J.M

Date __/__/ *Gravitation*

* Newton's law of gravitation (universal law of gravitation).

⇒ It states that every body in the universe attracts every other body with a force (F), which is

(i) directional proportional to the product of their masses and

(ii) inversely proportional to the square of the distance between them.

This force is along the line joining centres of two bodies.

$m_1$ → mass of one body in kg

$m_2$ → mass of other body in 'kg'

d → distance between two bodies in 'm'

F → gravitational force of attraction in 'N'

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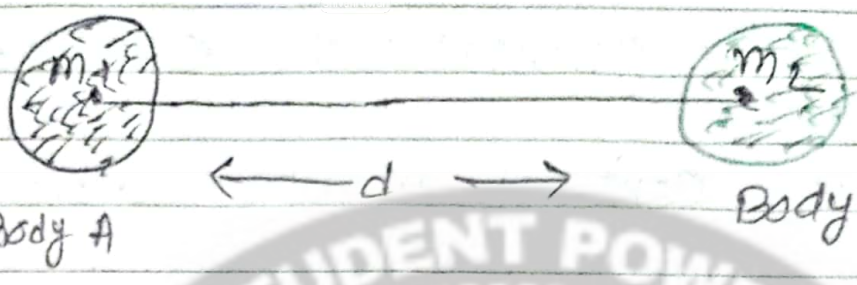


As per newton's universal law of gravitational

$$F \propto m_1 m_2 \quad \text{— (i)}$$

$$F \propto \frac{1}{d^2} \quad \text{— (ii)}$$

Combing these two equation -

$$F \propto \frac{m_1 m_2}{d^2}$$

$$F = \frac{G m_1 m_2}{d^2}$$

Where G → Constant of proportionality called Newton's gravitational constant

G → carries of value $6.67 \times 10^{-11}\ Nm^2/kg^2$

$$F = \frac{G m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \frac{m_1 m_2}{d^2}$$

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* Newton's gravitational constant (G):-

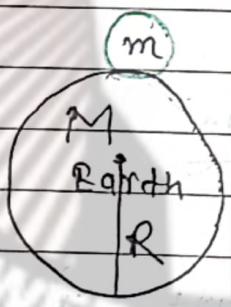
⇒ G is constant in the universe.

⇒ value of G is $6.67 \times 10^{-11} Nm^2/kg^2$

⇒ S.I unit of G is $Nm^2/kg^2$ or $Nm^2kg^{-2}$

* Acceleration due to gravity(g) and relation with G :-

Consider a body of mass 'm' on the surface of earth of mass 'M'




Let M → mass of the earth ($6 \times 10^{24}$ kg)

m → mass of body

R → radius of the earth

F → gravitational force of attraction.

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As per newton's law of gravitation.

$$F = G\frac{Mm}{r^2} \quad \text{— (I)}$$

As per newton's second law of motion, Force is the product of mass and acceleration.

$$F = m \times a$$

we know that acceleration with which body is falling towards centre of earth is called gravitation acceleration 'g'

$$F = mg \quad \text{— (II)}$$

Equating equation (I) and (II), we get

$$mg = G\frac{Mm}{R^2}$$

$$g = \frac{GM}{R^2}$$

$$\boxed{\frac{GM}{R^2} = g}$$

Acc. Due to gravity
The acc. of the free falling body due to the force of attract of another body is called Acc. due to

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* Calculation of gravitational acceleration 'g' due to earth

$$g = \frac{GM}{R^2}$$

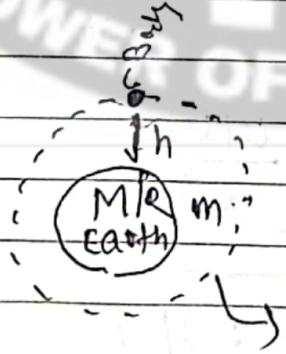
$$g = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24}}{(6400 \times 10^3)^2}$$

$$g = 9.81 \text{ m/s}^2$$

* VARIATION OF 'g' WITH ALTITUDE AND LATTITUDE

(a) Variation of 'g' with Altitude

⇒ Altitude is the distance measured in feet or meter above sea (mean) level.




↳ gravitational influence field.

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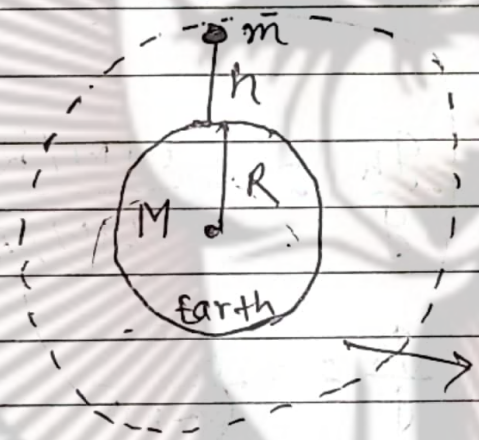
Let M — mass of the earth
R — radius of the earth
m — mass of the body
h — height of the body above earth's surface
g — earth's gravitational acceleration
$g_h$ — earth's gravitational acceleration at hight 'h' from earths surface




When body is on the earth's surface,

we have $mg = \frac{GMm}{R^2}$

$$g = \frac{GM}{R^2} \quad \text{— (1)}$$

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When body is at a height 'h' from earth's surface

$$mg_h = \frac{GMm}{(R+h)^2}$$

$$g_h = \frac{GM}{(R+h)^2} \quad \text{— (2)}$$

Dividing equation (2) by equation (1), we get

$$\frac{g_h}{g} = \frac{GM/(R+h)^2}{GM/R^2}$$

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\frac{g_h}{g} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\frac{g_h}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

$$\frac{g_h}{g} = \approx 1 - \frac{2h}{R}$$

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

## * Variation of 'g' with depth :-

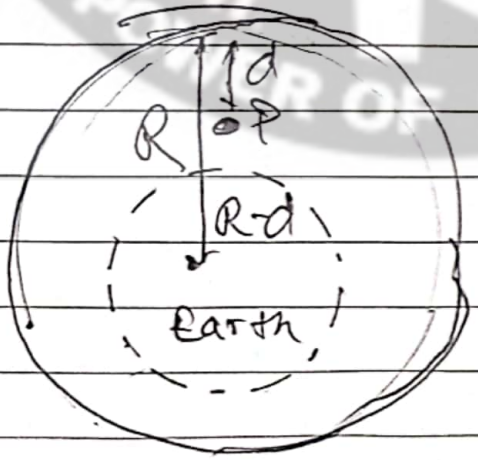
Let $g_d$ be the acceleration due to gravity at point 'P' which is below the surface of the earth at depth 'd' eg. mines.

The body experiences gravitational force due to inner shaded sphere of radius (R-d)

$$\frac{g_d}{g} = \frac{R-d}{R} = \frac{R(1-d/R)}{R}$$

$$\frac{g_d}{g} = 1\left(1 - \frac{d}{R}\right)$$

$$g_d = g\left(1 - \frac{d}{R}\right)$$



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## * Variation of g with Latitude :-

Latitude is the distance measured in degrees towards north and south from equator. (3 Marks)
Equator is the plane passing through the centre of earth and perpendicular to its axis of rotation. The latitude of a place is the angle made by line joining place and centre of the earth with the equator. The shape of the earth is not perfect sphere. It is bulged out at equator. Therefore radius from centre to north pole is 6350 km and radius along equator is 6372 km.

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PICTURE

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$EE' \rightarrow$ equator
$NS \rightarrow$ axis of spinning
$\omega \rightarrow$ angular velocity
$P \rightarrow$ Place on earth surface
$mg \rightarrow$ weight of body at P'
$mr\omega^2 \cos\theta \rightarrow$ component which opposes 'mg'
$mr\omega^2 \sin\theta \rightarrow$ component which has no effect on weight of body.

which P is the place which makes angle $\theta$ (latitude) with equator.

Thus at 'P' $\angle PCE = \theta$

at north or south pole $\angle NCE = \theta = 90^\circ$
at equator $\angle\theta = 0^\circ$

Earth is spining about its own axis, therefore every particle lying on its surface rotats around the axis in horizontal plane with same angular velocity $\omega$.

Thus effective weight of a body of mass 'm'

(i) at $P = mg_P = mg - mr\omega^2\cos\theta$

(ii) at $N = mg_N = mg - mr\omega^2\cos 90^\circ = mg$

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(iii) at $E = mg_E = mg_E - mr\omega^2\cos\theta = mg - mr\omega^2$

Thus effective gravitational acceleration

(i) at $P = g_P = g - r\omega^2\cos\theta$

(ii) at $N = g_N = g - r\omega^2\cos 90^\circ = g$

(iii) at $E = g_E = g - r\omega^2\cos 0^\circ = g - r\omega^2$

Thus g at equator is least and at pole it is highest.

This is one reason and orther reason is shape of earth. Earth is not exact sphere it is bulged out at equator and flatted at poles as shown. Therefore radius at equator is highest and radius at poles is least.

Because of two reasons (i) spinning of earth (ii) shape of earth; gravitational acceleration at equator is least and at poles is highest. Thus gravitational acceleration g is 0.5% more at poles than equator.