\cos\theta = -\frac{5}{13}$ and $\tan\phi = \frac{4}{3}$, where θ and ϕ belong to the second and third quadrants respectively. Find the quadrant to which $(\theta + \phi)$ lies.

$$\cos\theta = -\frac{5}{13} = \frac{b}{h}$$

$$p = \frac{12}{13}$$

$$\sin\theta = \frac{12}{13}$$

$$\tan\phi = \frac{4}{3} = \frac{p}{b}$$

$$h = 5$$

$$\sin\phi = -\frac{4}{5}$$

$$\cos\phi = -\frac{3}{5}$$

$$\sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\frac{-5}{13} \cdot \frac{-7}{25} - \left(-\frac{12}{13}\right) \cdot \left(\frac{-24}{25}\right)$$

$$\frac{35}{325} - \frac{288}{325}$$

$$\frac{35 - 288}{325} = -\frac{253}{325}$$

c) If $\cos\theta = -\frac{5}{13}$ and $\tan\phi = \frac{4}{3}$, where θ and ϕ belong to the second and third quadrants respectively. Find the quadrant to which $(\theta + \phi)$ lies.

$$\tan\phi = \frac{4}{3} = \frac{p}{b}$$

$$\cos\theta = \frac{-5}{13} = \frac{b}{h}$$

$$\sin\theta = \frac{p}{h} = \frac{12}{13}$$

$$\sin\phi = -\frac{4}{5}$$

$$\cos\phi = -\frac{3}{5}$$

$\sin(\theta+\phi)$

$\sin\theta\cdot\cos\phi+\cos\theta\cdot\sin\phi$

$\frac{12}{13}\times\left(\frac{-3}{5}\right)+\left(\frac{-5}{13}\right)\cdot\left(\frac{-4}{5}\right)$

$\frac{-36}{65}+\frac{20}{65}$

$\Rightarrow\frac{-16}{65}$

$\cos(\theta+\phi)$

$\cos(\theta+\phi)$

$\cos\theta\cdot\cos\phi-\sin\theta\cdot\sin\phi$

$\frac{-5}{13}\cdot\frac{-3}{5}-\frac{12}{13}\cdot\frac{-4}{5}$

$\frac{15}{65}+\frac{48}{65}=\frac{63}{65}$

in $(\theta+\phi)$ is negative but $\cos(\theta+\phi)$ is possitive

hich is possible only in fourth quadrant.

$(\theta+\phi)$ lies in fourth quadrant.

$\alpha-\beta=\frac{\pi}{4}$, then prove that: $(1+\tan\alpha)(1-\tan\beta)=2$

Taking tan both sides,

$\tan(\alpha-\beta)=\tan\frac{\pi}{4}$

$$\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}=\frac{1}{1}$$

$\tan\alpha - \tan\beta = 1 + \tan\alpha \cdot \tan\beta$

$\tan\alpha - \tan\beta - \tan\alpha \cdot \tan\beta = 1$

$(1+\tan\alpha) - \tan\beta - \tan\alpha \cdot \tan\beta = 1+1$

$(1+\tan\alpha) - \tan\beta\,(1+\tan\alpha) = 2$

$(1+\tan\alpha)\,(1-\tan\beta) = 2$

Q) $\alpha + \beta = \frac{\pi}{4}$

$(\cot\alpha - 1)\,(\cot\beta - 1) = 2$

Exercise:- 8·1

1) If $A = 45^\circ$, verify that

a) $\sin 3A = 3\sin A - 4\sin^3 A$

$\sin 3.45^\circ = 3\sin 45^\circ - 4\sin^3 45^\circ$

$\sin 135^\circ = 3 \cdot \frac{1}{\sqrt{2}} - 4 \times \left(\frac{1}{\sqrt{2}}\right)^3$

$\cos 45^\circ = \frac{3}{\sqrt{2}} - \frac{4^2}{2\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{3-2}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

App verified

$\sin 135$

$\sin(90+45)$

$\cos 45^\circ$

$\frac{1}{\sqrt{2}}$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$\tan 3\cdot 45^\circ = \frac{3\tan 45^\circ - \tan^3 45^\circ}{1 - 3\cdot\tan^2 45^\circ}$$

$$\tan 135^\circ = \frac{3\times 1 - 1}{1 - 3\cdot 1}$$

$$-1 = \frac{3-1}{1-3}$$

$$-1 = \frac{\cancel{2}}{-\cancel{2}}$$

$$-1 = -1$$

If $A = 30^\circ$, verify that :-

a) $$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$\sin 2\times 30^\circ = \frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$$

$$\sin 60^\circ = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\frac{\sqrt{3}}{2} = \frac{\frac{\cancel{2}^1}{\sqrt{3}}}{\frac{\cancel{4}2}{3}}$$

$$\frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}\times\frac{3}{2}$$

$$\frac{\sqrt{3}}{2} = \frac{\cancel{\sqrt{3}}\times\sqrt{3}}{\cancel{\sqrt{3}}\times 2}$$ Proved

b) $$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$\Rightarrow \cos 2\times 30^\circ = \frac{1-\tan^2 30}{1+\tan^2 30^\circ}$$

$$\Rightarrow \cos 60^\circ = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$\Rightarrow \frac{1}{2} = \frac{\frac{3-1}{\cancel{3}}}{\frac{3+1}{\cancel{3}}}$$

$$\Rightarrow \frac{1}{2} = \frac{\cancel{2}^1}{\cancel{4}2}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2}$$ Verified

3) If $\theta = 60^\circ$, verify that :-

a) $\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$\sin 2\times60^\circ = 2\sin 60 \cdot \cos 60^\circ$

$\sin 120^\circ = \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ verified

b) $\cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos 2\times60^\circ = 2\times\cos^2 60^\circ - 1$

$\Rightarrow \cos 120^\circ = 2\times\left(\frac{1}{2}\right)^2 - 1$

$\Rightarrow -\sin 30^\circ = \cancel{2}^{1} \times \frac{1}{\cancel{4}2} - \frac{1}{1}$

$\Rightarrow -\frac{1}{2} = \frac{1-2}{2}$

$\Rightarrow -\frac{1}{2} = -\frac{1}{2}$ verified

4) Do as directed :-

a) If $\sin A = \frac{1}{2}$ find $\sin 3A$

$\sin 3A = 3\sin A - 4\sin^3 A$

$= 3\times\frac{1}{2} - 4\times\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - \frac{4}{8}$

$= \frac{12-4}{8}$

$= \frac{8}{8}$

$= 1$

b) If $\sin A = 0.4$, find $\cos 3A$

$\Rightarrow \cos 3A = 4\cos^3 A - 3\cos A$

~~$= 4\times\cos 3\times 0.4 - 3$~~

$\sin A = \frac{0.4}{10} = \frac{p}{h}$, $\cos A = \frac{2\sqrt{21}}{10}$

$b = \sqrt{h^2 - p^2}$

$= \sqrt{10^2 - 4^2}$

$= \sqrt{100-16} = \sqrt{84} = 2\sqrt{21}$

$\cos 3A = 4\times\left(\frac{2\sqrt{21}}{10}\right)^3 - 3\times\left(\frac{2\sqrt{21}}{10}\right)$

$= 4\times 0.7698 - 3\times 0.9165$

$= 3.0792 - 2.7495$

$= 0.3297$

c) If $\cos A = -\frac{12}{13}$ and $\pi < A < \frac{3\pi}{2}$, find (i) $\sin 2A$ (ii) $\cos 2A$

$$\cos A = -\frac{12}{13} = \frac{b}{h}$$

$$\sin A = -\frac{5}{13}$$

i) $\sin 2A$

$$\Rightarrow 2\sin A \cdot \cos A$$

$$= 2\times\left(\frac{-5}{13}\right)\cdot\left(\frac{-12}{13}\right)$$

$$= -\frac{10}{13}\times\frac{-12}{13}$$

$$= \frac{120}{169}$$

ii) $\cos 2A$

$$= 1 - 2\sin^2 A$$

$$= 1 - 2\times\left(\frac{-5}{13}\right)^2$$

$$= \frac{1}{1} - 2\times\frac{25}{169}$$

$$= \frac{1}{1} - \frac{50}{169}$$

$$= \frac{169-50}{169} = \frac{119}{169}$$

If $\tan\theta = \frac{b}{a}$, find the value of $a\cos 2\theta + b\cdot\sin 2\theta$

$$a\cdot\cos 2\theta + b\cdot\sin 2\theta$$

$$= a\cdot\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\cdot\frac{2\tan\theta}{1+\tan^2\theta}$$

$$= \frac{a(1-\tan^2\theta) + 2b\tan\theta}{1+\tan^2\theta}$$

$$= \frac{a\left(\frac{1}{1}-\frac{b^2}{a^2}\right) + 2\cdot b\cdot\frac{b}{a}}{\frac{1}{1}+\frac{b^2}{a^2}}$$

$$\frac{a\left(\frac{a^2-b^2}{a^2}\right) + \frac{2b^2}{a}}{\frac{a^2+b^2}{a}}$$

$$= \frac{\frac{a(a^2-b^2)+2ab^2}{\cancel{a^2}}}{\frac{a^2+b^2}{\cancel{a}}}$$

$$= \frac{a^3 - ab^2 + 2\cdot ab^2}{a^2+b^2}$$

$$= \frac{a^3 + ab^2}{a^2+b^2}$$

$$\frac{a\cancel{(a^2+b^2)}}{\cancel{a^2+b^2}}$$

$= a = R.H.S$

Ans

e) If $\tan\theta = \frac{2}{3}$, find the value of $3\cos 2\theta + 2\sin 2\theta$

$\tan\theta = \frac{2}{3} = \frac{p}{b}$, $\cos\theta = \frac{b}{h} = \frac{3}{\sqrt{5}}$

$h = \sqrt{p^2 + b^2}$

$= \sqrt{2^2 - 3^2}$

$= \sqrt{4-9}$

$= -\sqrt{5}$

$\sin\theta = \frac{p}{h} = \frac{2}{\sqrt{5}}$

$\cos 2\theta = 1 - \sin^2\theta$

$= 1 - \left(\frac{2}{\sqrt{5}}\right)^2$

$= \frac{1}{1} - \frac{4}{5}$

$\Rightarrow \frac{5-4}{5} = \frac{1}{5}$

$\sin 2\theta = 2\sin\theta \cdot \cos\theta$

$= 2 \times \frac{2}{\sqrt{5}} \cdot \frac{3}{\sqrt{5}}$

$= \frac{4}{\sqrt{5}} \cdot \frac{3}{\sqrt{5}}$

$= \frac{12}{5}$

$3\cos 2\theta + 2\sin 2\theta$

$3 \times \frac{1}{5} + 2 \cdot \frac{12}{5}$

$\frac{3}{5} + \frac{12}{5} \Rightarrow \frac{3+12}{5} = \frac{15}{5} = 3$ Ans

If $\tan A = 3$ and $\tan B = 2$, find the values of (i) $\tan(2A+B)$ ii) $\tan(A+2B)$

$\tan A = 3$, $\tan B = 2$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\times 3}{1-3^2}$$

$$= -\frac{\cancel{6}\,3}{\cancel{8}\,4}$$

$$\tan 2B = \frac{2\tan B}{1-\tan^2 B}$$

$$= \frac{2\times 2}{1-4}$$

$$= -\frac{4}{3}$$

i) $\tan(2A+B)$

$$\Rightarrow \frac{\tan 2A + \tan B}{1-\tan 2A\cdot\tan B}$$

$$\Rightarrow \frac{-\frac{3}{4}+\frac{2}{1}}{1-\left(-\frac{3}{4}\right)\cdot 2}$$

$$\Rightarrow \frac{\frac{-3+8}{4}}{1+\frac{3}{4}\cdot 2}$$

$$= \frac{\frac{5}{4}}{\frac{1}{1}+\frac{6}{4}}$$

$$= \frac{\frac{\cancel{5}^1}{\cancel{4}_1}}{\frac{\cancel{10}_2}{\cancel{4}_1}} = \frac{1}{2}$$

ii) $\tan(A+2B)$

$$\Rightarrow \frac{\tan A + \tan 2B}{1-\tan A\cdot\tan 2B}$$

$$\Rightarrow \frac{3+\left(-\frac{4}{3}\right)}{1-3\cdot\left(-\frac{4}{3}\right)}$$

$$= \frac{\frac{3}{1}-\frac{4}{3}}{\frac{1}{1}+\frac{12}{3}}$$

$$= \frac{\frac{9-4}{\cancel{3}}}{\frac{3+12}{\cancel{3}}}$$

$$= \frac{\cancel{5}^1}{\cancel{15}_3} = \frac{1}{3}$$

5) Prove that

a) $$\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$$

L.H.S

$$\frac{\cancel{2}\sin\theta\cdot\cos\theta}{\cancel{1+}\cancel{2}\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \tan\theta = R.H.S$$

b) $$\frac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$$

$$L.H.S = \frac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta}$$

$$= \frac{\sin\theta + 2\sin\theta\cdot\cos\theta}{\cancel{1}+\cos\theta+2\cos^2\theta-\cancel{1}}$$

$$= \frac{\cancel{3}\sin\theta}{\cancel{3}\cos\theta}$$

$$= \tan\theta = R.H.S$$

c) $$\frac{\sin 2\theta + \cos\theta}{1+\sin\theta-\cos 2\theta} = \cot\theta$$

$$\frac{2\sin\theta\cdot\cos\theta + \cos\theta}{1-\cos 2\theta + \sin\theta}$$

$$\frac{\cancel{2\sin\theta}\cdot\cos\theta + \cos\theta}{\cancel{2}\sin^2\theta + \sin\theta}$$

$$= \frac{\cancel{2}\cos\theta}{\cancel{2}\sin\theta}$$

$$= \cot\theta \text{ } Js$$

d) $$\frac{\sin 9\theta}{\sin 3\theta} - \frac{\cos 9\theta}{\cos 3\theta} = 2$$

$$L.H.S :- \frac{\sin 9\theta}{\sin 3\theta} - \frac{\cos 9\theta}{\cos 3\theta}$$

$$= \frac{\sin 9\theta\cdot\cos 3\theta - \cos 9\theta\cdot\sin 3\theta}{\sin 3\theta\cdot\cos 3\theta}$$

$$= \frac{\sin(9\theta - 3\theta)}{\sin 3\theta\cdot\cos 3\theta}$$

$$= \frac{2\times\sin 6\theta}{2\times\sin 3\theta\cdot\cos 3\theta}$$

$$= \frac{2\cancel{\sin 6\theta}}{\cancel{\sin 6\theta}} = 2 = R.H.S$$

formula $[2\sin 3\theta \cdot \cos 3\theta = \sin 6\theta]$

$$\frac{1+\sec 2A}{\tan 2A} = \cot A$$

L.H.S :- $\dfrac{1+\sec 2A}{\tan 2A}$

$$= \frac{1}{\tan 2A} + \frac{\sec 2A}{\tan 2A}$$

$$= \cot 2A + \frac{\frac{1}{\cancel{\cos 2A}}}{\frac{\sin 2A}{\cancel{\cos 2A}}}$$

$$= \frac{\cos 2A}{\sin 2A} + \frac{1}{\sin 2A}$$

$$\Rightarrow \frac{\cos 2A + 1}{\sin 2A} = \frac{\cos 2A + 1}{2\sin A \cdot \cos A}$$

$$= \frac{\cancel{2}\cos^{\cancel{2}} A \cancel{-1} \cancel{+1}}{\cancel{2}\sin A \cdot \cancel{\cos A}} = \frac{\cos A}{\sin A}$$

$$= \cot A = R.H.S$$

f) $\dfrac{\sec 4\theta - 1}{\sec 2\theta - 1} = \dfrac{\tan 4\theta}{\tan \theta}$

L.H.S:- $\dfrac{\sec 4\theta - 1}{\sec 2\theta - 1}$

$= \dfrac{\dfrac{1}{\cos 4\theta} - \dfrac{1}{1}}{\dfrac{1}{\cos 2\theta} - \dfrac{1}{1}}$

$= \dfrac{\dfrac{1 - \cos 4\theta}{\cos 4\theta}}{\dfrac{1 - \cos 2\theta}{\cos 2\theta}}$

$= \dfrac{\dfrac{2\sin^2 2\theta}{\cos 4\theta}}{\dfrac{2\sin^2\theta}{\cos 2\theta}}$

$= \dfrac{\cancel{2}\sin^2 2\theta}{\cos 4\theta} \times \dfrac{\cos 2\theta}{\cancel{2}\sin^2\theta}$

$= \dfrac{\sin 2\theta \cdot \sin 2\theta \cdot \cos 2\theta}{\cos 4\theta \cdot \sin\theta \cdot \sin\theta}$

$= \dfrac{(2 \sin 2\theta \cdot \cos 2\theta) \cdot \sin 2\theta}{2 \cos 4\theta \cdot \sin\theta \cdot \sin\theta}$

$= \dfrac{\sin 4\theta \times \cancel{2}\,\cancel{\sin\theta} \cdot \cos\theta}{\cancel{2}\cos 4\theta \cdot \cancel{\sin\theta} \cdot \sin\theta}$

$= \tan 4\theta \cdot \cot\theta$

$= \tan 4\theta \cdot \dfrac{1}{\tan\theta}$

$= \dfrac{\tan 4\theta}{\tan\theta}$

$1 - \cos 2\theta = 2\sin^2\theta$

$1 - \cos 4\theta = 2\sin^2 2\theta$

$2\sin 2\theta \cdot \cos 2\theta = \sin 4\theta$

$1 + \cos 4\theta = 2\cos^2 2\theta$

$1 + \cos\theta = 2\cos^2\dfrac{\theta}{2}$

$$\sqrt{2+\sqrt{2+2\cos 4\theta}} = 2\cos\theta$$

L.H.S $= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$

$1+\cos 4\theta = 2\cos^2 2\theta$

$1+\cos 2\theta = 2\cos^2\theta$

$= \sqrt{2+\sqrt{2\cdot 2\cos^2 2\theta}}$

$= \sqrt{2+2\cos 2\theta}$

$= \sqrt{2(1+\cos 2\theta)}$

$= \sqrt{2+2\cos^2\theta}$

$= 2\cos\theta$ = R.H.S

$\cos A\cdot\cos(60^\circ - A)\cdot\cos(60^\circ + A) = \frac{1}{4}\cos 3A$

$\cos A\{\cos^2 60^\circ - \sin^2 A\}$

$\cos A\{(\frac{1}{2})^2 - \cos^2 A\}$

$\cos A\{\frac{1}{4} - \frac{\cos^2 A}{1}\}$

$\cos A\{\frac{1-4\cos^2 A}{4}\} = \frac{1}{4}\{\cos A - 4\cos^2 A\}$

$=$

h)

$\Rightarrow \cos A \left[\cos^2 60 - \sin^2 A\right]$

$\cos A \left[\frac{1}{4} - \left(1 - \cos^2 A\right)\right]$

$\cos A \left[\left(\frac{1}{4} - \frac{1}{1}\right) + \cos^2 A\right]$

$\cos A \left[-\frac{3}{4} + \frac{\cos^2 A}{1}\right]$

$\cos A \left[\frac{4\cos^2 A - 3}{4}\right]$

$\frac{4\cos^3 A - 3\cos A}{4}$

$\frac{\cos 3A}{4}$

$\frac{1}{4} \cos 3A = R.H.S$

$\cos(A+B) \cdot \cos(A-B)$
$= \cos^2 A - \sin^2 B$

$$L.H.S = \frac{\frac{\cos 2\theta}{\sin 2\theta}}{\frac{\cos\theta}{\sin\theta} - \frac{\cos 2\theta}{\sin 2\theta}}$$

$$= \frac{\frac{\cos 2\theta}{\sin 2\theta}}{\frac{\sin 2\theta \cdot \cos\theta - \cos 2\theta \cdot \sin\theta}{\sin\theta \cdot \sin 2\theta}}$$

$$= \frac{\frac{\cos 2\theta}{\sin 2\theta}}{\frac{\sin(2\theta - \theta)}{\sin\theta \cdot \sin 2\theta}}$$

$$= \frac{\cos 2\theta}{\cancel{\sin 2\theta}} \times \frac{\cancel{\sin\theta} \cdot \cancel{\sin 2\theta}}{\cancel{\sin\theta}}$$

$$= \cos 2\theta = R.H.S$$

J) $\dfrac{\sin 3\theta - \cos 3\theta}{\sin\theta + \cos\theta} = 2\sin 2\theta - 1$

$= \dfrac{(3\sin\theta - 4\sin^3\theta) - (4\cos^3\theta - 3\cos\theta)}{\sin\theta + \cos\theta}$

$= \dfrac{3\sin\theta + 3\cos\theta - 4\sin^3\theta - 4\cos^3\theta}{\sin\theta + \cos\theta}$

$= \dfrac{3(\sin\theta + \cos\theta) - 4(\sin^3\theta + \cos^3\theta)}{\sin\theta + \cos\theta}$

$= \dfrac{3(\sin\theta + \cos\theta) - 4[(\sin\theta + \cos\theta)\cdot(\sin^2\theta + \cos^2\theta - \sin\theta\cdot\cos\theta)]}{\sin\theta + \cos\theta}$

$= \dfrac{3(\sin\theta + \cos\theta) - 4(\sin\theta + \cos\theta)\cdot(1 - \sin\theta\cdot\cos\theta)}{\sin\theta + \cos\theta}$

$= \dfrac{\cancel{(\sin\theta + \cos\theta)}[3 - 4(1 - \sin\theta\cdot\cos\theta)]}{\cancel{\sin\theta + \cos\theta}}$

$= 3 - 4 + 4\sin\theta\cdot\cos\theta$

$= -1 + 2\cdot 2\sin\theta\cdot\cos\theta$

$= -1 + 2\sin 2\theta$

$= 2\sin 2\theta - 1 = R.S$

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

If $\cos\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$, then show that $\cos 3\theta = \frac{1}{2}\left(x^3 + \frac{1}{x^3}\right)$.

$\{\cos 3A = 4\cos^3 A - 3\cos A\}$

Sol^n :- $\cos 3\theta$

$= 4\cos^3\theta - 3\cos\theta$

$= 4\cdot\left\{\frac{1}{2}\left(x+\frac{1}{x}\right)\right\}^3 - 3\cdot\frac{1}{2}\left(x+\frac{1}{x}\right)$

$= \cancel{4}\cdot\frac{1}{\cancel{8}_2}\left(x+\frac{1}{x}\right)^3 - \frac{3}{2}\left(x+\frac{1}{x}\right)$

$= \frac{1}{2}\left(x+\frac{1}{x}\right)\left\{\left(x+\frac{1}{x}\right)^2 - 3\right\}$

$= \frac{1}{2}\left(x+\frac{1}{x}\right)\left\{x^2+\frac{1}{x^2}+2\cdot\cancel{x}\cdot\frac{1}{\cancel{x}} - 3\right\}$

$= \frac{1}{2}\left(x+\frac{1}{x}\right)\left\{x^2+\frac{1}{x^2} - 1\right\}$

$= \frac{1}{2}\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2} - x\cdot\frac{1}{x}\right)$

$= \frac{1}{2}\left(x^3+\frac{1}{x^3}\right)$

$\boxed{a^3+b^3 = (a+b)(a^2+b^2-ab)}$

$\therefore \cos 3\theta = \frac{1}{2}\left(x^3+\frac{1}{x^3}\right)$

$2S(A-B$

A^2B

$-\sin^2\theta$

Exercise

(1) $\tan\left(\frac{\alpha}{2}\right) = \sqrt{3}, \cos\alpha = ?$

$\tan\left(\frac{\alpha}{2}\right) = \tan\left(\frac{\pi}{3}\right)$

$\frac{\alpha}{2} = \frac{\pi}{3}$

$\alpha = \frac{2\pi}{3}$

$\cos\alpha = \cos\frac{2\pi}{3}$

$= \cos\left(\pi - \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3}$

$\boxed{\cos\alpha = -\frac{1}{2}}$

$\cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}$

$= \frac{1-(\sqrt{3})^2}{1+(\sqrt{3})^2}$

$= \frac{1-3}{1+3}$

$= -\frac{\cancel{2}\,1}{\cancel{4}\,2}$

$= -\frac{1}{2}$

(2) If $\cot\frac{A}{2} = \frac{1}{4}$, find $\operatorname{Cosec} A$.

$\frac{1}{\tan\frac{A}{2}} = \frac{1}{4}$

$\tan\frac{A}{2} = 4$

$\operatorname{Cosec} A = \frac{1}{\sin A}$

$= \frac{1}{\frac{2\tan A}{1+(\tan^2\frac{A}{2})}}$

$\frac{1+\tan^2\frac{A}{2}}{2\tan A}$

$= \frac{1+(4)^2}{2 \cdot 4}$

$= \frac{17}{8}$

$\cos 36^\circ = 1 + 2\cos^2 18^\circ$

$\cos 18^\circ = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$, $\cos 36^\circ$

$\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$

$\cos 2A = 1 - 2\sin^2 A$

$\cos 36^\circ = 1 - 2\sin^2 18^\circ$

$= 1 - 2\left(\dfrac{\sqrt{5}-1}{4}\right)^2$

$= 1 - 2\left(\dfrac{(\sqrt{5})^2 + (1)^2 - 2\cdot\sqrt{5}\cdot 1}{\cancel{8}\ 16}\right)$

$= 1 - \cancel{2}\left(\dfrac{6 - 2\sqrt{5}}{\cancel{16}\ 8}\right)$

$= \dfrac{1}{1} - \left(\dfrac{12 - 4\sqrt{5}}{32}\right)$

$= \dfrac{32 - 12 + 4\sqrt{5}}{32}$

$= \dfrac{20 + 4\sqrt{5}}{32}$

$=$

$= \dfrac{1}{1} - \left(\dfrac{6 - 2\sqrt{5}}{8}\right)$

$= \dfrac{8 - 6 + 2\sqrt{5}}{8}$

$= \dfrac{2 + 2\sqrt{5}}{8}$

$= \dfrac{\cancel{2}^{1}(1+\sqrt{5})}{\cancel{8}\ 4}$

$= \dfrac{\sqrt{5}+1}{4}$ Ans

$$\sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}$$

$$\cos\theta = 1-2\sin^2\frac{\theta}{2} = 2\cos^2\theta - 1 \quad \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$1+\cos\theta = 2\cos^2\frac{\theta}{2}$$

$$1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$2\sin^2 A = 1-\cos 2A$$

$$\sin^2 A = \frac{1-\cos 2A}{2}$$

$$\sin A = \sqrt{\frac{1-\cos 2A}{2}}$$

$$2\cos^2 A = 1+\cos 2A$$

$$\cos A = \sqrt{\frac{1+\cos 2A}{2}}$$

4a) $$\frac{\sin\frac{\theta}{2}+\sin\theta}{1+\cos\frac{\theta}{2}+\cos\theta} = \tan\frac{\theta}{2}$$

L.H.S $$\frac{\sin\frac{\theta}{2}+2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{(1+\cos\theta)+\cos\frac{\theta}{2}}$$

$$= \frac{\sin\frac{\theta}{2}\left(1+2\cos\frac{\theta}{2}\right)}{2\cos^2\frac{\theta}{2}+\cos\frac{\theta}{2}}$$

$$\frac{\sin\frac{\theta}{2}\cdot\cancel{\left(1+2\cos\frac{\theta}{2}\right)}}{\cos\frac{\theta}{2}\cancel{\left(2\cos\frac{\theta}{2}+1\right)}} = \tan\frac{\theta}{2}$$

b) $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \cot\dfrac{\theta}{2}$

L.H.S $\rightarrow$ $\dfrac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$

$$\Rightarrow \frac{\dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta} - \dfrac{1}{1}}{\dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta} + \dfrac{1}{1}} = \frac{\dfrac{\cos\theta + 1 - \sin\theta}{\cancel{\sin\theta}}}{\dfrac{\cos\theta - 1 + \sin\theta}{\cancel{\sin\theta}}}$$

$$= \frac{1 + \cos\theta - \sin\theta}{-(1 - \cos\theta) + \sin\theta}$$

$$= \frac{2\cos^2\frac{\theta}{2} - 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{-2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}$$

$$= \frac{2\cos\frac{\theta}{2}\left\{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right\}}{-2\sin\frac{\theta}{2}\left\{\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right\}}$$

$$= \frac{\cancel{2}\cos\frac{\theta}{2}\cdot\cancel{\left\{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right\}}}{\cancel{2}\sin\frac{\theta}{2}\cdot\cancel{\left\{-\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right\}}}$$

$\cot\frac{\theta}{2} = R.H.S$

proved

c) $\dfrac{1}{\sec\theta - \tan\theta} = \cot\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right)$

L.H.S :- $\dfrac{1}{\sec\theta - \tan\theta}$

$$= \frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} = \frac{1}{\frac{1-\sin\theta}{\cos\theta}}$$

$$= \frac{\cos\theta}{1-\sin\theta}$$

$$\Rightarrow \frac{\sin\left(\frac{\pi}{2} - \theta\right)}{1 - \cos\left(\frac{\pi}{2} - \theta\right)}$$

$$= \frac{\cancel{2}\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\cdot\cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\cancel{2}\sin^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$$

$$= \frac{\cancel{\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}\cdot\cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\cancel{\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}\cdot\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)} = \frac{\cos\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}{\sin\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$$

$$= \cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$$

d) $\sec 2\theta + \tan 2\theta = \tan\left(\frac{\pi}{4}+\theta\right)$

L.H.S :- $\sec 2\theta + \tan 2\theta$

$$= \frac{1}{\cos 2\theta} + \frac{\sin 2\theta}{\cos 2\theta}$$

$$= \frac{1+\sin 2\theta}{\cos 2\theta}$$

$$= \frac{1-\cos\left\{\frac{\pi}{2}+2\theta\right\}}{\sin\left\{\frac{\pi}{2}+2\theta\right\}}$$

$$= \frac{\cancel{2}\sin^{\cancel{2}}\left\{\frac{\pi}{4}+\frac{\cancel{2}\theta}{\cancel{2}}\right\}}{\cancel{2}\,\cancel{\sin}\left\{\frac{\pi}{4}+\frac{\cancel{2}\theta}{\cancel{2}}\right\}\cdot\cos\left(\frac{\pi}{4}+\frac{\cancel{2}\theta}{\cancel{2}}\right)}$$

$$= \frac{\sin\left(\frac{\pi}{4}+\theta\right)}{\cos\left(\frac{\pi}{4}+\theta\right)}$$

$$= \tan\left(\frac{\pi}{4}+\theta\right) = \text{R.H.S}$$

) $\frac{1+\sin A}{1-\sin A} = \cot^2\left(\frac{\pi}{4}-\frac{A}{2}\right)$

L.H.S :- $\frac{1+\sin A}{1-\sin A}$

$$\frac{1+\cos\left(\frac{\pi}{2}-A\right)}{1-\cos\left(\frac{\pi}{2}-A\right)}$$

$\cos(90-A)$
$= \sin A$
$\cos(90+A)$
$= -\sin A$

$$= \frac{\cancel{2}\cos^2\left(\frac{\frac{\pi}{2}-A}{2}\right)}{\cancel{2}\sin^2\left(\frac{\frac{\pi}{2}-A}{2}\right)}$$

$$= \cot^2\left(\frac{\pi}{4}-\frac{A}{2}\right)$$

b) $\tan\left(45^\circ-\frac{A}{2}\right) = \sqrt{\frac{1-\sin A}{1+\sin A}}$

$$\tan^2\left(45^\circ-\frac{A}{2}\right) = \frac{1-\sin A}{1+\sin A}$$

$$\Rightarrow \frac{1-\cos(90-\theta)}{1+\cos(90-\theta)} \Rightarrow \cancel{1-2\cos^2}($$

$$\Rightarrow \frac{\sin^2\left(\frac{\pi}{4}-\frac{A}{2}\right)}{\cos^2\left(\frac{\pi}{4}-\frac{A}{2}\right)}$$

$$\Rightarrow \tan^2\left(\frac{\pi}{4}-\frac{A}{2}\right)$$

$$\Rightarrow \tan^2\left(45^\circ-\frac{A}{2}\right)$$ **Proved**

Q) $\sec\left(\frac{\pi}{4}+\theta\right)\cdot\sec\left(\frac{\pi}{4}-\theta\right) = 2\sec 2\theta$

$$\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

L.H.S :-
$$\frac{1}{\cos\left(\frac{\pi}{4}+\theta\right)\cdot\cos\left(\frac{\pi}{4}-\theta\right)}$$

$$= \frac{1}{\cos^2\frac{\pi}{4} - \sin^2\theta}$$

$$= \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2\theta}$$

$$= \frac{1}{\frac{1}{2} - \frac{\sin^2\theta}{1}} = \frac{2}{1 - 2\sin^2\theta}$$

$$= \frac{2}{(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}$$

$$= \frac{2}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{2}{\cos 2\theta}$$

$= 2\sec 2\theta$ Proved

$\cos^2 36^\circ + \sin^2 18^\circ$

$\left(\frac{\sqrt{5}+1}{4}\right)^2 + \left(\frac{\sqrt{5}-1}{4}\right)^2$

$\frac{2\left[(\sqrt{5})^2 + (1)^2\right]}{16}$

$= \frac{\cancel{2}^{1} \times \cancel{6}^{3}}{\cancel{16}\ \cancel{8}\ 4}$

$= \frac{3}{4}$ Ans

$(a+b)^2$

$\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}}$

$= \sqrt{\frac{2-\sqrt{3}}{2} \times \frac{1}{2}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}}$

Formula

$$2\sin A \cdot \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cdot \sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \cdot \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right) \cdot \cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{D-C}{2}\right)$$

$$= -2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

Exercise

1. Express the following as the Sum or difference of trigonometric functions:-

a) $2\sin 4\theta \cdot \cos 2\theta$

$\Rightarrow \sin(4\theta + 2\theta) + \sin(4\theta - 2\theta)$

$\sin 6\theta + \sin 2\theta$

b) $2\cos 117^\circ \cdot \sin 53^\circ$

$= \sin(117^\circ + 53^\circ) - \sin(117^\circ - 53^\circ)$

$= \sin 170^\circ - \sin 64^\circ$

c) $2\cos 4\theta \cdot \cos 2\theta$

$\cos(4\theta + 2\theta) + \cos(4\theta - 2\theta)$

$\cos 6\theta + \cos 2\theta$

d) $\sin \frac{\theta}{4} \cdot \sin \frac{3\theta}{4}$

$= \frac{1}{2}\left\{2\sin\frac{\theta}{4} \cdot \sin\frac{3\theta}{4}\right\}$

$= \frac{1}{2}\left[\cos\left(\frac{\theta}{4} - \frac{3\theta}{4}\right) - \cos\left(\frac{\theta}{4} + \frac{3\theta}{4}\right)\right]$

$= \frac{1}{2}\left[\cos\frac{-2\theta}{4} - \cos\frac{4\theta}{4}\right]$

$= \frac{1}{2}\left[\left(\cos -\frac{\theta}{2}\right) - \cos\theta\right]$

$= \frac{1}{2}\left[\cos\frac{\theta}{2} - \cos\theta\right]$ Ans

$2\sin\left(\frac{2\theta+4\theta}{2}\right)\cdot\cos\left(\frac{2\theta-4\theta}{2}\right)$ $\Rightarrow 2\cos\left(\frac{7\theta+5\theta}{2}\right)\cdot\sin\left(\frac{7\theta-5\theta}{2}\right)$

$2\sin 3\theta\cdot\cos(-\theta)$ $\Rightarrow 2\cos 6\theta\cdot\sin\theta$

$2\sin 3\theta\cdot\cos\theta$

c) $\cos 70^\circ+\cos 20^\circ$

$2\cos\left(\frac{70+20^\circ}{2}\right)\cdot\cos\left(\frac{70^\circ-20^\circ}{2}\right)$

$2\cos 45^\circ\cdot\cos 25^\circ$

d) $\cos\frac{\pi}{13}-\cos\frac{2\pi}{13}$

$\Rightarrow 2\sin\left(\frac{\frac{\pi}{13}+\frac{2\pi}{13}}{2}\right)\cdot\sin\left(\frac{\frac{2\pi}{13}-\frac{\pi}{13}}{2}\right)$

$\Rightarrow 2\sin\left(\frac{3\pi\times 1}{13\times 2}\right)\cdot\sin\left(\frac{\pi}{13\times 2}\right)$

$\Rightarrow 2\sin\left(\frac{3\pi}{26}\right)\cdot\sin\left(\frac{\pi}{26}\right)$

3) Do as directed :-

a) $\sin 80^\circ+\sin 50^\circ = 2\sin\alpha\cdot\cos\beta$

$2\sin\left(\frac{80^\circ+50^\circ}{2}\right)\cdot\cos\left(\frac{80^\circ-50^\circ}{2}\right) = 2\sin\alpha\cdot\cos\beta$

$\cancel{2}\sin 65^\circ\cdot\cos 15^\circ = \cancel{2}\sin\alpha\cdot\cos\beta$

$\sin 65^\circ = \sin\alpha$	$\cos 15^\circ = \cos\beta$
$\alpha = 65^\circ$	$\beta = 15^\circ$

b) Evaluate :-

$\sin\frac{3\pi}{2}\cdot\sin\frac{7\pi}{2}-\cos 5\pi\cdot\cos 6\pi$

$\frac{1}{2}\left[2\cdot\sin\frac{3\pi}{2}\cdot\sin\frac{7\pi}{2}-2\cos 5\pi\cdot\cos 6\pi\right]$

$$\frac{1}{2}\left[\cos\left(-\frac{3\pi}{2}-\frac{7\pi}{2}\right)-\cos\left(\frac{3\pi+7\pi}{2}\right)-\cos(5\pi+6\pi)+\cos(5\pi-6\pi)\right]$$

$$\frac{1}{2}\left[\cos\left(-\frac{4\pi}{2}\right)-\cos\left(\frac{10\pi}{2}\right)-\cos(11\pi)+\cos(-\pi)\right]$$

$$\frac{1}{2}\left[\cos 2\pi-\cos 5\pi-\cos 11\pi+\cos\pi\right]$$

$$\cancel{\frac{1}{2}\left[\cos 5\pi -\right.}$$

$$=\frac{1}{2}\left[1-\cos 5\pi-\cos 11\pi+1\right]$$

$$=\frac{1}{2}\left[2-\cos 5\pi-\cos 11\pi\right]$$

$$=\frac{1}{2}\left[2+1+1\right]$$

$$=\frac{1}{\cancel{2}}\times\cancel{4}^{\,2}$$

$$=2$$

$\cos 2\pi$
$\cos 360^\circ$
$=1$

$\cos 180^\circ$
$=-1$

$\cos 5\pi=-1$
$\cos 6\pi=1$

c) Show that:-

$$\frac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = \tan 5A$$

Can this result be considered true if $A = 18^\circ$?

L.H.S:- $$\frac{(\sin 9A + \sin A) + \sin 5A}{(\cos 9A + \cos A) + \cos 5A}$$

$$= \frac{2\sin\left(\frac{9A+A}{2}\right)\cdot\cos\left(\frac{9A-A}{2}\right) + \sin 5A}{2\cos\left(\frac{9A+A}{2}\right)\cdot\cos\left(\frac{9A-A}{2}\right) + \cos 5A}$$

$$= \frac{2\sin 5A\cdot\cos 4A + \sin 5A}{2\cos 5A\cdot\cos 4A + \cos 5A}$$

$$= \frac{\sin 5A\ \cancel{(2\cos 4A + 1)}}{\cos 5A\ \cancel{(2\cos 4A + 1)}}$$

$= \tan 5A = R.H.S$

Proved

d) Prove that:-

$$\cos 20^\circ + \cos 60^\circ + \cos 100^\circ + \cos 140^\circ = \frac{1}{2}$$

L.H.S:- $$\frac{1}{2} + (\cos 20^\circ + \cos 100^\circ) + \cos 140^\circ$$

$$= \frac{1}{2} + 2\cdot\cos 60^\circ\cdot\cos 40^\circ + \cos 140^\circ$$

$$8\cdot\frac{1}{2} + \cancel{2}\cdot\frac{1}{\cancel{2}}\cdot\cos 40^\circ + \cos 140^\circ$$

$$= \frac{1}{2} + \cos 90^\circ + \cos 50^\circ$$

$$= \frac{1}{2} + 2\cdot 0 + \cos 50^\circ$$

$$= \frac{1}{2} + 0$$

$$= \frac{1}{2} \text{ As}$$

e) If $\sin\theta = n\cdot\sin(\theta + 2\alpha)$, Show that

$$\tan(\theta+\alpha) = \frac{1+n}{1-n}\cdot\tan\alpha$$

Sol:- $\sin\theta = n \times \sin(\theta+2\alpha)$

$$n = \frac{\sin\theta}{\sin(\theta+2\alpha)}$$

$$R.H.S = \frac{1+n}{1-n}\cdot\tan\alpha$$

$$= \frac{\frac{1}{1} + \frac{\sin\theta}{\sin(\theta+2\alpha)}}{\frac{1}{1} - \frac{\sin\theta}{\sin(\theta+2\alpha)}}\cdot\tan\alpha$$

$$= \frac{\frac{\sin(\theta+2\alpha) + \sin\theta}{\cancel{\sin(\theta+2\alpha)}}}{\frac{\sin(\theta+2\alpha) - \sin\theta}{\cancel{\sin(\theta+2\alpha)}}}\cdot\tan\alpha$$

$$\frac{\cancel{2}\sin\left(\frac{\theta+2\alpha+\theta}{2}\right)\cdot\cos\left(\frac{\cancel{\theta}+2\alpha-\cancel{\theta}}{\cancel{2}}\right)}{\cancel{2}\cos\left(\frac{\theta+2\alpha+\theta}{2}\right)\cdot\sin\left(\frac{\cancel{\theta}+2\alpha-\cancel{\theta}}{\cancel{2}}\right)}\cdot\tan\alpha$$

$$=\frac{\sin\left(\frac{2\theta+2\alpha}{2}\right)\cdot\cos\alpha}{\cos\left(\frac{2\theta+2\alpha}{2}\right)\cdot\sin\alpha}\cdot\tan\alpha$$

$$=\tan\left(\frac{2\theta+2\alpha}{2}\right)\cdot\cancel{\cot\alpha}\cdot\frac{1}{\cancel{\cot\alpha}}$$

$$=\tan\left(\frac{\cancel{2}(\theta+\alpha)}{\cancel{2}}\right)$$

$$=\tan(\theta+\alpha)\ \text{Ans}$$

f) If $\sin(2A+B)=5\sin B$, then, show that $\tan(A+B)=\frac{3}{2}\tan A$

$$\frac{\sin(2A+B)}{\sin B}=\frac{5}{1}$$

Using Componendo and dividendo

$$\frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B}=\frac{5+1}{5-1}$$

$$=\frac{2\sin\left\{\frac{2A+B+B}{2}\right\}\cdot\cos\left\{\frac{2A+\cancel{B}-\cancel{B}}{2}\right\}}{2\cos\left\{\frac{2A+B+B}{2}\right\}\cdot\sin\left\{\frac{2A+\cancel{B}-\cancel{B}}{2}\right\}}=\frac{3}{2}$$

$$\frac{\cancel{2}\sin(A+B)\cdot\cos A}{\cancel{2}\cos(A+B)\cdot\sin A}=\frac{3}{2}$$

$$\tan(A+B)\cdot\frac{1}{\tan A}=\frac{3}{2}$$

$$\tan(A+B)=\frac{3}{2}\tan A$$

Proved

4) Prove that

a) $$\frac{\sin A + \sin B}{\sin A - \sin B} = \tan\left(\frac{A+B}{2}\right) \cdot \cot\left(\frac{A-B}{2}\right)$$

L.H.S :- $$\frac{\sin A + \sin B}{\sin A - \sin B}$$

$$= \frac{\cancel{2} \sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)}{\cancel{2} \cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right)}$$

$$= \tan\left(\frac{A+B}{2}\right) \cdot \cot\left(\frac{A-B}{2}\right) = R.H.S$$

Ans

b) $$\frac{\sin 2A + \sin 2B}{\sin 2A - \sin 2B} = \tan(A+B) \cdot \cot(A-B)$$

L.H.S :- $$\frac{2 \sin\left(\frac{2A+2B}{2}\right) \cdot \cos\left(\frac{2A-2B}{2}\right)}{2 \cos\left(\frac{2A+2B}{2}\right) \cdot \sin\left(\frac{2A-2B}{2}\right)}$$

$$= \frac{\cancel{2} \sin\left\{\frac{\cancel{2}(A+B)}{\cancel{2}}\right\} \cdot \cos\left\{\frac{\cancel{2}(A-B)}{\cancel{2}}\right\}}{\cancel{2} \cos\left\{\frac{\cancel{2}(A+B)}{\cancel{2}}\right\} \cdot \sin\left\{\frac{\cancel{2}(A-B)}{\cancel{2}}\right\}}$$

$$= \tan(A+B) \cdot \cot(A-B)$$

c) $\dfrac{\sin 5A + \sin 3A}{\cos 5A + \cos 3A} = \tan 4A$

L.H.S :- $\dfrac{\cancel{2} \sin\left(\frac{5A+3A}{2}\right) \cdot \cancel{\cos\left(\frac{5A-3A}{2}\right)}}{\cancel{2} \cos\left(\frac{5A+3A}{2}\right) \cdot \cancel{\cos\left(\frac{5A-3A}{2}\right)}}$

$= \dfrac{\sin 4A}{\cos 4A}$

$= \tan 4A = R.H.S$

Proved

d) $\dfrac{\sin 7A + \sin 3A}{\cos 3A - \cos 7A} = \cot 2A$

L.H.S :- $\dfrac{\cancel{2 \sin\left(\frac{7A+3A}{2}\right)} \cdot \cos\left(\frac{7A-3A}{2}\right)}{\cancel{2 \sin\left(\frac{3A+7A}{2}\right)} \cdot \sin\left(\frac{7A-3A}{2}\right)}$

$= \dfrac{\cos 2A}{\sin 2A}$

$= \cot 2A = R.H.S$

Proved

e) $\dfrac{\sin 5A - \sin 3A}{\cos 3A + \cos 5A} = \tan A$

L.H.S :- $\dfrac{\sin 5A - \sin 3A}{\cos 3A + \cos 5A}$

$$\frac{\cancel{2\cos\left(\frac{5A+3A}{2}\right)}\cdot\sin\left(\frac{5A-3A}{2}\right)}{\cancel{2\cos\left(\frac{5A+3A}{2}\right)}\cdot\cos\left(\frac{5A-3A}{2}\right)}$$

$$\Rightarrow \frac{\sin A}{\cos A}$$

$$= \tan A = R.H.S$$

f) $$\frac{\sin A - \sin B}{\cos B - \cos A} = \cot\left(\frac{A+B}{2}\right)$$

L.H.S :- $$\frac{\cancel{2}\cos\left(\frac{A+B}{2}\right)\cdot\cancel{\sin\left(\frac{A-B}{2}\right)}}{\cancel{2}\sin\left(\frac{B+A}{2}\right)\cdot\cancel{\sin\left(\frac{A-B}{2}\right)}}$$

$$= \cot\left(\frac{A+B}{2}\right)$$

g) $\dfrac{\cos 2B - \cos 2A}{\sin 2A + \sin 2B} = \tan(A-B)$

L.H.S:- $\dfrac{\cancel{2}\sin\left(\frac{2B+2A}{2}\right)\cdot\sin\left(\frac{2A-2B}{2}\right)}{\cancel{2}\sin\left(\frac{2A+2B}{2}\right)\cdot\cos\left(\frac{2A-2B}{2}\right)}$

$= \dfrac{\sin\left\{\frac{\cancel{2}(A-B)}{\cancel{2}}\right\}}{\cos\left\{\frac{\cancel{2}(A-B)}{\cancel{2}}\right\}}$

$= \tan A-B = R.H.S$

) $\cos\frac{\pi}{5} - \cos\frac{2\pi}{5} = \frac{1}{2}$

$2\sin\left(\dfrac{\frac{\pi}{5}+\frac{2\pi}{5}}{2}\right)\cdot\sin\left(\dfrac{\frac{2\pi}{5}-\frac{\pi}{5}}{2}\right)$

$2\sin\left(\dfrac{\frac{3\pi}{5}}{2}\right)\cdot\sin\left(\dfrac{\frac{\pi}{5}}{2}\right)$

$2\sin\frac{3\pi}{10}\cdot\sin\frac{\pi}{10}$

i) $\dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$

$\sin 11^\circ$

$\sin(90^\circ - 79)$

$\cos 79^\circ$

$\cos(90^\circ - 79^\circ)$

$\sin 11^\circ$

L.H.S :- $\dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$

$= \dfrac{\cos 11^\circ + \cos 79^\circ}{\sin 79^\circ - \sin 11^\circ}$

$= \dfrac{\cancel{2}\, \cancel{\cos\left(\frac{11^\circ + 79^\circ}{2}\right)} \cdot \cos\left(\frac{11^\circ - 79^\circ}{2}\right)}{\cancel{2}\, \cancel{\cos\left(\frac{11^\circ + 79^\circ}{2}\right)} \cdot \sin\left(\frac{79^\circ - 11^\circ}{2}\right)}$

$= \dfrac{\cos\left(\frac{-68}{2}\right)}{\sin\left(\frac{68}{2}\right)} = \dfrac{\cos 34^\circ}{\sin 34^\circ} = \cot 34^\circ$.

$= \cot 34^\circ$

$= \cot(90^\circ - 56^\circ)$

$= \tan 56^\circ = R.H.S$

2nd method

dividing numerator and denominator by $\cos 11^\circ$

$\dfrac{\frac{\cos 11^\circ}{\cos 11} + \frac{\sin 11^\circ}{\cos 11}}{\frac{\cos 11^\circ}{\cos 11^\circ} - \frac{\sin 11^\circ}{\cos 11^\circ}}$

$= \dfrac{1 + \tan 11^\circ}{1 - \tan 11^\circ} = \dfrac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \cdot \tan 11^\circ}$

$= \tan(45^\circ + 11^\circ)$

$= \tan 56^\circ$ Ps

J) $$\frac{\sin 80^\circ - \cos 50^\circ}{\sin 80^\circ + \sin 40^\circ} = \frac{1}{\sqrt{3}} \cot 70^\circ$$

L.H.S :- $$\frac{\sin 80^\circ - \sin 40^\circ}{\sin 80^\circ + \sin 40^\circ}$$

$$= \frac{\cancel{2}\cos\left(\frac{80^\circ + 40^\circ}{2}\right) \cdot \sin\left(\frac{80^\circ - 40^\circ}{2}\right)}{\cancel{2}\sin\left(\frac{80^\circ + 40^\circ}{2}\right) \cdot \cos\left(\frac{80^\circ - 40^\circ}{2}\right)}$$

$$= \frac{\cos 60^\circ \cdot \sin 20^\circ}{\sin 60^\circ \cdot \cos 20^\circ}$$

$$= \cancel{\cot} 60^\circ \cdot \tan 20^\circ$$

$$= \frac{1}{\sqrt{3}} \cdot \tan(90^\circ - 70^\circ)$$

$$= \frac{1}{\sqrt{3}} \cdot \cot 70^\circ$$

5) Prove that :-

a) $$\frac{\sin A + \sin 3A + \sin 5A + \sin 7A}{\cos A + \cos 3A + \cos 5A + \cos 7A} = \tan 4A$$

L.H.S :- $$\frac{\{\sin A + \sin 7A\} + \{\sin 3A + \sin 5A\}}{\{\cos A + \cos 7A\} + \{\cos 3A + \cos 5A\}}$$

$$= \frac{\left\{2\cdot\sin\left(\frac{A+7A}{2}\right)\cdot\cos\left(\frac{A-7A}{2}\right)\right\}\cdot\left\{2\cdot\sin\left(\frac{3A+5A}{2}\right)\cdot\cos\left(\frac{3A-5A}{2}\right)\right\}}{\left\{2\cdot\cos\left(\frac{A+7A}{2}\right)\cdot\cos\left(\frac{A-7A}{2}\right)\right\}\cdot\left\{2\cdot\cos\left(\frac{3A+5A}{2}\right)\cdot\cos\left(\frac{3A-5A}{2}\right)\right\}}$$

$$= \frac{2\cdot\sin 4A\cdot\cos(-3A) + 2\cdot\sin 4A\cdot\cos(-A)}{2\cdot\cos 4A\cdot\cos(-A) + 2\cdot\cos 4A\cdot\cos(-A)}$$

$$= \frac{\cancel{2}\cdot\sin 4A\,(\cancel{\cos A+\cos A})}{\cancel{2}\cdot\cos 4A\,(\cancel{\cos A+\cos A})}$$

$= \tan 4A =$ R.H.S proved

b) $$\frac{\sin 4x+\sin 5x+\sin 6x}{\cos 4x+\cos 5x+\cos 6x} = \tan 5x$$

L.H.S:- $$\frac{\{\sin 4x+\sin 6x\}+\sin 5x}{\{\cos 4x+\cos 6x\}+\cos 5x}$$

$$= \frac{2\cdot\sin\left(\frac{4x+6x}{2}\right)\cdot\cos\left(\frac{4x-6x}{2}\right)+\sin 5x}{2\cdot\cos\left(\frac{4x+6x}{2}\right)\cdot\cos\left(\frac{4x-6x}{2}\right)+\cos 5x}$$

$$= \frac{\cancel{2}\sin 5x\cdot\cos(-x)+\sin 5x}{\cancel{2}\cos 5x\cdot\cos(-x)+\cos 5x}$$

$$= \frac{\sin 5x \cancel{(\cos x + 1)}}{\cos 5x \cancel{(\cos x + 1)}}$$

$$= \tan 5x = R.H.S$$

Proved

c) $$\frac{\cos 3A + 2\cos 5A + \cos 7A}{\cos A + 2\cos 3A + \cos 5A} = \cos 2A - \sin 2A \cdot \tan 3A$$

L.H.S:- $$\frac{\{\cos 3A + \cos 7A\} + 2\cos 5A}{\{\cos A + \cos 5A\} + 2\cos 3A}$$

$$= \frac{\cancel{2}\cos\left(\frac{3A+7A}{2}\right)\cdot\cos\left(\frac{3A-7A}{2}\right) + 2\cos 5A}{\cancel{2}\cos\left(\frac{A+5A}{2}\right)\cdot\cos\left(\frac{A-5A}{2}\right) + 2\cos 3A}$$

$$= \frac{2\cos 5A \cdot \cos 2A + 2\cos 5A}{2\cos 3A \cdot \cos 2A + 2\cos 3A}$$

$$= \frac{\cancel{2}\cos 5A \cancel{(\cos 2A + 1)}}{\cancel{2}\cos 3A \cancel{(\cos 2A + 1)}}$$

$$= \frac{\cos(3A + 2A)}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos 2A - \sin 3A \cdot \sin 2A}{\cos 3A}$$

$$= \frac{\cancel{\cos 3A} \cdot \cos 2A}{\cancel{\cos 3A}} - \frac{\sin 3A \cdot \sin 2A}{\cos 3A}$$

$$\cos 2A - \sin 2A \cdot \tan 3A = R.H.S$$

Proved

d) $$\frac{\sin A + 2\sin 3A + \sin 5A}{\sin 3A + 2\sin 5A + \sin 7A} = \cos 2A - \cot 5A \cdot \sin 2A$$

L.H.S :- $$\frac{\{\sin A + \sin 5A\} + 2\sin 3A}{\{\sin 3A + \sin 7A\} + 2\sin 5A}$$

$$= \frac{2\sin\left(\frac{A+5A}{2}\right)\cdot\cos\left(\frac{A-5A}{2}\right) + 2\sin 3A}{2\sin\left(\frac{3A+7A}{2}\right)\cdot\cos\left(\frac{3A-7A}{2}\right) + 2\sin 5A}$$

$$= \frac{2\sin 3A\cdot\cos 2A + 2\sin 3A}{2\sin 5A\cdot\cos 2A + 2\sin 5A}$$

$$= \frac{\cancel{2}\sin 3A\,\cancel{(\cos 2A + 1)}}{\cancel{2}\sin 5A\,\cancel{(\cos 2A + 1)}}$$

$$= \frac{\sin 3A}{\sin(3A + 2A)}$$

$$= \frac{\sin 3A}{\sin 3A\cdot\cos 2A + \cos 3A\cdot\sin 2A}$$

$$= \frac{\cancel{\sin 3A}}{\cancel{\sin 3A}\cdot\cos 2A} + \frac{\sin 3A}{\cos 3A\cdot\sin 2A}$$

$$= \cos 2A + \tan 3A\cdot\sin 2A$$

e) $$\frac{\sin(\theta+\phi) - 2\cos\theta - \sin(\theta-\phi)}{\cos(\theta+\phi) + 2\sin\theta - \cos(\theta-\phi)} = -\cot\theta$$

L.H.S:- $$\frac{\sin(\theta+\phi) - 2\cos\theta - \sin(\theta-\phi)}{\cos(\theta+\phi) + 2\sin\theta - \cos(\theta-\phi)}$$

$$= \frac{\cancel{\sin\theta\cdot\cos\phi} + \cancel{\cos\theta\cdot\sin\phi} - 2\cos\theta - \cancel{\sin\theta\cdot\cos\phi} - \cancel{\cos\theta\cdot\sin\phi}}{\cancel{\cos\theta\cdot\cos\phi} - \cancel{\sin\theta\cdot\sin\phi} + 2\sin\theta - \cancel{\cos\theta\cdot\cos\phi} + \cancel{\sin\theta\cdot\sin\phi}}$$

$$= -\frac{\cancel{2}\cos\theta}{\cancel{2}\sin\theta}$$

$$= -\cot\theta = R.H.S$$ proved

f) $$\frac{\sin 11x\cdot\sin x + \sin 7x\cdot\sin 3x}{\cos 11x\cdot\sin x + \cos 7x\cdot\sin 3x} = \tan 8x$$

L.H.S:- Multiple both numr or denor by 2

$$\Rightarrow \frac{2\sin 11x\cdot\sin x + 2\sin 7x\cdot\sin 3x}{2\cos 11x\cdot\sin x + 2\cos 7x\cdot\sin 3x}$$

$$\Rightarrow \frac{\cos(11x-x) - \cos(11x+x) + \cos(7x-3x) - \cos(7x+3x)}{\sin(11x+x) - \sin(11x-x) + \sin(7x+3x) - \sin(7x-3x)}$$

$$= \frac{\cancel{\cos 10x} - \cos 12x + \cos 4x - \cancel{\cos 10x}}{\sin 12x - \cancel{\sin 10x} + \cancel{\sin 10x} - \sin 4x}$$

$$= \frac{\cos 4x - \cos 12x}{\sin 12x - \sin 4x}$$

$$= \frac{\sin 12x + \sin 4x}{}$$

$$\frac{\cancel{2}\sin\left(\frac{4x+12x}{2}\right)\cdot \cancel{\sin\left(\frac{12x-4x}{2}\right)}}{\cancel{2}\cos\left(\frac{12x+4x}{2}\right)\cdot \cancel{\sin\left(\frac{12x-4x}{2}\right)}}$$

$$= \cancel{\tan}\ \frac{\sin 8x}{\cos 8x}$$

$$= \tan 8x$$

g) $$\frac{\sin x\cdot\sin 2x + \sin 3x\cdot\sin 6x}{\sin x\cdot\cos 2x + \sin 3x\cdot\cos 6x} = \tan 5x$$

L.H.S:- $$\frac{2\sin x\cdot\sin 2x + 2\sin 3x\cdot\sin 6x}{2\sin x\cdot\cos 2x + 2\sin 3x\cdot\cos 6x}$$

$$= \frac{\cos(x-2x) - \cos(x+2x) + \cos(3x-6x) - \cos(3x+6x)}{\sin(x+2x) + \sin(x-2x) + \sin(3x+6x) + \sin(3x-6x)}$$

$$= \frac{\cos x - \cancel{\cos 3x} + \cancel{\cos 3x} - \cos 9x}{\cancel{\sin 3x} - \sin x + \sin 9x - \cancel{\sin 3x}}$$

$$= \frac{\cos x - \cos 9x}{\sin 9x - \sin x}$$

$$\frac{\cancel{2}\sin\left(\frac{x+9x}{2}\right)\cdot\cancel{\sin\left(\frac{9x-x}{2}\right)}}{\cancel{2}\cos\left(\frac{9x+x}{2}\right)\cdot\cancel{\sin\left(\frac{9x-x}{2}\right)}}$$

$$= \frac{\sin 5x}{\cos 5x}$$

$$= \tan 5x = R.H.S$$

b) $2\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = 0$

L.H.S :- $2\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7} + 2\cos\left(\frac{\frac{4\pi}{7}+\frac{6\pi}{7}}{2}\right)\cdot\cos\left(\frac{\frac{4\pi}{7}-\frac{6\pi}{7}}{2}\right)$

$= 2\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7} + 2\cos\frac{\cancel{10}^{5}\pi}{\cancel{14}_{7}}\cdot\cos\frac{\cancel{2}\pi}{\cancel{14}_{7}}$

$= 2\cos\frac{2\pi}{7}\cdot\cos\frac{\pi}{7} + 2\cos\frac{5\pi}{7}\cdot\cos\frac{\pi}{7}$

$2\cos\frac{\pi}{7}\left(\cos\frac{2\pi}{7} + 2\cos\frac{5\pi}{7}\right)$

$2\cos\frac{\pi}{7}\left[2\cos\left(\frac{\frac{2\pi}{7}+\frac{5\pi}{7}}{2}\right)\cdot\cos\left(\frac{\frac{2\pi}{7}-\frac{5\pi}{7}}{2}\right)\right]$

$2\cos\frac{\pi}{7}\left[2\cos\frac{\cancel{7}\pi}{\cancel{14}_{2}}\cdot\cos\frac{3\pi}{14}\right]$

$$= 2\cos\frac{\pi}{7}\left[0\cdot\cos\cancel{\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)}+\cos\cancel{\left(\frac{\pi}{2}+\frac{3\pi}{14}\right)}\right]$$

$$= 0 \qquad \left[\because \cos\frac{\pi}{2} = 0\right]$$

$$(\cos A+\cos B)^2+(\sin A-\sin B)^2 = 4\cos^2\left(\frac{A+B}{2}\right)$$

$$= \left(2\cos\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)\right)^2+\left(2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)\right)^2$$

$$= 4\cos^2\frac{A+B}{2}\cdot\cos^2\frac{A-B}{2}+4\cos^2\frac{A+B}{2}\cdot\sin^2\frac{A-B}{2}$$

$$= 4\cos^2\left(\frac{A+B}{2}\right)\left[\cos^2\frac{A-B}{2}+\sin^2\frac{A-B}{2}\right]$$

$$= 4\cos^2\left(\frac{A+B}{2}\right)\times 1$$

$$= 4\cos^2\left(\frac{A+B}{2}\right) = R.H.S$$

Proved

k) Prove that :-

$$\frac{\cos 2A + 2\cos 4A + \cos 6A}{\cos A + 2\cos 3A + \cos 5A} = \cos A - \sin A \cdot \tan 3A$$

$$= \frac{\cos 6A + \cos 2A + 2\cos 4A}{\cos 5A + \cos A + 2\cos 3A}$$

$$= \frac{2\cos\left(\frac{6A+2A}{2}\right)\cdot\cos\left(\frac{6A-2A}{2}\right) + 2\cos 4A}{2\cos\left(\frac{5A+A}{2}\right)\cdot\cos\left(\frac{5A-A}{2}\right) + 2\cos 3A}$$

$$= \frac{2\cos 4A \cdot \cos 2A + 2\cos 4A}{2\cos 3A \cdot \cos 2A + 2\cos 3A}$$

$$= \frac{\cancel{2}\cos 4A\,\cancel{(\cos 2A + 1)}}{\cancel{2}\cos 3A\,\cancel{(\cos 2A + 1)}}$$

$$= \frac{\cos(3A + A)}{\cos 3A}$$

$$= \frac{\cos 3A \cdot \cos A - \sin 3A \cdot \sin A}{\cos 3A}$$

$$= \frac{\cancel{\cos 3A}\cdot\cos A}{\cancel{\cos 3A}} - \frac{\sin 3A \cdot \sin A}{\cos 3A}$$

$$= \cos A - \sin A \cdot \tan 3A = R.H.S$$

Proved

6) Prove that :-

a) $8 \sin 20^\circ \cdot \sin 40^\circ \cdot \cos 10^\circ = \sqrt{3}$

$\Rightarrow 4 \cos 10^\circ \left(2 \sin 40^\circ \cdot \sin 20^\circ\right)$

$= 4 \cos 10^\circ \left[\cos(40^\circ - 20^\circ) - \cos(40^\circ + 20^\circ)\right]$

$= 4 \cos 10^\circ \left[\cos 20^\circ - \cos 60^\circ\right]$

$= 4 \cos 10^\circ \left[\cos 20^\circ - \frac{1}{2}\right]$

$= 4 \cos 10^\circ \cdot \cos 20^\circ - 2 \cos 10^\circ$

$= 2 \left[\cos(10 + 20^\circ) + \cos(10 - 20)\right] - 2 \cos 10^\circ$

$= 2 \left[\cos 30^\circ + \cos 10^\circ\right] - 2 \cos 10^\circ$

$= \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2} + \cancel{2\cos 10^\circ} - \cancel{2 \cos 10^\circ}$

$= \sqrt{3}$ Proved

b) $\cos 15^\circ \cdot \cos 30^\circ \cdot \cos 60^\circ \cdot \cos 75^\circ = \frac{\sqrt{3}}{16}$

$\cos 75^\circ \cdot \cos 15^\circ \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

$= \frac{1}{2} \left[2 \cos 75^\circ \cdot \cos 15^\circ\right] \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

$$\Rightarrow \left[\cos\left(75^\circ + 15^\circ\right) + \cos\left(75^\circ - 15^\circ\right)\right] \cdot \frac{\sqrt{3}}{8}$$

$$= \cos 90^\circ + \cos 60^\circ \cdot \frac{\sqrt{3}}{8}$$

$$= 0 + \frac{1}{2} \cdot \frac{\sqrt{3}}{8}$$

$$= \frac{\sqrt{3}}{16} \quad \underline{\text{Proved}}$$

c) $\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 80^\circ = \frac{\sqrt{3}}{8}$

<u>L.H.S</u> $\left[2 \sin 80^\circ \cdot \sin 20^\circ\right] \cdot \sin 40^\circ \times \frac{1}{2}$

$$= \left[-\cos\left(80^\circ + 20^\circ\right) + \cos\left(80^\circ - 20^\circ\right)\right] \cdot \sin 40^\circ \times \frac{1}{2}$$

$$= \left[-\cos 100^\circ + \cos 60^\circ\right] \cdot \sin 40^\circ \times \frac{1}{2}$$

$$= -\left[\cos 100^\circ - \frac{1}{2}\right] \cdot \sin 40^\circ \times \frac{1}{2}$$

$$= \left[\cos 100^\circ \cdot \sin 40^\circ - \sin 40^\circ \cdot \frac{1}{2}\right] - \frac{1}{2}$$

$$= \left[\frac{1}{2}\left\{2 \sin 40^\circ + \cos 100^\circ\right\} - \sin 40^\circ \cdot \frac{1}{2}\right] - \frac{1}{2}$$

$$= \left[\left\{\sin\left(40^\circ + 100^\circ\right) + \sin\left(40 - 100^\circ\right)\right\} - \sin 40^\circ\right\} - \frac{1}{4}$$

$$\left[(\sin 140^\circ - \sin 60^\circ) - \sin 40^\circ\right] \times \left(-\frac{1}{4}\right)$$

$$\left[\sin 140^\circ - \frac{\sqrt{3}}{2} - \sin 40^\circ\right] \cdot \frac{-1}{4}$$

$$\Rightarrow \left[\sin 140^\circ - \sin 40^\circ - \frac{\sqrt{3}}{2}\right] \cdot \frac{-1}{4}$$

$$\Rightarrow \left[2\cos\left(\frac{140^\circ + 40^\circ}{2}\right) \cdot \sin\left(\frac{140^\circ - 40^\circ}{2}\right) - \frac{\sqrt{3}}{2}\right] \times \frac{-1}{4}$$

$$= \left[\underbrace{2\cos 90^\circ \cdot \sin 50^\circ}_{0} - \frac{\sqrt{3}}{2}\right] \times \frac{-1}{4}$$

$$= \left(0 - \frac{\sqrt{3}}{2}\right) \times \frac{-1}{4}$$

$$= -\frac{\sqrt{3}}{2} \times \frac{-1}{4}$$

$$= \frac{\sqrt{3}}{8} = R.H.S$$

Proved

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d) $\sin 10^\circ \cdot \cos 40^\circ \cdot \cos 20^\circ = \frac{1}{8}$

$$= (2\cos 40^\circ \cdot \cos 20^\circ) \cdot \frac{1}{2}\sin 10^\circ$$

$$= \left[\cos(40^\circ + 20^\circ) + \cos(40^\circ - 20^\circ)\right] \cdot \frac{1}{2}\sin 10^\circ$$

$$\cos 60^\circ + \cos 20^\circ \cdot \frac{1}{2} \sin 10^\circ$$

$$\left[\frac{1}{2} + \cos 20^\circ\right] \cdot \frac{1}{2} \sin 10^\circ$$

$$= \left[\frac{1}{2} \sin 10^\circ + \cos 20^\circ \cdot \sin 10^\circ\right] \frac{1}{2}$$

$$= \left[\frac{1}{2} \sin 10^\circ + \frac{1}{2}\left(2 \cos 20^\circ \cdot \sin 10^\circ\right)\right]$$

$$= \left[\sin 10^\circ + \left\{\sin(20^\circ + 10^\circ) - \sin(20^\circ - 10^\circ)\right\}\right] \cdot \frac{1}{4}$$

$$= \left[\cancel{\sin 10^\circ} + \frac{1}{2} - \cancel{\sin 10^\circ}\right] \frac{1}{4}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} = R.H.S$$

Pooved

$$\sin 10^\circ \cdot \sin 30^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ = \frac{1}{16}$$

$$\frac{1}{2} \cdot \frac{1}{2}\left(2 \sin 10^\circ \cdot \sin 50^\circ\right) \cdot \sin 70^\circ$$

$$\Rightarrow \frac{1}{4}\left(-\cos(10^\circ + 50^\circ) + \cos(10^\circ - 50^\circ)\right) \cdot \sin 70^\circ$$

$$\Rightarrow \frac{1}{4}\left(-\cos 60^\circ + \cos 40^\circ\right) \cdot \sin 70^\circ$$

$$\Rightarrow \frac{1}{4}\left(-\frac{1}{2} \cdot \sin 70^\circ + \cos 40^\circ \cdot \sin 70^\circ\right)$$

$$\Rightarrow \frac{1}{4}\left[-\frac{1}{2}\cdot \sin 70^\circ + \frac{1}{2}\left(2\cos 40^\circ \cdot \sin 70^\circ\right)\right]$$

$$= \frac{1}{8}\left[-\sin 70^\circ + \sin\left(40^\circ+70^\circ\right) - \sin\left(40^\circ-70^\circ\right)\right]$$

$$= \frac{1}{8}\left[\sin 110^\circ - \sin 70^\circ + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[2\cos\left(\frac{110^\circ+70^\circ}{2}\right)\cdot \sin\left(\frac{110^\circ-70^\circ}{2}\right) + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[2\cos 90^\circ \cdot \sin 20^\circ + \frac{1}{2}\right]$$

$$= \frac{1}{8}\cdot\left(0+\frac{1}{2}\right)$$

$$= \frac{1}{16}$$ Proved

$$\Rightarrow \frac{1}{4}\left[-\frac{1}{2}\cdot \sin 70^\circ + \frac{1}{2}\left(2\cos 40^\circ \cdot \sin 70^\circ\right)\right]$$

$$= \frac{1}{8}\left[-\sin 70^\circ + \sin\left(40^\circ + 70^\circ\right) - \sin\left(40^\circ - 70^\circ\right)\right]$$

$$= \frac{1}{8}\left[\sin 110^\circ - \sin 70^\circ + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[\left(2\cos\left(\frac{110^\circ + 70^\circ}{2}\right)\cdot \sin\left(\frac{110^\circ - 70^\circ}{2}\right)\right) + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[\left(2\cos 90^\circ \cdot \sin 20^\circ\right) + \frac{1}{2}\right]$$

$$= \frac{1}{8}\cdot\left(0 + \frac{1}{2}\right)$$

$$= \frac{1}{16}$$ Proved

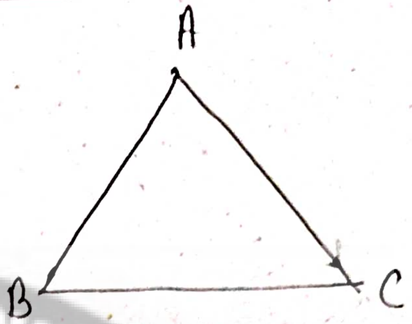
$\frac{1}{8}\left(\frac{1}{2}\right)$

16

d

Formula

$\angle A + \angle B + \angle C = 180^0$



$A + B + C = 180^0$

$A + B = \pi - C$

$\sin(A+B) = \sin(\pi - C)$

* $\sin(A+B) = \sin C$

* $\sin(A+C) = \sin B$

* $\sin(B+C) = \sin A$

$A + B + C = \pi$

$A + B = \pi - C$

$\cos(A+B) = \cos(\pi - C)$

* $\cos(A+B) = -\cos C$

* $\cos(B+C) = -\cos A$

* $\cos(A+C) = -\cos B$

$\frac{A+B+C}{2} = \frac{\pi}{2}$

$\frac{B+C}{2} = \frac{\pi}{2} - \frac{A}{2}$

$\sin\left(\frac{B+C}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{A}{2}\right)$

* $\sin\left(\frac{B+C}{2}\right) = -\cos\frac{A}{2}$

* $\sin\left(\frac{A+C}{2}\right) = -\cos\frac{B}{2}$

$\sin\left(\frac{A+B}{2}\right) = -\cos\frac{C}{2}$