Trigonometry

(1) (i) $Sin(A+B)=\sin A\cos B+\cos A\sin B$ (ii) $Sin(A-B)=\sin A\cos B-\cos A\sin B$

(iii) $\cos(A+B)=\cos A\cos B-\sin A\sin B$ (iv) $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(v) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ (vi) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

(vii) $\cot(A+B)=\dfrac{\cot A\cot B-1}{\cot B+\cot A}$ (viii) $\cot(A-B)=\dfrac{\cot A\cot B+1}{\cot B-\cot A}$

(ix) $\tan(A+B+C)=\dfrac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan c\tan A}$

(2) (i) $\tan\left(\dfrac{\pi}{4}+\theta\right)=\dfrac{1+\tan\theta}{1-\tan\theta}$ (ii) $\tan\left(\dfrac{\pi}{4}-\theta\right)=\dfrac{1-\tan\theta}{1+\tan\theta}$

(iii) $\cot\left(\dfrac{\pi}{4}+\theta\right)=\dfrac{\cot\theta-1}{\cot\theta+1}$ (iv) $\cot\left(\dfrac{\pi}{4}-\theta\right)=\dfrac{\cot\theta+1}{\cot\theta-1}$

(3) (i) $\sin(A+B)\sin(A-B)=\sin^2A-\sin^2B=\cos^2B-\cos^2A$

(ii) $\cos(A+B)\cos(A-B)=\cos^2A-\sin^2B=\cos^2B-\sin^2A$

(4) (i) $\sin 2\theta=2\sin\theta.\cos\theta$ (ii) $\sin 2\theta=\dfrac{2\tan\theta}{1+\tan^2\theta}$ (iii) $\sin 2\theta=\dfrac{2\cot\theta}{1+\cot^2\theta}$

(5) (i) $\cos 2\theta=\cos^2\theta-\sin^2\theta$ (ii) $\cos 2\theta=1-2\sin^2\theta$

(iii) $\cos 2\theta=2\cos^2\theta-1$ (iv) $1+\cos 2\theta=2\cos^2\theta$

(v) $1+\cos\theta=2\cos^2\dfrac{\theta}{2}$ (vi) $1-\cos 2\theta=2\sin^2\theta$

(vii) $1-\cos\theta=2\sin^2\dfrac{\theta}{2}$

(6) (i) $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$ (ii) $\tan\theta=\dfrac{2\tan\dfrac{\theta}{2}}{1-\tan^2\dfrac{\theta}{2}}$

(7) (i) $\cot 2\theta=\dfrac{\cot^2\theta-1}{2\cot\theta}$ (ii) $\cot\theta=\dfrac{\cot^2\dfrac{\theta}{2}-1}{2\cot\dfrac{\theta}{2}}$

(8) (i) $\sin 3\theta=3\sin\theta-4\sin^3\theta$ (ii) $\cos 3\theta=4\cos^3\theta-3\cos\theta$

(iii) $\tan 3\theta=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

(9) (i) $2\sin A\cos B=\sin(A+B)+\sin(A-B)$ (ii) $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

(iii) $2\cos A\cos B=\cos(A-B)+\cos(A+B)$ (iv) $2\sin A\sin B=\cos(A-B)-\cos(A+B)$

(10) (i) $\sin C+\sin D=2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)$

(ii) $\sin C-\sin D=2\cos\left(\frac{C+D}{2}\right).\sin\left(\frac{C-D}{2}\right)$

(iii) $\cos C+\cos D=2\cos\left(\frac{C+D}{2}\right).\left(\frac{C-D}{2}\right)$

(iv) $\cos C-\cos D=2\sin\left(\frac{C+D}{2}\right).\sin\left(\frac{D-C}{2}\right)$

(v) $\tan C+\tan D=\frac{\sin(C+D)}{\cos C.\cos D}$ (vi) $\tan C-\tan D=\frac{\sin(C-D)}{\cos C.\cos D}$

(vii) $\cot C+\cot D=\frac{\sin(C+D)}{\sin C.\sin D}$ (viii) $\cot C-\cot D=\frac{\sin(D-C)}{\sin C.\sin D}$

Inverse trigonometric Function

(1) Domain , Co-domain and Range (प्रांत , सह-प्रांत तथा परिसर)

Function	Domain	Range
$y=\sin^{-1}x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\cos^{-1}x$	$-1\le x\le 1$	$0\le y\le\pi$
$y=\tan^{-1}x$	$-\infty<x<\infty$	$-\frac{\pi}{2}<y<\frac{\pi}{2}$
$y=\cot^{-1}x$	$-\infty<x<\infty$	$0<y<\pi$
$y=\sec^{-1}x$	$x\le -1$ or $x\ge 1$	$0\le y\le\pi, y\ne\frac{\pi}{2}$
$y=\cos ec^{-1}x$	$x\le -1$ or $x\ge 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}, y\ne 0$

(2) (i) $\sin(\sin^{-1}x)=x \;,\; x\in[-1,1]$ (ii) $\cos(\cos^{-1}x)=x \;,\; x\in[-1,1]$

(iii) $\tan(\tan^{-1}x)=x \;,\; x\in R$ (iv) $\cot(\cot^{-1}x)=x \;,\; x\in R$

(v) $\sec(\sec^{-1}x)=x \;,\; x\in(-\infty,-1]\cup[1,\infty)$

(vi) $\cos ec(\cos ec^{-1}x)=x \;,\; x\in(-\infty,-1]\cup[1,\infty)$

(3) (i) $\sin^{-1}(\sin x)=x \;,\; x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (ii) $\cos^{-1}(\cos x)=x \;,\; x\in[0,\pi]$

(iii) $\tan^{-1}(\tan x)=x \;,\; x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (iv) $\tan^{-1}(\tan x)=x \;,\; x\in(0,\pi)$

(v) $\sec^{-1}(\sec x)=x \;,\; x\in\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

(vi) $\cos ec^{-1}(\cos ecx)=x \;,\; x\in\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

(4.) (i) $\sin^{-1} x = \cos ec^{-1}\left(\frac{1}{x}\right)$ (ii) $\cos ec^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$

(iii) $\cos^{-1} x = \sec^{-1}\left(\frac{1}{x}\right)$ (iv) $\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right)$

(v) $\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right), x > 0 = -\pi + \cot^{-1}\left(\frac{1}{x}\right), x < 0$

(vi) $\cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right), x > 0 = \pi + \tan^{-1}\left(\frac{1}{x}\right), x < 0$

(5.) (i) $\sin^{-1}(-x) = -\sin^{-1} x$ (ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

(iii) $\tan^{-1}(-x) = -\tan^{-1} x$ (iv) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1} x$ (vi) $\cos ec^{-1}(-x) = -\cos ec^{-1} x$

(6.) (i) $\sin^{-1} + \cos^{-1} x = \frac{\pi}{2}$ (ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$

(iii) $\sec^{-1} + \cos ec^{-1} x = \frac{\pi}{2}$

(7.) (i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

(ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$

(iii) $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{(1-x^2)(1-y^2)}\right)$

(iv) $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left(xy + \sqrt{(1-x^2)(1-y^2)}\right)$

(v) $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(vi) $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(vii) $\cot^{-1} x + \cot^{-1} y = \cot^{-1}\left(\frac{xy-1}{y+x}\right)$

(viii) $\cot^{-1} x - \cot^{-1} y = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$

* $\tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) & , \text{ यदि } xy < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & , \text{ यदि } x > 0, y > 0 \text{ तथा } xy > 1 \\ -\pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) & , \text{ यदि } x < 0, y < 0 \text{ तथा } xy > 1 \end{cases}$

* $$\tan^{-1}x-\tan^{-1}y=\begin{cases}\tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{यदि } xy>-1\\ \pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{यदि } x>0,y<0 \text{ तथा } xy<-1\\ -\pi+\tan^{-1}\left(\frac{x-y}{1+xy}\right), & \text{यदि } x<0,y>0 \text{ तथा } xy<-1\end{cases}$$

(8.) (i) $2\tan^{-1}x=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ (ii) $2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(iii) $2\tan^{-1}x=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (iv) $2\cot^{-1}x=\cot^{-1}\left(\frac{x^2-1}{2x}\right)$

(9.) (i) $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ (ii) $2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right)$

(10.) (i) $\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{x}{\sqrt{1-x^2}}=\cos^{-1}\frac{\sqrt{1-x^2}}{x}=\sec^{-1}\frac{1}{\sqrt{1-x^2}}=\operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$

(ii) $\cos^{-1}x=\sin^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{\sqrt{1-x^2}}{x}=\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\sec^{-1}\frac{1}{x}=\operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$

(iii) $\tan^{-1}x=\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)=\cot^{-1}\left(\frac{1}{x}\right)=\sec^{-1}\sqrt{1+x^2}=\operatorname{cosec}^{-1}\left(\frac{\sqrt{1+x^2}}{x}\right)$

(11.) (i) $3\sin^{-1}x=\sin^{-1}(3x-4x^3)$ (ii) $3\cos^{-1}x=\cos^{-1}(4x^3-3x)$

(iii) $3\tan^{-1}x=\tan^{-1}\left(\frac{3x-3x^3}{1-3x^2}\right)$

Matrix

(1.)

Trick (i) $(adjA)A=A(adjA)=|A|In$

Eg :- यदि $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}_{2\times2}$ हो तो $(adjA)A$ बराबर होगा।

$$|A|=14-15=-1$$

Trick :- $(adjA)A=|A|In$, $(adjA)A=-1\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$

Trick (ii) यदि $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ तो $adjA=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$ Eg :- $A=\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$ तो $adjA=\begin{bmatrix}7 & -3\\ -5 & 2\end{bmatrix}$

Trick (iii) $adj(AB)=(adjB)(adjA)$

Trick (iv) यदि आव्यूह A,n कोटि का वर्ग आव्यूह है तथा $|A|$ आव्यूह A का सारणिक है, तो

(a) $|adjA| = |A|^{n-1}$ (b) $adj(adjA) = |A|^{n-2} A$ (c) $|adj(adjA)| = |A|^{(n-$

(2.) ***Trick***

(i) $A^{-1} = \frac{adjA}{|A|}, A \neq 0$ (ii) $(AB)^{-1} = B^{-1}A^{-1}$

(ii) $(AA)^{-1} = A^{-1}A = I$ (iii) यदि $\det A \neq 0$ तो $\det(A^{-1}) = (\det A)^{-1} = \frac{1}{\det A}$

Differentiation

(1.) (i) $\frac{d}{dx}(x^n) = nx^{n-1}$ (ii) $\frac{dx}{dx} = 1$ (iii) $\frac{d\left(\frac{1}{x}\right)}{dx} = \frac{-1}{x^2}$ (iv) $\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$

Trick :- $\frac{d\left(\frac{1}{x^n}\right)}{dx} = \frac{-n}{x^{n+1}}$

(2.) (i) $\frac{d\sin x}{dx} = \cos x$ (ii) $\frac{d\cos x}{dx} = -\sin x$ (iii) $\frac{d\tan x}{dx} = \sec^2 x$

(iv) $\frac{d\cot x}{dx} = -\cos ec^2 x$ (v) $\frac{d\sec x}{dx} = \sec x.\tan x$ (vi) $\frac{d\cos ecx}{dx} = -\cos ecx.\cot x$

(3.) (i) $\frac{d(e^x)}{dx} = e^x$ (ii) $\frac{da^x}{dx} = a^x.\log_e a$ (iii) $\frac{d}{dx}(\log_e a) = \frac{1}{x}, x > 0$

(iv) $\frac{d\log_e|x|}{dx} = \frac{1}{x}, x \neq 0$ (v) $\frac{d\log_a x}{dx} = \frac{\log_a e}{x}$

(4.) (i) $\frac{d\sin^{-1}x}{dx} = \frac{1}{\sqrt{1-x^2}}$ (ii) $\frac{d\cos^{-1}x}{dx} = \frac{-1}{\sqrt{1-x^2}}$ (iii) $\frac{d\tan^{-1}x}{dx} = \frac{1}{1+x^2}$

(iv) $\frac{d\cot^{-1}x}{dx} = \frac{-1}{1+x^2}$ (v) $\frac{d\sec^{-1}x}{dx} = \frac{1}{x\sqrt{x^2-1}}$ (vi) $\frac{d\cos ec^{-1}x}{dx} = \frac{-1}{x\sqrt{x^2-1}}$

(5.) (i) $\frac{d|x|}{dx} = \frac{|x|}{x} = \frac{x}{|x|}, x \neq 0$ (ii) $\frac{d|x|}{dx} = 1, x > 0$ (iii) $\frac{d|x|}{dx} = -1, x < 0$

(iv) $\frac{d\sqrt{x^2}}{dx} = \frac{d|x|}{dx}$

(6.) (i) $\frac{dc}{dx} = 0$ (ii) $\frac{dcu}{dx} = c\frac{du}{dx}$ (iii) $\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

(iv) $\frac{d(u-v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$ (v) $\frac{duv}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ (vi) $\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(vii) $\frac{d(uvw)}{(dx)} = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$

Trick:- यदि $y=[u(x)]^{v(x)}$ तो $\frac{dy}{dx}=[u(x)]^{v(x)}\left[\frac{v(x)}{u(x)}\times u'(x)+v'(x)\log[u(x)]\right]$

Integration

(1.) (i) $\int x^n dx=\frac{x^{n+1}}{n+1}+c, n\neq -1$ (ii) $\int\frac{1}{x}dx=\log|x|+c$ (iii) $\int dx=x+c$

(iv) $\int\frac{1}{\sqrt{x}}dx=2\sqrt{x}+c$ Trick :- $\int\frac{1}{x^n}=\frac{-1}{(n-1)x^{n-1}}+c$

(2.) (i) $\int\sin x dx=-\cos x+c$ (ii) $\int\cos x dx=\sin x+c$ (iii) $\int\sec^2 x dx=\tan x+c$

(iv) $\int\cos ec^2 x dx=-\cot x+c$ (v) $\int\sec x.\tan x dx=\sec x+c$ (vi) $\int\cos ecx.\cot x=-\cos ecx+c$

(vii) $\int\tan x dx=\log|\sec x|+c$ (viii) $\int\cot x dx=\log|\sin x|+c$ (ix) $\int\sec x dx=\log|\sec x+\tan x|+c$

(x) $\int\cos ecx dx=\log|\cos ecx-\cot x|+c=\log\left|\tan\frac{x}{2}\right|+c$

(3.) (i) $\int\frac{1}{\sqrt{1-x^2}}dx=\sin^{-1}x+c$ (ii) $\int\frac{dx}{1+x^2}=\tan^{-1}x+c$ (iii) $\int\frac{dx}{x\sqrt{x^2-1}}=\sec^{-1}x+c$

(iv) $\int e^x dx=e^x+c$ (v) $\int a^x dx=\frac{a^x}{\log_e a}+c$

Integration by Parts

(i) $\int u.vdu=u\int vdu-\int\left(\frac{du}{dx}\int vdx\right)dx$ (ii) $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+c$

Some Important formula

(i) $\int\frac{dx}{x^2-a^2}=\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$ (ii) $\int\frac{dx}{a^2-x^2}=\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

(iii) $\int\frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$ (iv) $\int\frac{dx}{\sqrt{x^2-a^2}}=\log\left|x+\sqrt{x^2-a^2}\right|+c$

(v) $\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$ (vi) $\int\frac{dx}{\sqrt{a^2+x^2}}=\log\left|x+\sqrt{a^2+x^2}\right|+c$

(vii) $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$

(viii) $\int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

(ix) $\int\sqrt{a^2+x^2}dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right|+c$

Trick:- $\int\frac{a\cos x+b\sin x}{p\cos x+q\sin x}dx=\left(\frac{ap+bq}{p^2+q^2}\right)x+\left(\frac{aq-bp}{p^2+q^2}\right)\log|p\cos x+q\sin x|+c$

E.g (i)

$$\int\frac{2\cos x+3\sin x}{4\cos x+3\sin x}dx=\left(\frac{8+15}{4^2+5^2}\right)x+\left(\frac{10-12}{4^2+5^2}\right)\log|4\cos x+5\sin x|+c=\frac{23}{41}x-\frac{2}{41}\log|4\cos x+5\sin x|+c$$