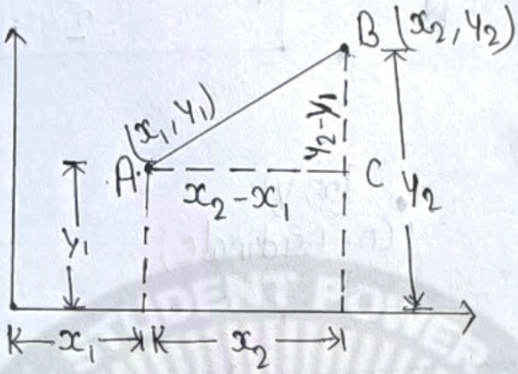
① Date – 17/12/2023

Co-ordinate Geometry

Straight line

Formula

a) Distance formula



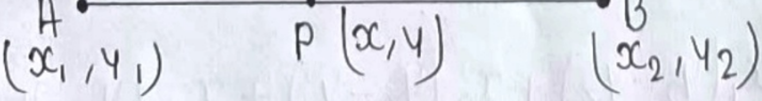
Distance AB

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$AB = \sqrt{(AC)^2 + (BC)^2}$$

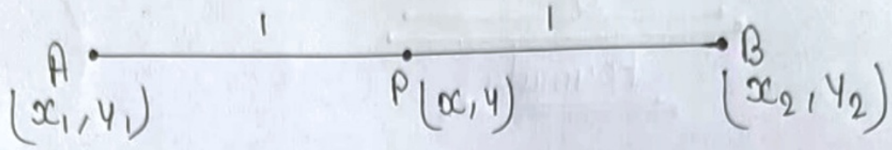
$$\boxed{AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

b) Section formula



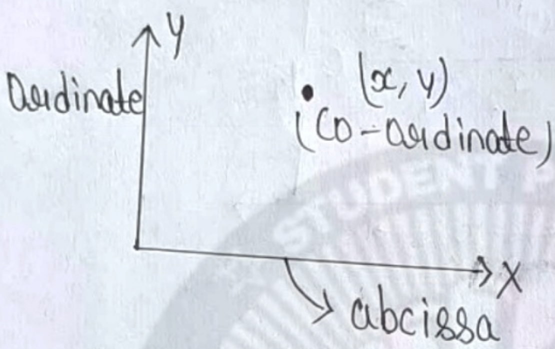
$$\boxed{x = \frac{mx_2 + nx_1}{m+n}} \quad , \quad \boxed{y = \frac{my_2 + nx_1}{m+n}}$$

* Mid point formula

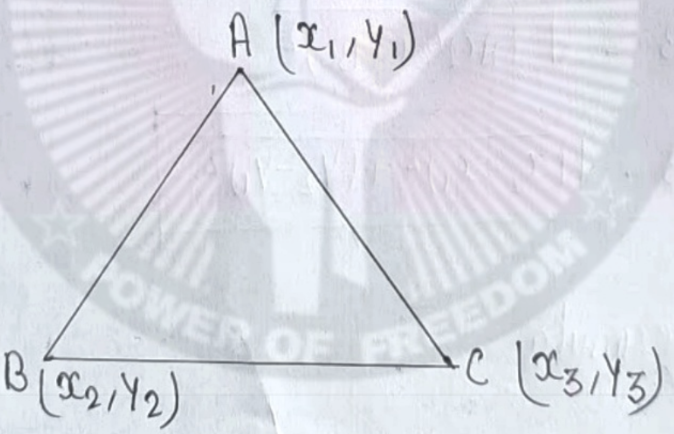


$$\boxed{x = \frac{x_1 + x_2}{2}} \quad , \quad \boxed{y = \frac{y_1 + y_2}{2}}$$

☆



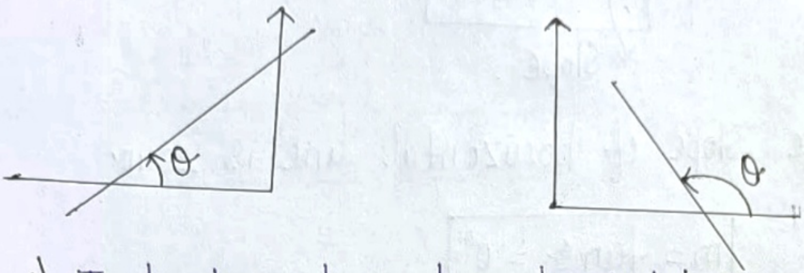
☆ Area of Δ whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is



$$\boxed{\Delta = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]}$$

☆ Inclination of a line ☆

→ The angle of inclination of a line is the angle θ which the part of the line above the x-axis makes with the positive direction of x-axis.



** i) Inclination of a line parallel (||) to x-axis is zero.

ii) Inclination of a line parallel (||) to y axis is $90°$.

* Horizontal line :— Any line parallel to x-axis or x-axis itself is called a horizontal line.

* vertical line :— Any line parallel to y-axis or y-axis itself is called a vertical line.

* Oblique line :— A line which is neither horizontal nor vertical is called a oblique line.

(4)

* Slope or Gradient of a line *

→ If θ is the inclination of non-vertical line, then $m = \tan\theta$ is called the slope of the line.

$$\boxed{m = \tan\theta}$$

m → Slope

** i) The slope of horizontal line is zero.

$$\boxed{m = \tan\theta = 0^\circ}$$

ii) The slope of vertical line is not defined.

$$\boxed{m = \tan 90^\circ = \text{not defined}}$$

***

a) $m = 0 \Rightarrow \tan\theta = 0$

$\theta = 0^\circ$

b) $m > 0 \Rightarrow \tan\theta > 0$

$= \theta$ lies b/w 0° & 90°

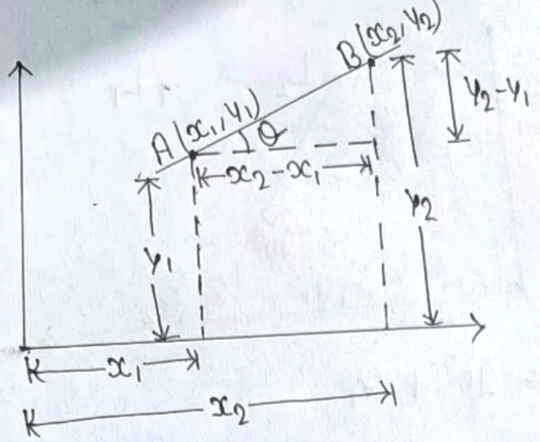
$= \theta$ is acute angle

c) $m < 0 \Rightarrow \tan\theta < 0$

$= \theta$ lies b/w 90° & 180°

$= \theta$ is obtuse

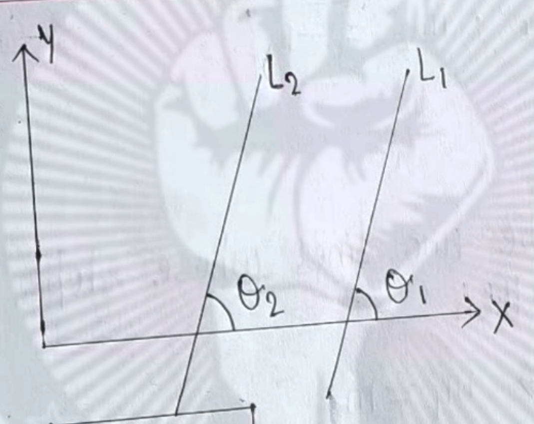
* Slope of a line passing through two given point *



$m = \tan\theta$

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$

☆ Slope of parallel line



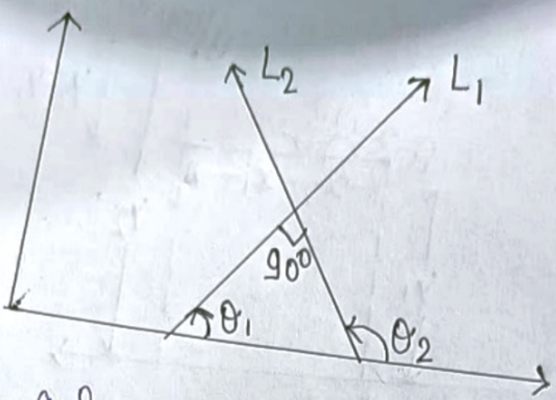
$$\boxed{m_1 = m_2}$$

$\tan\theta_1 = \tan\theta_2$

$$\boxed{\theta_1 = \theta_2}$$

6

☆ Slope of perpendicular ($\perp^r$) line



$$\theta_2 = 90^\circ + \theta_1$$

$$\tan\theta_2 = \tan(90^\circ + \theta_1)$$

$$\tan\theta_2 = -\cot\theta_1$$

$$\tan\theta_2 = \frac{-1}{\tan\theta_1}$$

$$\tan\theta_1 \cdot \tan\theta_2 = -1$$

$$\boxed{m_1 \cdot m_2 = -1}$$

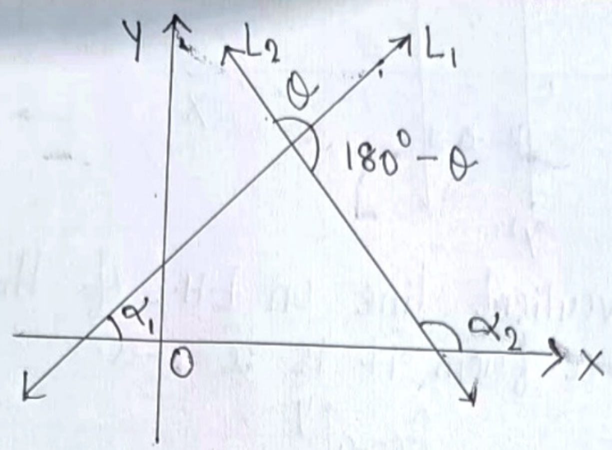
☆☆ Summary

Let L_1 & L_2 be two lines whose slope is m_1 & m_2 respectively, then

(i) $L_1 \parallel L_2 \iff m_1 = m_2$

(ii) $L_1 \perp L_2 \iff m_1 \cdot m_2 = 1$

☆ Angle b/w to non-vertical & non-perpendicular lines ☆



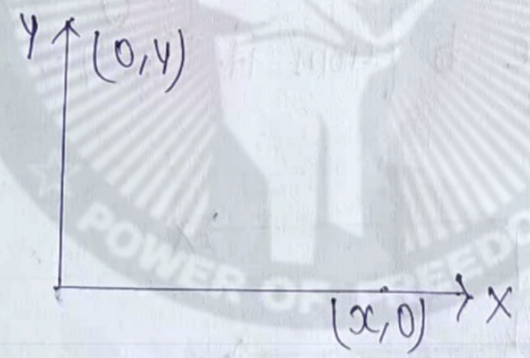
Let $\alpha = \min(\theta, (180^\circ - \theta))$

$$\therefore \boxed{\tan\alpha = \left|\frac{m_2 - m_1}{1 + m_1 \cdot m_2}\right|}$$

* various form of equation of a line *

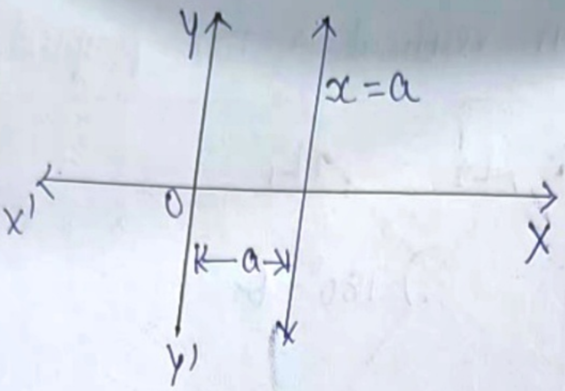
i) eq^n of x-axis is $y = 0$

ii) eq^n of y-axis is is $x = 0$

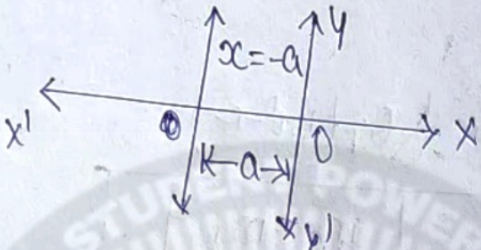


iii) eq^n of a vertical line on R.H.S of the y-axis at a distance a from it is $x = a$

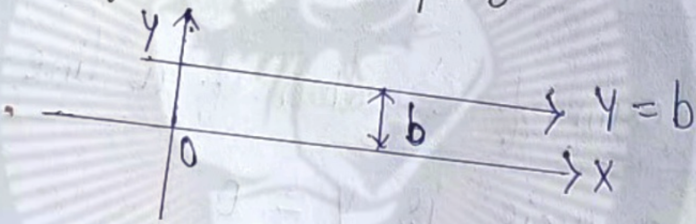
$= r^2$ radius with Centre at the r is



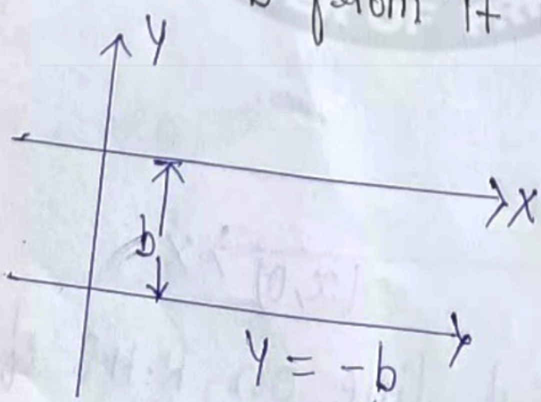
iv) eq^n of a vertical line on L.H.S of the x-axis at a distance a from it is $x = -a$



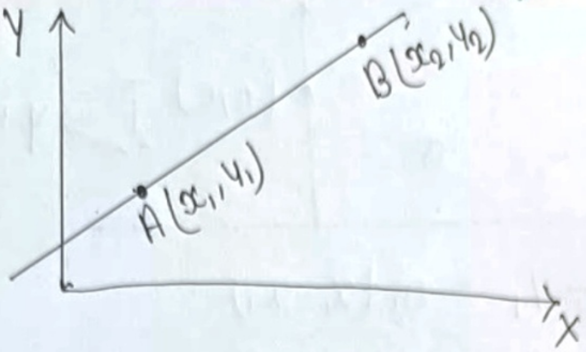
v) eq^n of a horizontal line lying above the x-axis at a distance b from it is $y = b$



vi) Eq^n of a horizontal line lying below the x-axis at a distance b from it is $y = -b$



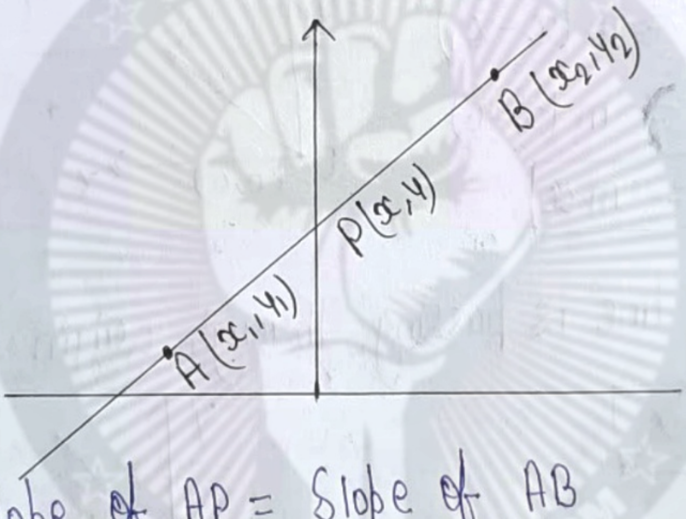
☆ Eqⁿ of a line in point slope form :-



$$m = \frac{y-y_1}{x-x_1}$$

$$\boxed{y-y_1 = m(x-x_1)}$$

☆ Eqⁿ of a line in two point form :-

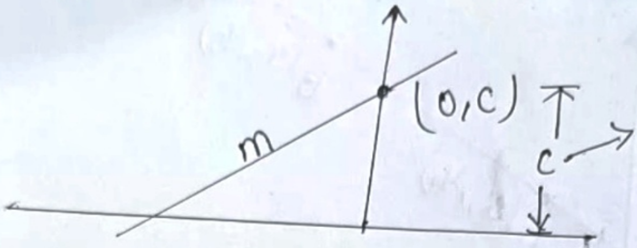


Slope of AP = Slope of AB

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\boxed{y-y_1 = \frac{y_2-y_1}{x_2-x_1} \cdot x-x_1}$$

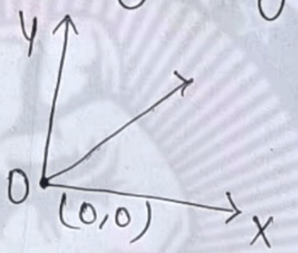
* Equation of a line in slope intercept form :-



$$y - y_1 = m(x - x_1)$$
$$y - c = m(x - 0)$$
$$y - c = mx$$
$$\boxed{y = mx + c}$$

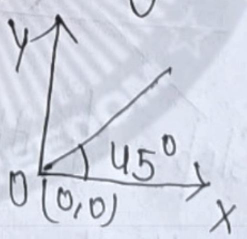
** If the line is passing through origin

$$y - y_1 = m(x - x_1)$$
$$y - 0 = m(x - 0)$$
$$\boxed{y = mx}$$

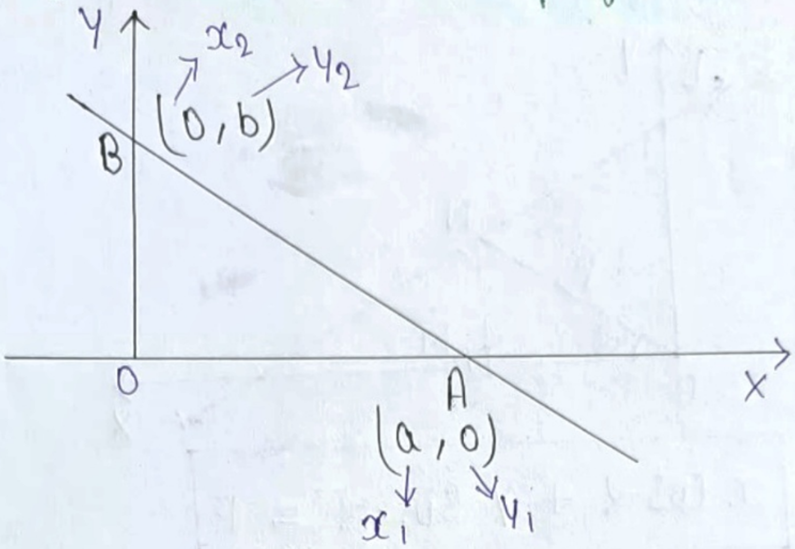


** If the line is passing through origin and $\theta = 45^0$

$$\boxed{y = x}$$



* Equation of a line in intercept form



$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - 0}{x - a} = \frac{b - 0}{0 - a}$$

$$\frac{y}{x - a} = \frac{b}{-a}$$

$$-ay = b(x - a)$$

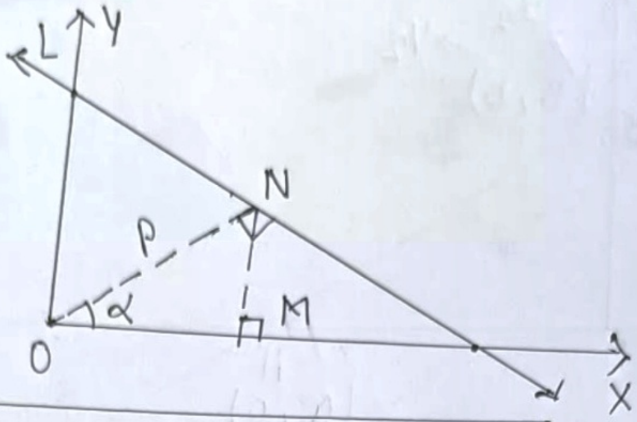
$$-ay = bx - ba$$

$$bx + ay = ab$$

$$\frac{\cancel{b}x}{a\cancel{b}} + \frac{\cancel{a}y}{\cancel{a}b} = \frac{ab}{ab}$$

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

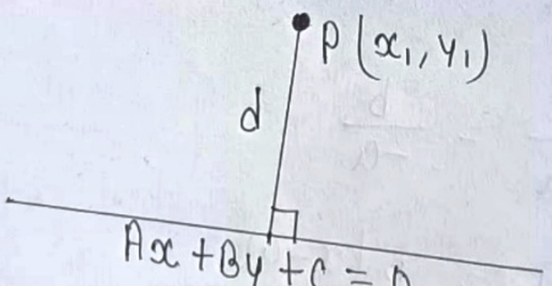
* Equation of a line in normal form :—



$$\boxed{x \cos\alpha + y \sin\alpha = p}$$

* Distance of a point from a line

☆ Length of perpendicular from a given point P(x,
on the line.



$$\boxed{d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}}$$

☆ Length of perpendicular from the origin (0,0).

$$d = \frac{|Ax_1 + By_1 + c|}{\sqrt{A^2+B^2}} = \frac{|A \times 0 + B \times 0 + c|}{\sqrt{A^2+B^2}}$$

$$\boxed{d = \frac{|c|}{\sqrt{A^2+B^2}}}$$

* Distance b/w two parallel (||) line $Ax + By + c_1 = 0$ & $Ax + By + c_2 = 0$ is

$$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{A^2 + B^2}}}$$

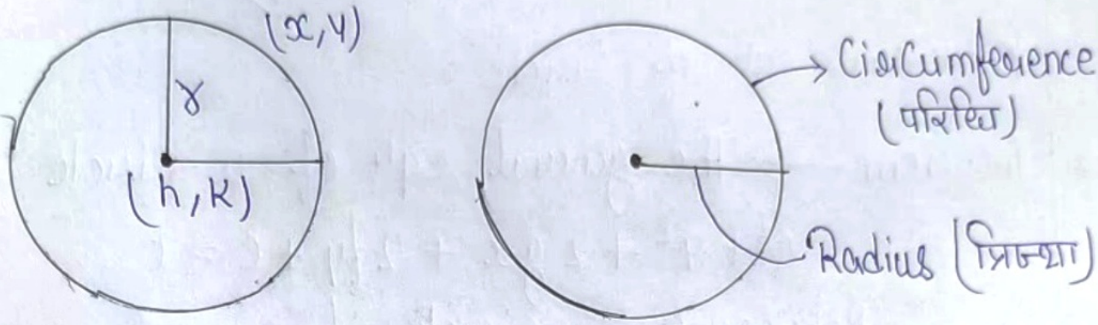
* Distance b/w $y = mx + c_1$ & $y = mx + c_2$

$$\boxed{d = \frac{|c_2 - c_1|}{\sqrt{1 + m^2}}}$$

Circle

Formula

* Circle :- A circle is the set of all point in a plane which are at a Constant distance form a fixed point in the plane



* Equation of a circle in standard form *

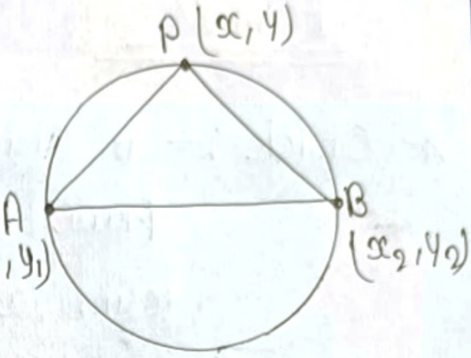
Theorem 1 → The eq^n of circle with Centre (h, K) and radius r is

$$\boxed{(x-h)^2 + (y-K)^2 = r^2}$$

Theorem 2 → The eq^n of Circle with Centre at the Origin and radius r is

$$\boxed{x^2 + y^2 = r^2}$$

☆ Theorem 3 — The eqⁿ of circle with A (x_1, y_1) and B (x_2, y_2) as the end points of a diameter of circle.



$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$$

* General eqⁿ of Circle

☆ Theorem ⟶ The general eqⁿ of a circle of the form

$$x^2+y^2+2gx+2fy+c=0$$

Represent a circle if

$$(g^2+f^2-c) > 0$$

Its Centre is $(-g, -f)$ and Radius is $\sqrt{g^2+f^2-c} = 0$