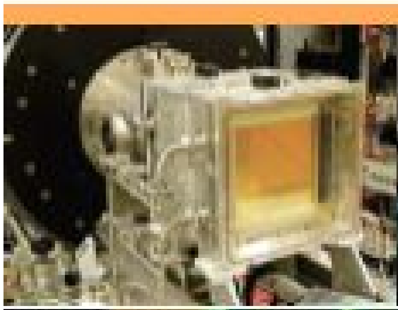
As Per New Syllabus of SBTE, Jharkhand
First Year Diploma • Semester I
Engineering and Technology

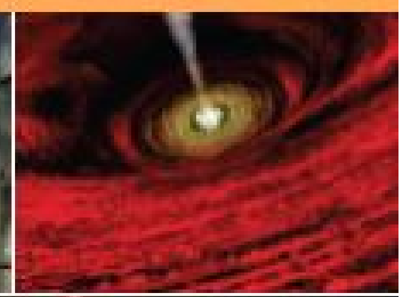
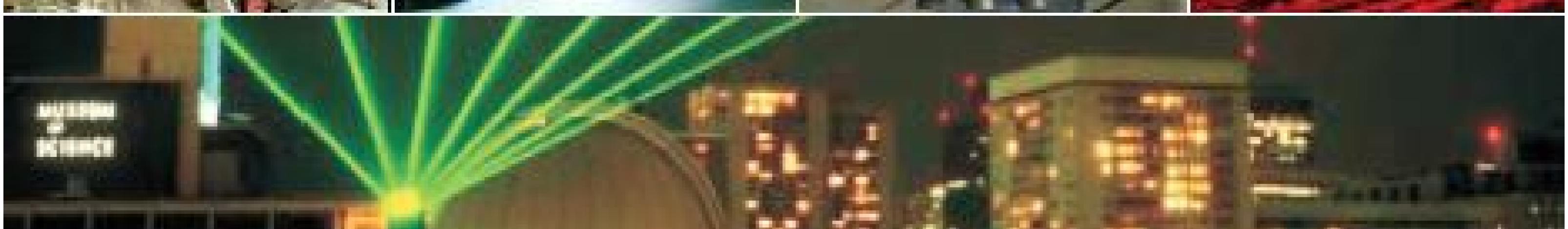
New Syllabus

Engineering Physics-I

(Basic Physics)

Dr. M. S. Pawar
Dr. M. A. Sutar



NIRALI® PRAKASHAN
ADVANCEMENT OF KNOWLEDGE

A TEXT BOOK OF

ENGINEERING PHYSICS-I

(BASIC PHYSICS)

SEMESTER – I

FIRST YEAR DIPLOMA COURSES IN ENGINEERING AND TECHNOLOGY

AS PER SBTE'S NEW REVISED SYLLABUS

FOR JHARKHAND, JULY 2017

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Preface

It gives us a sense of satisfaction to bring out this New revised edition of **Engineering Physics - I**.

This book aims at providing a complete coverage of the needs of First Year students as per S.B.T.E's. revised syllabus. The entire revised syllabus has been covered keeping in view the non-availability of the complete subject matter through a single source.

The difficult articles have been explained in a simple language providing, wherever necessary, neat and well explained diagrams so that even an average student may be able to follow it independently. A sufficient number of solved examples and problems with answers and SBTE questions are given at the end of each topic. Formulae specifying symbol meaning are enlisted before solving the examples. Synopsis is given at the starting of each topic which will give a fair idea about the task to be studied in the chapter. Solving problem is the big problem for most of the students. In this book, attempt has been made to simplify this difficulty by solving examples in simple style and in sufficient details to follow the developments.

We are thankful to Prof. Vidya S. Byakod, Principal, Pimpri-Chinchwad Educational Trust's, Pimpri-Chinchwad Polytechnic, for giving valuable suggestions while writing this book. We are also thankful to the management of Pimpri-Chinchwad Educational Trust, especially to Late Shri. S. B. Patil (President), Shri. Dnyaneshwarji Landge (Vice President) and Shri. V. S. Kalbhor (Secretary), Shri. P. D. Patil (Director Dr. D. Y. Patil Pratishthan), Principal Hiremath Sir (Y. B. Patil Polytechnic) and our colleagues from Y. B. Patil Polytechnic and Dr. D. Y. Patil Institute of Technology, who encouraged us for the successful completion of this venture.

We are grateful to the publisher Shri. Dineshbhai Furia, Shri. Jignesh Furia and staff of Nirali Prakashan, Pune for providing all facilities required and bringing out this book in the short span of time at their disposal.

During the progress of writing this book, Prof. S. S. Deo, Prof. Handore, Prof. Shirshetti, Mrs. Rohini M. Pawar, Shri. S. D. Jagtap from P. C. Polytechnic, Nigadi. Prof. B. S. Salunkhe, from N. M. V. Polytechnic, Talegaon, Prof. More, Prof. Bhosale, Prof. Saindane, from J. N. I. O. T. Katraj extended their valuable help, co-operation and encouragement. We are really thankful for valuable corrections and suggestions to Prof. Dumne Sharadchandra Ex. H.O.D. in Lal Bahadur Shastri Sr. College in District Nanded. We are really thankful to all of them.

We sincerely hope this edition will be well received by students and teachers of all polytechnics.

The authors will appreciate suggestions from teachers and students for the improvement of the book.

We take this opportunity to extend our good wishes to the students and teachers of all polytechnics.

– Authors

Syllabus

Chapter 1 : UNITS AND MEASUREMENTS **(Hours 04, Marks 06)**

1.1 Need of measurement and unit in engineering and science, definition of unit, requirements of standard unit, systems of units - CGS, MKS and SI, fundamental and derived quantities and their units.

1.2 Definition of dimensions with examples, principle of homogeneity of dimensions, limitations of dimensions.

1.3 Definition of accuracy, precision and error, estimation of errors - absolute error, relative error and percentage error, rules and identification of significant figures.

(Numericals on percentage error and significant figures)

Chapter 2 : MECHANICS **(Hours 04, Marks 10)**

2.1 Motion along a Straight Line and Force

Concept of scalar and vector quantities, Equations of motion with constant acceleration (derivation and requirement), Equations of motion of falling body under gravity, Newton's laws of motion, Force, Inertia, Action and reaction, Tension, Momentum, Impulse and impulsive force with practical examples (basic idea), Conservation of linear momentum.

(Simple problems on linear motion)

2.2 Angular Motion

Definition of angular displacement, angular velocity and angular acceleration, Relation between linear velocity and angular velocity, Definition of simple harmonic motion (SHM), SHM as a projection of uniform circular motion on any diameter, Equation of SHM, Derivation of displacement, velocity and acceleration of a body executing SHM.

Chapter 3 : GRAVITATION **(Hours 03, Marks 06)**

Newton's law of gravitation, Newton's gravitational constant (G) and its SI unit, Acceleration due to gravity (g) and its relation with "G", Variation of g with altitude and latitude (deduction not required).

(Simple problems)

Chapter 4 : WORK, ENERGY AND POWER **(Hours 02, Marks 06)**

Definition of work, energy and power, Equation for P.E. and K.E., Work-energy principle, Representation of work by using graph, Work done by a torque (no derivation).

(Numericals on work, potential and kinetic energy)

Chapter 5 : GENERAL PROPERTIES OF MATTER **(Hours 04, Marks 08)**

5.1 Elasticity

Deforming force, Restoring force, Elastic and plastic body, Stress and strain with their types, Elastic limit, Hooke's law, Young's modulus, Bulk modulus, Modulus of rigidity and relation between them (no derivation)

(Numericals on stress, strain and Young's modulus)

5.2 Surface Tension

Molecular force, Cohesive and adhesive force, Molecular range, Sphere of influence, Laplace's molecular theory, Definition of surface tension and its S.I. unit, Angle of contact, Capillary action with examples, Shape of meniscus for water and mercury, Relation between surface tension, Capillary rise and radius of capillary (no derivation), Effect of impurity and temperature on surface tension.

(Numericals on relation between surface tension, capillary rise and radius)

5.3 Viscosity

Definition of viscosity, viscous force, velocity gradient, Newton's law of viscosity, Coefficient of viscosity and its S.I. unit, Streamline and turbulent flow with examples, Critical velocity, Reynold's number and its significance, Derivation of viscous force for free fall of spherical body through viscous medium, upthrust, terminal velocity, Stoke's law (statement for formula).

(Numericals on coefficient of viscosity, Reynold's number and Stoke's formula)

Chapter 6 : HEAT **(Hours 04, Marks 08)**

Transmission of Heat and Expansion of Solids :

Three modes of transmission of heat – Conduction, Convection and Radiation, Good and bad conductor of heat with examples, Law of thermal conductivity, Coefficient of thermal conductivity and its S.I. unit, Definition of linear, aerial and cubical expansion and relation between them. (No derivation)

(Numericals on law of thermal conductivity and coefficients of expansions)

Chapter 7 : ACOUSTICS

7.1 Sound **(Hours 04, Marks 06)**

Definition of wave motion, amplitude, period, frequency and wavelength, Relation between velocity, frequency and wavelength, Longitudinal and transverse wave, Definition of stationary wave, Node and antinode, Forced and free vibrations, Definition of resonance with examples, Derivation of formula for velocity of sound with end correction.

(Numericals on relation $v = n\lambda$ and resonance)

7.2 Acoustics of Building **(Hors 04, Marks 06)**

Acoustics : Concept and definition, Intensity and loudness of sound, Echo, Reverberation, standard reverberation time, Sabine's formula, Conditions for good acoustics, Factors affecting acoustical planning and auditorium.

(Numericals on Sabine's formula)

Practicals :

Skills to be developed

1. Intellectual skills :

Proper selection of measuring instruments on the basis of range, least count, precision and accuracy required for measurement.

Analyze properties of matter and their use for the selection of material.

To verify the principles, laws, using given instruments under different conditions.

To read and interpret the graph.

To interpret the results from observations and calculations.

To use these results for parallel problems.

2. Motor skills :

Proper handling of instruments.

Measuring physical quantities accurately.

To observe the phenomenon and to list the observations in proper tabular form.

To adopt proper procedure while performing the experiment.

To plot the graphs.

List of Experiments :

1. To know your Physics Laboratory.
2. To use Vernier Callipers for the measurement of dimension of given object.
3. To use Micrometer Screw Gauge for the measurement of dimensions (Length, Thickness, Diameter) of given object.
4. To use spherometer for the measurement of thickness of a given glass piece.
5. To calculate Young's modulus of elasticity of steel wire by Vernier method.
6. To study capillary phenomenon and to verify that the height of liquid in capillary is inversely proportional to the radius of capillary.
7. To determine coefficient of viscosity of given liquid using Stoke's method.
8. To calculate the linear thermal coefficient of expansion for copper by using Pullinger's apparatus.
9. To determine refractive index of a glass using glass slab by pin method (sin i/sin r = μ).
10. To determine the velocity of sound by using resonance tube.

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Contents

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Chapter 1

UNITS AND MEASUREMENTS

1.1 UNITS AND MEASUREMENTS

1.1.1 Introduction

In physics we are required to measure the physical quantities. Accurate measurements of physical quantities are needed. Measurement consists of the **comparison of an unknown quantity with a known fixed (standard) quantity**. Development of physics is mostly due to very accurate and precise instruments which can measure the size of physical quantity to high degree of accuracy.

The main purpose of measurement in engineering and science is to determine whether a job has been manufactured to the requirements of specification. Measurement is compulsory part of development technology. Accuracy of measurement depends on

- Method of measurement.
- Measuring instrument.

Measurement consists of the comparison of given quantity with standard.

e.g. Length of table is 3 metre.

i.e. Any measurement consists of two parts.

The **first is the number** which **indicates the magnitude of quantity** and **second indicates the standard**. In the above example, 3 is the magnitude and metre is the standard (unit) of that quantity. It gives exact sense that the length of the table is 3 times the standard.

Thus 3 (↓ magnitude ↓ i.e. how many times of standard unit) metre (↓ name of the unit (standard))

All the quantities measured must be clearly defined so that there should not be any confusion or doubt in between two parties. e.g. customer asks for 3 metre cloth to a shopkeeper. Here metre is the standard for the length and which is fixed and well accepted by universe (society). Hence, there is no chance for confusion between shopkeeper and customer.

1.1.2 Need of Measurement in Engineering and Science

Measurement is required in day to day life, for example, Mahendra's school is 2 km from his house, I can swim 90 sec under water without breathing, etc.

The main purpose of measurement in engineering and science is to determine whether a job has been manufactured to the requirement of specification. Measurement is compulsory part of development technology. Measurement is also required in commerce, agriculture, geography, etc.

Scientist will not be able to perform experiments, derive laws, form theories without accurate measurement. Accurate measurement is required in science and engineering. Accuracy of measurement depends on : (1) Method of measurement, (2) Measuring instrument.

Many times discrepancies observed in experimental and theoretical values led to development of new theories and relations.

Accurate measurement is very important in science and engineering in the following ways :

Need in Science :

(1) To predict scientific laws :

The correlation between the two more quantities can be found by measuring the changes occurred in their magnitudes under different conditions.

For example, consider a gas of fixed mass (m) and with constant absolute temperature (T). While predicting the relation between its pressure and volume, one has to check the changes occurred in these quantities by accurate measurement. "With the measurement of pressure of different volumes of gas, it is found that as pressure of a gas increases, its volume decreases in the same proportion" i.e. if P is doubled, volume becomes half the earlier. This becomes the law (Boyle's law).

(2) To verify the law (To verify theory) :

If some one wants to verify the law or scientific statements, the concerned quantities should measure accurately under given conditions.

For example, to verify Archimedie's principle, one should calculate volume of solid by using mathematical formula and comparing this value with the volume of liquid displaced by the same solid.

Need in Engineering :

(1) Correct sense of quantities : For example, measurement of mass, length, diameter etc. must be clear for development of engineering product.

(2) In quality assurance of a product : Quality depends on various factors, like strength, finishing, precision, correctness, user friendliness, etc. These factors should be measured prior to launching the product in the market. In every industry, this care is taken by the department of metrology and quality control (quality assurance department).

1.1.3 Unit, Requirements of Standard Unit

1.1.3.1 Unit of a Physical Quantity

Any physical quantity can be measured and represented in terms of number and unit.

Unit (Definition) : The standard used for measurement of a physical quantity is called unit of that quantity.

In the above example, 3 metre is the length of the table. Here metre is the standard (unit) used for the measurement of the length.

1.1.3.2 Requirements of Standard Unit

The unit selected should have following characteristics :

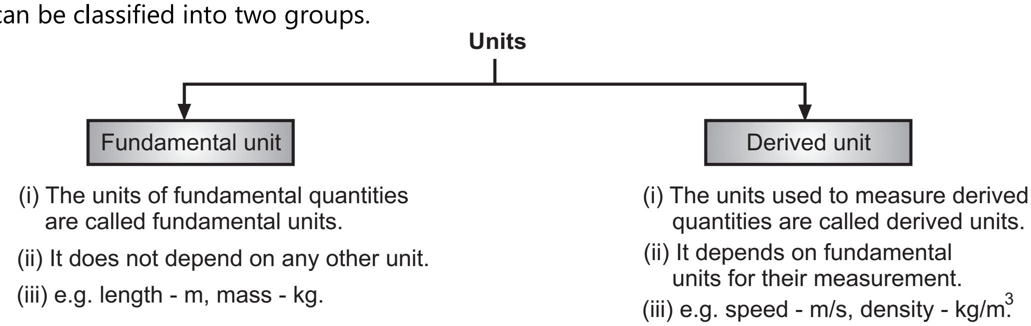
1. It should be universally accepted (i.e. accepted by all).
2. It should be definite and well defined.
3. It should be invariable (fixed) with time and place.
4. It should be easily reproducible and non-perishable.
5. It should be easily comparable with other similar units.
6. Its size should be such that the quantities measured with it should not be too large or too small.
7. It should be readily available.

A body named Conference Generale des poids et measure (CGMP).

OR

A body named General Conference on Weight and Measures has authority to decide units by international agreement.

Units can be classified into two groups.



1.1.4 Physical Quantities

Physical Quantity (Definition) : A physical quantity is a quantity which can be measured (computed, quantified or enumerated).

OR

Physical quantities are the quantities which are used to describe the property of a physical phenomenon.

OR

Any quantity, which can be measured, is called a physical quantity.

Examples of physical quantities :

Length, mass, time, current, force, work, power ..... etc.

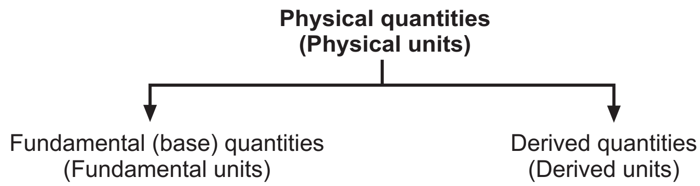
e.g. Length of table is **3 m**.

1.1.4.1 Fundamental (Base) Quantities, Derived Quantities and their Units

As we know that in coloured printer cartridge of computer, there are only three colours viz. Red, Green and Blue are called primary colours (basic colours).

But using these three colours in different proportions, we can derive any colour.

Similarly in physics, there are seven basic quantities (fundamental quantities), using which we can derive any physical quantity.



1.1.4.1.1 Fundamental Quantities and Fundamental Units

Fundamental Quantities (Definition) : The physical quantities which do not depend on any other physical quantities for their measurements are called fundamental quantities or base quantities.

Fundamental Units (Definition) : The units used to measure fundamental quantities are called fundamental units i.e. the unit of fundamental quantity is called fundamental unit. It does not depend on any other unit.

There are seven fundamental (basic) physical quantities : Length, mass, time, temperature, electric current, luminous intensity and amount of a substance and their units are fundamental units.

Following are the fundamental quantities with their units and symbol of units.

Table 1.1

Fundamental (basic) quantity	Fundamental unit (S.I.)	Symbol of unit
1. Length	metre	m
2. Mass	kilogram	Kg
3. Time	second	s
4. Temperature	kelvin	K
5. Electric current	ampere	A
6. Luminous intensity	candela	cd
7. Amount of a substance	mole	mol

There are two supplementary quantities (units) to fundamental quantities.

Supplementary quantity	Supplementary unit	Symbol of unit
1. Plane angle	radian	rad
2. Solid angle	steradian	sr

1.1.4.1.2 Derived Quantities and Derived Units

Derived Quantities (Definition) :

Physical quantities which depend on one or more fundamental quantities for their measurements are called derived quantities.

OR

The physical quantities which are derived using one or more fundamental quantities are called derived quantities.

Derived Units (Definition) :

The units used to measure derived quantities are called derived units.

OR

The units of derived quantities which depend on fundamental units for their measurement are called derived units.

Thus units of derived quantities are derived units.

As we have seen there are seven fundamental quantities. **The remaining all quantities are derived quantities.**

Examples :

(1) Density $= \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{l \times b \times h} = \frac{\text{Mass}}{L \times L \times L}$ …… kg/m^3

It is derived using two fundamental quantities i.e. mass and length.

(2) Force = mass × [acceleration] = mass × [(velocity)/time]

= mass × [(displacement/time)/time] …… $kg\ m/s^2$ …… N

It is derived using mass, length and time.

Following are some derived quantities with their units and symbol of unit.

Table 1.2

Sr. No.	**Derived physical quantity**	**Derived unit (S.I.)**	**Symbol of unit**
1.	Area	square metre	m^2
2.	Volume	cubic metre	m^3
3.	Velocity	metre/sec	m/s
4.	Acceleration	metre/sec^2	m/s^2
5.	Force	newton	N
6.	Pressure	newton/metre^2	N/m^2
7.	Density	kilogram/metre^3	kg/m^3
8.	Speed	metre/sec	m/s
9.	Work	joule	J

1.1.5 Systems of Unit

System of unit (Definition) : A systems of unit, is a class of units consisting components from a fundamental (base) set of units.

Units are broadly categorized into two groups i.e. fundamental units and derived units.

In the year 1832, the German mathematician Carl Friedrich Gauss advised to select three fundamental quantities. Considering **length**, **mass** and **time** as base **units**, available systems of units are C.G.S., M.K.S., F.P.S. and S.I. systems.

Many systems of base units are available in engineering. They are divided into different groups. The system of units which are in use are (i) C.G.S., (ii) M.K.S., (iii) F.P.S. and (iv) S.I.

Here **length**, **mass** and **time** are taken as basic physical quantities. Using this basic quantities, many physical quantities can be derived.

(i) C.G.S. system : In this system, the units of length, mass and time are **C**entimetre, **G**ram and **S**econd respectively.

(ii) M.K.S. system : In this system, the units of length, mass and time are **M**etre, **K**ilogram and **S**econd respectively.

(iii) F.P.S. system : In this system, the units of length, mass and time are **F**oot, **P**ound and **S**econd respectively.

(iv) S.I. system : S.I. is the abbreviation of the French name **System Internationale** (International system of units).

Different systems of units are very confusing for sharing scientific information between different regions of world. Hence common system of units called System International (SI system) or International system of units was accepted in International General Conference of weights and measures in Paris. There are seven base units in the system, from which other units are derived. This system was formerly called the MKS system.

Table 1.3

System	**Length**	**Mass**	**Time**
1. CGS system	centimetre	gram	second
2. MKS system	metre	kilogram	second
3. FPS system	foot	pound	second
4. S.I. system	means System International (International system of units)		

1.1.5.1 Advantages of S.I. System

S.I. system has some advantages, which are as under :

(1) It is internationally accepted system.

(2) S.I. system is rational system of units i.e. it uses only unit for given physical quantity e.g. joule (J) is used for all types of energies.

(3) S.I. system is coherent system of units : only by multiplying or dividing the fundamental units, derived units are obtained.

(4) **S.I. unit is metric system :** Multiples of S.I. units can be represented as a power of 10.

1.1.5.2 S.I. Prefixes – Metric Prefixes to Power of 10

The magnitudes of physical quantities changed over broad range.

As an engineer we should be able to convert the units of same quantity in different systems and forms.

The CGPM recommended standard prefixes of multiples and sub-multiples of units are as shown below.

The entire SI system consists of units which are multiples and submultiples of MKS units in steps of 10^3 or 10^{-3}.

Table 1.4 : Prefixes for SI units

Value indicated	Prefix	Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	**giga**	**G**
10^{6}	**mega**	**M**
10^{3}	**kilo**	**k**
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-2}	**centi**	**c**
10^{-3}	**milli**	**m**
10^{-6}	**micro**	**μ**
10^{-9}	**nano**	**n**
10^{-12}	**pico**	**p**
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

The bold type prefixes (giga, mega, kilo, centi, milli, micro, nano, pico) shown above are used more oftenly. For example, how to convert

$$1 \text{ kg} = 1 \times 10^3 \text{ gm}$$

$$1 \text{ MW} = 1 \times 10^6 \text{ W}$$

$$1 \text{ cm} = 1 \times 10^{-2} \text{ m}$$

$$1 \text{ μm} = 1 \times 10^{-6} \text{ m}$$

$$1 \text{ nF} = 1 \times 10^{-9} \text{ F}$$

Table 1.5

Some examples	CGS	MKS	FPS	SI
1. Speed	cm/s	m/s	ft/s	m/s
2. Area	cm^2	m^2	ft^2	m^2
3. Density	gm/cm^3	kg/m^3	lb/ft^3	kg/m^3
4. Force	dyne dyn ($gm.cm/s^2$)	newton N ($kg.m/s^2$)	pound force $l$bf	newton N kg m/s
5. Pressure	dyn/cm^2	N/m^2	lb/ft^2 (pound per square foot)	N/m^2

1.2 DIMENSIONS

1.2.1 Introduction

As we have seen that derived quantities are those quantities which are derived using one more fundamental quantities. In mechanics every derived quantity is derived using length, mass and time (basic quantities). Therefore, length, mass, time i.e. L, M, T are taken as fundamental quantities and every derived quantity can be expressed in terms of L, M and T.

1.2.2 Dimensions, Dimensional Formula

Dimension (Definition) : The dimensions of physical quantity are the powers to which fundamental (base) units must be raised to obtain the unit of a given physical quantity.

All the physical quantities can be derived from the fundamental (base) quantities. When a physical quantity is represented in terms of base quantities, it can be represented as a product of different powers of base quantities.

Dimension : The **exponent** of a base quantity which enters into the expression, is called **dimension** of the quantity in that base.

To decide the dimensions of a physical quantity, the units of fundamental quantities are expressed by the following :

length can be expressed by 'L'
mass by 'M'
time by 'T'
temperature by 'K'
current by 'I'
luminous intensity by 'C'
and amount of substance by 'mol'

1.2.3 Dimensional Formula (Equation)

Dimensional formula (equation) (Definition) : An equation, which gives the relation between fundamental units and derived units in terms of dimensions is called dimensional formula (equation). In mechanics the length, mass and time are taken as three base dimensions and are represented by letters L, M, T respectively.

The derived unit of all physical quantities can be represented in terms of the base (fundamental) unit of length, mass and time raised to some power (exponent).

Examples :

(i) Dimensional formula (equation) for area :

We have, Area = length × breadth = length × length = $[L] \times [L] = [L^2]$

$\therefore$ Dimensional formula (equation) for area $[A] = [L^2 M^0 T^0]$

Thus, $[L^2 M^0 T^0] \rightarrow$ is called dimensional formula (equation)

$[^2, ^0, ^0] \rightarrow$ are called dimensions.

Thus, dimensions of area are

2 in length
0 in mass
and 0 in time

(ii) Dimensional formula (equation) for speed :

We have, $$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{length}}{\text{time}} = \frac{[L^1 M^0 T^0]}{[L^0 M^0 T^1]} = [L^1 M^0 T^{-1}]$$

Thus, $[L^1 M^0 T^{-1}]$ is the dimensional formula (equation) for speed and $[^1, ^0, ^{-1}]$ are dimensions of speed.

i.e. dimensions of speed are

1 in length

0 in mass

and –1 in time

(iii) Dimensional formula (equation) for density :

We have, Density $= \dfrac{\text{mass}}{\text{volume}} = \dfrac{\text{mass}}{\text{length} \times \text{breadth} \times \text{height}} = \dfrac{\text{mass}}{\text{length} \times \text{length} \times \text{length}} = \dfrac{[L^0\ M^1\ T^0]}{[L^3\ M^0\ T^0]}$

$= [L^{-3}\ M^1\ T^0]$

Thus, $[L^{-3}\ M^1\ T^0]$ is the dimensional formula (equation) for density and $[^{-3},\ ^{1},\ ^{0}]$ are dimensions of density.

i.e. Dimensions of density are

–3 in length

1 in mass

and 0 in time

Physical quantities with formula, dimensional formula and SI unit symbols :

Table 1.6

Sr. No.	Physical quantity	Formula or Relation	Dimensional formula (equation)	S.I. unit symbol
1.	Length	length	$[L^1\ M^0\ T^0]$	m
2.	Mass	mass	$[L^0\ M^1\ T^0]$	kg
3.	Time	time	$[L^0\ M^0\ T^1]$	s
4.	Area	length × breadth = length × length	$[L^2\ M^0\ T^0]$	m^2
5.	Volume	length × breadth × height = length × length × length	$[L^3\ M^0\ T^0]$	m^3
6.	Density	$\dfrac{\text{mass}}{\text{volume}} = \dfrac{\text{mass}}{\text{length} \times \text{breadth} \times \text{height}} = \dfrac{\text{mass}}{\text{length} \times \text{length} \times \text{length}}$	$[L^{-3}\ M^1\ T^0]$	kg/m^3
7.	Speed (velocity)	$\dfrac{\text{distance}}{\text{time}} = \dfrac{\text{length}}{\text{time}}$	$[L^1\ M^0\ T^{-1}]$	m/s
8.	Acceleration	$\dfrac{\text{velocity}}{\text{time}} = \dfrac{\frac{\text{distance}}{\text{time}}}{\text{time}}$	$[L^1\ M^0\ T^{-2}]$	m/s^2
9.	Force	$\text{mass} \times \text{acceleration} = \text{mass} \times \dfrac{\text{velocity}}{\text{time}} = \text{mass} \times \left(\dfrac{\frac{\text{distance}}{\text{time}}}{\text{time}}\right)$	$[L^1\ M^1\ T^{-2}]$	N ($kg.m/s^2$)
10.	Pressure	$\dfrac{\text{force}}{\text{area}} = \dfrac{\text{mass} \times \text{acceleration}}{\text{area}} = \dfrac{\text{mass} \times \text{velocity}}{\text{area} \times \text{time}} = \dfrac{\text{mass} \times \frac{\text{distance}}{\text{time}}}{\text{length} \times \text{breadth} \times \text{time}} = \dfrac{\text{mass} \times \text{length/time}}{\text{length} \times \text{length} \times \text{time}}$	$[L^{-1}\ M^1\ T^{-2}]$	N/m^2 ($kg/m.s^2$)
11.	Impulse	$\text{force} \times \text{time} = \text{mass} \times \text{acceleration} \times \text{time} = \text{mass} \times \dfrac{\text{velocity}}{\text{time}} \times \text{time} = \text{mass} \times \text{velocity} = \text{mass} \times \dfrac{\text{distance}}{\text{time}}$	$[L^1\ M^1\ T^{-1}]$	Ns (kg.m/s)

… Contd.

12.	Work	(force) × displacement = (mass × acceleration) × distance $= (\text{mass} \times \frac{\text{velocity}}{\text{time}}) \times \text{distance}$ $= \left(\text{mass} \times \frac{\frac{\text{distance}}{\text{time}}}{\text{time}}\right) \times \text{distance}$	$[L^2\ M^1\ T^{-2}]$	J $(kg.m^2/s^2)$
13.	K.E.	$\frac{1}{2}mv^2 = \frac{1}{2} \times \text{mass} \times \left(\frac{\text{distance}}{\text{time}}\right)^2$	$[L^2\ M^1\ T^{-2}]$	J $(kg.m^2/s^2)$
14.	P.E	$mgh = \text{mass} \times \frac{\text{velocity}}{\text{time}} \times \text{length}$ $= \left(\text{mass} \times \frac{\frac{\text{distance}}{\text{time}}}{\text{time}} \times \text{length}\right) = \left(\text{mass} \times \frac{\frac{\text{length}}{\text{time}}}{\text{time}} \times \text{length}\right)$	$[L^2\ M^1\ T^{-2}]$	J $(kg.m^2/s^2)$
15.	Power	$\frac{\text{work}}{\text{time}}$ $= \frac{(\text{force}) \times \text{displacement}}{\text{time}}$ $= \frac{(\text{mass} \times \text{acceleration}) \times \text{distance}}{\text{time}}$ $= \frac{\left(\text{mass} \times \frac{\text{velocity}}{\text{time}}\right) \times \text{distance}}{\text{time}}$ $= \frac{\text{mass} \times \frac{\text{distance}}{\text{time}^2} \times \text{distance}}{\text{time}}$	$[L^2\ M^1\ T^{-3}]$	W or J/s
16.	Momentum	$\text{mass} \times (\text{velocity}) = \text{mass} \times \frac{\text{distance}}{\text{time}}$	$[L^1\ M^1\ T^{-1}]$	kg.m/s
17.	Stress (Note : dimensions of stress and pressure are same $\because \frac{\text{Force}}{\text{Area}}$)	$\frac{\text{Force}}{\text{Area}} = \frac{(\text{mass} \times \text{acceleration})}{(\text{length} \times \text{breadth})}$ $= \frac{\left(\text{mass} \times \frac{\text{velocity}}{\text{time}}\right)}{(\text{length} \times \text{length})} = \frac{\left(\text{mass} \times \frac{\text{distance}}{\text{time}^2}\right)}{(\text{length} \times \text{length})}$	$[L^{-1}\ M^1\ T^{-2}]$	N/m^2
18.	Strain	$\frac{\text{change in dimensions}}{\text{original dimension}}$	$[L^0\ M^0\ T^0]$	No unit
19.	Frequency	$\frac{1}{\text{time period}}$	$[L^0\ M^0\ T^{-1}]$	Hz or /s
20.	Wavelength	length of one wave	$[L^1\ M^0\ T^0]$	m

1.2.4 Principle of Homogeneity of Dimensions

It states that the dimensions on both the sides of physical equation must be same i.e. dimensions of terms involved on left side and dimensions of terms involved on right side of equation must be same. Two or more physical quantities can be added or subtracted if and only if their dimensions are same. Example,

(1) Length + Length → [L] + [L] = [L] and not 2 [L].

(2) Length – Length → [L] – [L] = [L] and not zero.

(3) Length + Mass → cannot be added.

(4) Consider the equation of motion

$$v = u + at$$

where u = initial velocity
v = final velocity
a = uniform acceleration
t = time

$$[L^1M^0T^{-1}] = [L^1M^0T^{-1}] + [L^1M^0T^{-2}] [L^0M^0T^1]$$

i.e. $$[L^1M^0T^{-1}] = [L^1M^0T^{-1}] + [L^1M^0T^{-1}]$$

i.e. balanced as per principle of homogeneity.

(5) Consider equation of motion as

$$v^2 = u^2 + 2as$$

where u = initial velocity
v = final velocity
a = uniform acceleration
s = displacement

$$[L^1M^0T^{-1}] [L^1M^0T^{-1}] = [L^1M^0T^{-1}] [L^1M^0T^{-1}] + [L^1M^0T^{-2}] [L^1M^0T^0]$$

i.e. $$[L^2M^0T^{-2}] = [L^2M^0T^{-2}] + [L^2M^0T^{-2}]$$

i.e. balanced as per principle of homogeneity.

1.2.5 Applications of Dimensional Analysis

1. To find the correctness of physical equation.
2. To convert physical quantity from one system of units to other.
3. To establish relationship between related physical quantities.

1.2.6 Limitations of Dimensions

1. It cannot find relation or formula if a physical quantity depends on more than three factors having dimensions. i.e. apart from L, M and T.
2. It cannot derive a formula containing trigonometric function, logarithmic function and exponential function.
3. It gives no information about pure numbers and non-dimensional constants.

1.3 ACCURACY AND ERRORS IN MEASUREMENT AND SIGNIFICANT FIGURES

1.3.1 Introduction

We have seen, measurement consists of comparison of unknown quantity with a known fixed (standard) quantity. Development of physics is mostly due to accurate and precise instruments which can measure size of physical quantity to high degree of accuracy. Along with high degree of accuracy, high precision of instrument is also required. Even we take utmost care, some fault in the measurement occurs.

Defects in the measurement of physical quantity lead to errors and mistakes. Mistakes can be avoided by observer by taking utmost care. But errors cannot be avoided or eliminated completely but can be minimized.

Accuracy of measurement depends on : (1) Method of measurement, (2) Measuring instrument, (3) Systematicness while taking readings.

1.3.2 Accuracy and Precision of Instruments

Accuracy : In any measurement, the possibility of error is bound to arise. No measurement is exact. Hence, accuracy of measurement is most important aspect.

Accuracy is the agreement of the result of a measurement with the true value of the measured quantity.

The accuracy of the instrument is its ability to give correct results.

The accuracy of measurement depends upon quality of instrument and selection of proper instrument e.g. diameter of ball bearing can be measured more accurately by micrometer than that of vernier.

Accuracy of measurement depends on human limitations i.e. sense of hearing, sense of touch, sense of sight and systematicness.

Accuracy also depends on conditions of surroundings like temperature, pressure, humidity etc.

Precision : Precision is defined as the **repeatability** of a measuring process.

In physics, for any measurement, we take number of readings and find out the average (mean) i.e. set of measurements. In any set of measurements, the individual measurements are scattered about the mean and precision tells us about the performance of measurement. Less repeatability is a sign of less **precision** and more repeatability is a sign of more **precision**. If number of readings are concentrated near average reading then precision is said to be more and if readings are scattered about mean reading then precision is said to be less.

Following set of readings illustrate distinction between accuracy and precision.

e.g. Exact diameter of ball bearing is **2.5 mm (true value)**.

Table 1.7

Set 1	Set 2	Set 3
2.4 mm 2.4 mm 2.45 mm 2.45 mm 2.5 mm	2.3 mm 2.4 mm 2.48 mm 2.56 mm 2.66 mm	2.49 mm 2.49 mm 2.5 mm 2.51 mm 2.51 mm
Mean = 2.44 mm	Mean = 2.48 mm	Mean = 2.5 mm
It is more precise but not accurate.	It is not precise (since scattered) but accurate.	It is more precise and also accurate (since matching with true value).

1.3.3 Errors

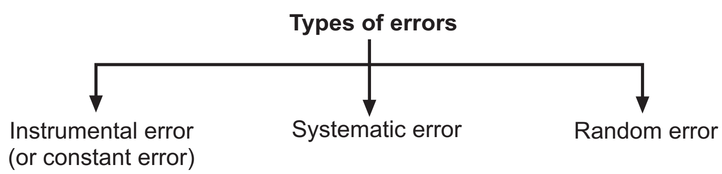
1.3.3.1 Errors, Types of Errors

Error (Definition) : An error is a fault, which may occur even in the most careful observation.

Or Error is a deviation of measurement from standard value.

Or Error is an uncertainty in a given measurement.

Error arises due to human limitations and instrumental limitations. Errors cannot be completely eliminated but can be reduced.



(1) Instrumental error or constant error :

Instrumental error (Definition) : An error caused due to faulty instrument is called instrumental error.

Fault in the instrument may be because of faulty construction or faulty calibration.

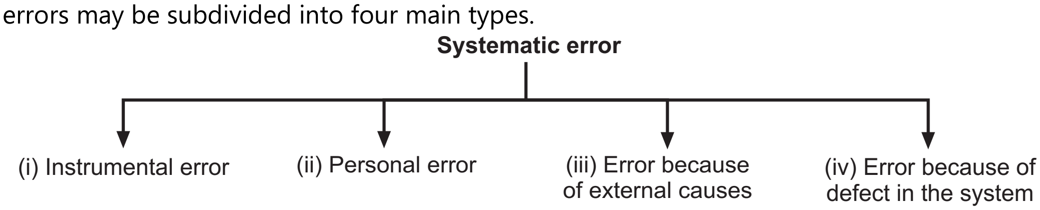
Constant error (Definition) : When number of observations taken by instrument possess **same amount of error** then the error is said to be a constant error. e.g. Local made measuring scale having faulty calibration on it, say 1 cm on scale actually corresponds to 1.1 cm. The length measured by such scale will differ from its value by a constant amount. In this case, measured length will always be smaller than true value by a constant amount. Hence such error is called constant error or instrumental error.

(2) Systematic error or Persistent error (Definition) :

Error caused due to defective setting of instrument or defective adjustment or unsystematicness of the experimentor is called systematic error.

This error exists due to known causes. Such errors can be reduced or eliminated by identifying the sources of error and applying the rules governing the error.

Systematic errors may be subdivided into four main types.



(i) Instrumental error (Definition) : The error because of wrong adjustment or defective setting of the instrument is called as instrumental error.

e.g. (1) Zero error in micrometer screw gauge.

(2) Pointer of the voltmeter if not pivoted at exact zero of the calibrated scale.

These types of errors can be minimized by identifying its causes.

(ii) Personal error : As the name suggests this error is due to carelessness or fault of an observer while taking readings.

For example, while taking readings of ammeter, the eye sight of an observer should be exactly perpendicular to the display. For this, removal of parallax is necessary i.e. the pointer and its image in mirror should exactly match with each other. Thus the error because of non-removal of parallax between pointer and its image in case of ammeter is the personal error.

(iii) Error because of external causes : Such errors arise due to changes in environmental conditions i.e. change in temperature, humidity, pressure etc. For example if metal meter scale is used it shows different readings in summer and winter, due to change in length of scale itself due to change in temperature.

(iv) Error due to defect in the system : This error is because of defective (imperfect) experimental setup.

For example, while performing experiment to determine coefficient of linear expansion by Pullinger's apparatus, there may not be proper thermal contact between thermometer and the metal rod.

(3) Random errors (Definition) : The error caused due to experimental conditions and human limitations is called random error.

For example

(a) While performing electrical experiments if there is voltage fluctuation then some error may be included.

(b) While performing Stoke's experiment to determine coefficient of viscosity of liquid : when metal sphere crosses marking 'A', stop watch should be started and as soon as it crosses marking 'B', stop watch should be stopped. Same person may get different readings because of human limitations.

Errors can be minimized by :

(i) Taking large magnitude of the physical quantity which is to be measured.

(ii) Taking large number of readings.

(iii) Using smaller least count instrument.

1.3.4 Estimation of Errors

As we know, errors cannot be completely eliminated but can be reduced or minimized.

Estimation of errors helps to minimize the error, and also to understand accuracy and precision of the measurement process. It also helps to calibrate the instrument. In quality control department, it helps to take decision whether the measured job is to be selected or rejected with known tolerance.

Steps for estimation of errors :

(1) Calculate corrected readings (if possible)

(2) Calculate mean (average) value of readings.

(3) Calculate absolute error.

(4) Calculate mean (average) absolute error.

(5) Calculate relative error.

(6) Calculate percentage error.

To understand the process of estimation of errors and different steps involved in it, we will go through one example.

Example : *The diameter of a rod is measured with micrometer screw gauge for three times as* x_1 *= 1.223 cm,* x_2 *= 1.224 cm and* x_3 *= 1.224 cm. Calculate percentage error in measuring diameter of rod if zero error of micrometer is +0.002 cm.*

(**Note :** Actually, sufficient large number of readings should be taken, but here for simplicity only three readings are taken.)

Solution : (1) Corrected reading :

$$\text{Corrected reading} = \text{reading} \pm \text{constant error}$$
$$= \text{reading} \pm \text{zero error}$$

If zero error is +ve, correction is –ve and if zero error is –ve, correction is +ve.

In the given example, zero error is +ve therefore correction is –ve (z = +0.002 cm)

∴ Corrected reading

$$x_{c1} = x_1 - z = 1.223 - 0.002 = 1.221 \text{ cm}$$
$$x_{c2} = x_2 - z = 1.224 - 0.002 = 1.222 \text{ cm}$$
$$x_{c3} = x_3 - z = 1.224 - 0.002 = 1.222 \text{ cm}$$

(2) Mean (average) reading : To get mean reading add all corrected readings and divide it by total number of readings. In many cases, mean reading is also called as most probable value.

Mean (average) reading, $x_m = \dfrac{x_{c1} + x_{c2} + x_{c3}}{3}$

$$x_m = \frac{(1.221 + 1.222 + 1.222)}{3}$$

$$\boxed{x_m = 1.2217 \text{ cm}}$$

(3) Absolute error (Definition) : The difference between reading or (corrected reading) and the mean (average) reading is called absolute error.

Absolute error,

$$\delta x_1 = |x_{c1} - x_m| = |1.221 - 1.2217| = 0.0007 \text{ cm}$$
$$\delta x_2 = |x_{c2} - x_m| = |1.222 - 1.2217| = 0.0003 \text{ cm}$$
$$\delta x_3 = |x_{c3} - x_m| = |1.222 - 1.2217| = 0.0003 \text{ cm}$$

(4) Average (mean) absolute error (Definition) :

The average of absolute error is called average absolute error.

∴ Average absolute error,

$$\delta x_{avg} = \frac{(\delta x_1 + \delta x_2 + \delta x_3)}{3} = \frac{(0.0007 + 0.0003 + 0.0003)}{3}$$

$$\boxed{\delta x_{avg} = 0.00043 \text{ cm}}$$

(5) Relative error (Definition) : It is the ratio of average absolute error to mean (average) reading.

$$\text{Relative error} = \frac{\delta x_{avg}}{x_m} = \frac{0.00043}{1.2217} = 0.000352$$

(6) Percentage error : It is the product of relative error and 100.

Definition : Percentage error is defined as a way of expressing a relative error as a fraction of 100.

$$\text{Percentage error} = \text{Relative error} \times 100$$
$$= \frac{\delta x_{avg}}{x_m} \times 100 = 0.000352 \times 100 = 0.0352\%$$

1.3.5 Significant Figures, Rules and Identification of Significant Figures

Using calculators, we compute multiplication, divisions and we get answers in many digits after decimal point. i.e. too long answers (e.g. 0.2430687692). But we have to think that how many digits are really meaningful and where to restrict this figure.

Also if answer is too long, then there are more chances of misentering these digits.

Hence, the final result must always be restricted and rounded to a number of digits consistent with accuracy. In simple language, consider only those digits which are meaningful i.e. significant figures.

A significant figure is defined as a figure in any place (in number) which is reasonably trustworthy (meaningful).

Significant digits : Digits that are meaningful in assigning a true value or realistic value to a result.

Rules to determine significant figures :

(1) The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are significant figures. OR all non-zero digits are significant.

(2) **Zero is significant** if it comes in between two non-zero numbers. e.g. 106 km

(3) All zeros to the right side of decimal point and left side of non-zero digits are **not significant** e.g. 0.0123 → it has only three significant figures. i.e. zero is not significant if it is used to fix the decimal point.

(4) **Zero is significant** if it is to the right side of non-zero digit after decimal point. e.g. 0.2300 has four significant figures, 10.00 has four significant figures.

(5) If zero is on right side of non zero numbers then it may be significant or may not.

Refer following examples to clear this.

Some examples,

1. **0.00476 :** Here significant figures are 4, 7 and 6. Zero is not significant (meaningful) since it is used to fix decimal point.
2. **1024 :** Here, all digits including zero i.e. 1, 0, 2, 4 are significant (meaningful).
3. **Bangalore is at a distance of 1200 km from Mumbai.**

In this case 1 and 2 are significant, zero may or may not be significant since distance may or may not be exact 1200 km.

Case 1 : If actual distance is exact 1200 km (or 1199.5 km) then all the **four** digits are significant (meaningful), hence significant figure is **4**.

To convey this properly, we can write this as **1.200×10^3 km**.

Case 2 : If exact distance is 1196 km and if it is rounded to 1200 km, then 1, 2 and left zero is significant. Hence, significant figure is **3**.

To convey this properly, we can write this as **1.20×10^3 km.**

Case 3 : If exact distance is 1173 km and if it is rounded to 1200 km, then both the zeros are not significant (meaningful). Only 1, 2 are significant.

Hence, significant figure is **2**. To convey this we can write **1.2×10^3 km**.

If we express speed of sound as 320 m/s (at 22°C), it means it is closer to 320 than to 319 or 321. And if we express speed of sound as 320.0 m/s it means that it is closer to 320.0 than to 319.9 or 320.1. **Thus, 320.0 is 10 times precise than 320.**

Some examples :

Value	Number of significant figures
1. 1.080	4
2. 0.0018	2
3. 0.00180	3
4. 0.001800	4
5. 4.38×10^{11}	3

Rounding of approximate numbers :

It is the discarding of insignificant digits in a number, discarding of insignificant digits on the right of the decimal point.

To round a number to 'n' significant figures, discard all digits to the right of n^{th} place.

e.g. $$\pi = 3.141592654 \xrightarrow[\text{3 significant figures}]{\text{to round it to}} 3.14.$$

Rules or conventions for rounding of approximate numbers :

1. If the first digit of discarded number is less than 5, leave the n^{th} digit unchanged.

e.g. $$1.12345678 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 1.123$$

($\because$ first discarded digit is 4 which is less than 5 keep (n^{th}) i.e. 4^{th} digit (here 3) unchanged.)

2. If the first digit of discarded number is greater than 5 then increase the n^{th} digit by 1.

   e.g. $6.124756 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 6.125$

   (∵ first discarded digit is 7 which is greater than 5, hence increase (n^{th}) 4^{th} digit by 1 i.e. 4 + 1 = 5.)

3. If the first digit of discarded number is exact 5, then leave n^{th} digit unchanged if it is an even and add 1 to it if it is odd.

   e.g. $3.142519 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 3.142$ (Here n^{th} digit is 2 i.e. even ∴ keep unchanged)

   e.g. $6.4475 \xrightarrow[\text{4 significant figures}]{\text{to round it to}} 6.448$ (Here n^{th} digit is 7 i.e. odd, hence add 1)

Comparison between fundamental and derived quantities

Fundamental (base) quantities	Derived quantities
(1) The physical quantities which **do not depend** on any other physical quantities for their measurement are called fundamental or base quantities.	(1) The physical quantities which **depend** on one or more fundamental quantities for their measurement are called derived quantities.
(2) According to SI system, **there are seven** fundamental quantities.	(2) Excluding seven fundamental quantities, **remaining all** physical quantities are derived quantities.
(3) Fundamental quantities are used to derive derived quantities.	(3) Derived quantities are derived using one or more fundamental quantities.
(4) e.g. Length, mass, time, temperature, electric current, luminous intensity and amount of a substance.	(4) e.g. Area, volume, velocity, force, energy, work, power .....etc.

Comparison between fundamental and derived units

Fundamental (base) units	Derived units
(1) Units (standard) used to measure fundamental quantities are called fundamental units.	(1) Units (standard) used to measure derived quantities are called derived units.
(2) It does not depend on any other unit.	(2) It depends on one or more fundamental units.
(3) Fundamental units are used for deriving derived units.	(3) Derived units are derived using fundamental units.
(4) e.g. length – m mass – kg time – s temperature – K current – A	(4) e.g. area – m^2 volume – m^3 density – kg/m^3 velocity – m/s force – N work – J

Comparison between accuracy and precision

Accuracy	Precision
1. Accuracy is the agreement of the result of measurement with the true value of the measured quantity.	1. Precision is the repeatability of the measuring process.
2. Accuracy depends on least count, range of instrument, human limitations and conditions of surroundings.	2. Precision depends on quality of instrument, human limitations and conditions of surroundings.
3. Accurate measurement may not be precise.	3. Precise measurement may not be accurate.
4. Less is the percentage error, more is the accuracy of the instrument.	4. Less is the percentage error then more is the precision in measurement.
5. It can be determined by calibration against standard.	5. It can be determined by comparative test measurement.

Important Points

- The standard used for measurement of a physical quantity is called unit of that quantity.
- Requirements of standard unit are that it should be universally accepted, definite, well defined, easily reproducible, invariable, readily available.
- Seven fundamental quantities are length, mass, time, temperature, current, luminous intensity and amount of substance.
- Plane angle and solid angle are the two supplementary quantities.
- Remaining all are derived quantities e.g. volume, density, pressure.
- System of units are C.G.S., M.K.S., F.P.S., and S.I. Length, mass and time are the basic quantities preferred in system of units.
- Dimensions of the physical quantity are the powers to which fundamental (base) units must be raised to obtain the unit of a given physical quantity.
- $[L^a M^b T^c] \rightarrow$ is called dimensional formula (equation). $[^a, {}^b, {}^c] \rightarrow$ are called dimensions.
- Error is a fault, which may occur even in the most careful observations, which arises due to human limitations and instrumental limitations.
- Errors cannot be completely eliminated but can be reduced.
- Three types of errors are : (1) instrumental (constant), (2) systematic and (3) random errors.
- A significant figure is defined as a figure in any place (in number) which is reasonably trustworthy (meaningful).
- Significant digits : Digits that are meaningful in assigning a true value or realistic value to a result.

Formulae

(1) Corrected reading = ± constant error

i.e. $x_c = x \pm z$

(2) Mean (average) reading : $x_m = \dfrac{(x_{c1} + x_{c2} + \ldots\ldots + x_{cn})}{n}$

(3) Absolute error : $\delta x = | x_{c1} - x_m|$

(4) Average (mean) absolute error :

$$\delta x_{avg} = \frac{(\delta x_1 + \delta x_2 + \ldots\ldots + \delta x_n)}{n}$$

(5) Relative error : $R = \dfrac{\delta x_{avg}}{x_m}$

(6) Percentage error $= R \times 100 = \dfrac{\delta x_{avg}}{x_m} \times 100$

(7) If $x = a + b$ OR $x = a - b$ OR $x = a.b$ OR $x = \dfrac{a}{b}$

Then in all these cases, $\delta x = \delta a + \delta b$

$$\% \text{ error in } x = \frac{\delta x}{x} \times 100 = \frac{\delta a}{a} \times 100 + \frac{\delta b}{b} \times 100$$

(8) If $x = a^n$ then relative error $= \dfrac{\delta x}{x} = n\dfrac{\delta a}{a}$

$$\% \text{ error in } x = \frac{\delta x}{x} \times 100 = \frac{n\delta a}{a} \times 100$$

SOLVED EXAMPLES

EXAMPLES ON ESTIMATION OF ERRORS OR ERRORS

Example 1 : *Thickness of metal sheet is measured with micrometer screen gauge for five times as 0.623 cm, 0.624 cm, 0.622 cm, 0.624 cm, 0.623 cm. Calculate (i) average corrected reading, (ii) average absolute error and (iii) percentage error in measuring the thickness of sheet if zero error of micrometer is –0.003 cm*

Solution : As given zero error is –ve, correction is +ve i.e. to obtain corrected reading, add 0.003 cm.

$z = -0.003$ cm $\therefore x_c = (x + 0.003)$ cm

Sr. No	Given reading x_{cm}	Corrected reading $x_c = x + z$ $= (x + 0.003)$	Corrected average reading x_m	Absolute error δx $= \lvert x_c - x_m \rvert$ $= \lvert x_c - 0.6262 \rvert$	Average absolute error δx_{avg}	Relative error $R = \frac{\delta x_{avg}}{x_m}$	Percent error = $R \times 100$
1.	0.623 cm	= 0.626 cm		0.0002 cm			
2.	0.624 cm	= 0.627 cm	$= \left(\frac{3.131}{5}\right)$	0.0008	0.00064	0.001022	0.1022
3.	0.622 cm	= 0.625 cm		0.0012			
4.	0.624 cm	= 0.627 cm	= 0.6262 cm	0.0008			
5.	0.623 cm	x_{c_5} = 0.626 cm		0.0002			
		Total → 3.131					

Example 2 : *A student measures the diameter of a bob three times using vernier caliper. The measurements are 2.35 cm, 2.36 cm and 2.31 cm. Estimate error in the measurement (Calculate absolute error and percentage error.)*

Solution :

Sr. No.	Given reading x	Average reading x_m	Absolute error $\delta x = \lvert x - x_m \rvert$	Average absolute error δx_{avg}	Relative error $R = \frac{\delta x_{avg}}{x_m}$	% error $= R \times 100$
1.	2.35	$\frac{7.02}{3}$	0.01	$\frac{0.06}{3}$		
2.	2.36		0.02		$\frac{0.02}{2.34}$	
3.	2.31	= 2.34	0.03	= 0.02		0.855
	Total = 7.02		Total = 0.06		= 0.00855	

Example 3 : *The diameter of a rod is measured with micrometer screw gauge for three times as 1.223 cm, 1.224 cm and 1.224 cm. Calculate percentage error in measuring the diameter of rod if zero error of micrometer is + 0.002 cm.*

Solution : As given zero error is positive, correction is negative i.e. to obtain corrected reading subtract 0.002 cm from given readings. $z = +0.002$ cm $\therefore x_c = x - z$

Sr. No.	Given reading x	Corrected reading cm $x_c = x - z$	Corrected average reading x_m	Absolute error $\delta x = \lvert x_c - x_m \rvert$	Average absolute error δx_{avg}	Relative error 'R' $= \frac{\delta x_{avg}}{x_m}$	% error $= R \times 100$ $= \frac{\delta x_{avg}}{x_m} \times 100$
1.	1.223	1.221		0.0007			
2.	1.224	1.222	1.2217	0.0003	0.00043	0.00035	0.035
3.	1.224	1.222		0.0003			

Example 4 : *The mass of object is 37.6 ± 0.02 gm. Estimate the percentage error in this measurement.*

Solution : $$\text{Percentage error} = \frac{\text{Average absolute error}}{\text{Average reading}} \times 100 = \frac{0.02}{37.6} \times 100$$

$\therefore$ $\boxed{\%\ \text{error} = 0.053\ \%}$

Example 5 : *Which of the following is more accurate reading and why ?*
(a) 2.33 ± 0.002 mm (b) 2.55 ± 0.0015 mm.

Solution : We have,

$$\text{Percentage (\%) error} = \frac{\text{Average absolute error}}{\text{Average reading}} \times 100$$

(a) $$\% \text{ error} = \frac{0.002}{2.33} \times 100 = \mathbf{0.0858\%}$$

(b) $$\% \text{ error} = \frac{0.0015}{2.55} \times 100 = \mathbf{0.0588\ \%}$$

Percentage error in the second case is less, hence accuracy is more. Thus 2.55 ± 0.0015 mm is more accurate.

Example 6 : *The mass of the final job is 179.5 ± 0.03 gm. Estimate the percentage error in this measurement.*

Solution : $$\text{Percentage error} = \frac{\text{Average absolute error}}{\text{Average reading}} \times 100 = \frac{\text{tolerance}}{\text{reading}} \times 100 = \frac{0.03}{179.5} \times 100$$

$$\boxed{\% \text{ error} = 0.0167\ \%}$$

Example 7 : *Diameter of the wire measured by micrometer screw gauge is 0.345 cm. If the least count (L.C.) of micrometer screw gauge is 0.001 cm, calculate percentage error.*

Solution : $$\text{Percentage error} = \frac{\text{Average absolute error}}{\text{Average reading}} \times 100 = \frac{\text{Least count}}{\text{reading}} \times 100 = \frac{0.001}{0.345} \times 100$$

$$\boxed{\text{Percentage error} = 0.289\ \%}$$

Example 8 : *The length of an object measured by vernier caliper is 4.78 cm. If the L.C. of vernier caliper is 0.01 cm, calculate the percentage error.*

Solution : Given : L = 4.78 cm, L.C. = 0.01 cm

$$\text{Percentage error} = \frac{0.01}{4.78} \times 100$$

$$\boxed{\text{Percentage error} = 0.209\ \%}$$

Example 9 : *Calculate percentage error in kinetic energy of a body of mass 23 ± 0.1 g and moving with velocity of 46 ± 0.2 cm/s.*

Solution : Given : m = 23 ± 0.1 g, i.e. m = 23 g, δm = 0.1 g, v = 46 ± 0.2 cm/s, i.e. v = 46 cm/s, δv = 0.2 cm/s

We have kinetic energy $$\text{K.E.} = \frac{1}{2}mv^2$$

$$\therefore \quad \text{Percentage error in K.E.} = \left(\frac{\delta m}{m} \times 100\right) + \left(\frac{2\delta v}{v} \times 100\right) = \left(\frac{0.1}{23} \times 100\right) + \left(\frac{2 \times 0.2}{46} \times 100\right) = 0.435 + 0.8696$$

$$\boxed{\text{Percentage error in K.E.} = 1.304\ \%}$$

Example 10 : *Calculate percentage error in measurement of area of land. The length and breadth are given by l = 27.37 ± 0.01 km, b = 18.45 ± 0.01 km.*

Solution : We have, Area = $A = l \times b$

$$\% \text{ error in area} = \frac{\delta A}{A} \times 100 = \left(\frac{\delta l}{l} \times 100\right) \times \left(\frac{\delta b}{b} \times 100\right) = \left(\frac{0.01}{27.37} \times 100\right) + \left(\frac{0.01}{18.45} \times 100\right)$$

$$= 0.0365 + 0.0542$$

$$\boxed{\text{Percentage error in area} = 0.0907\ \%}$$

Example 11 : *Calculate percentage error in the measurement of density of cube, if mass of the cube has 2% error and length has 3% error.*

Solution : We have, $$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{(\text{length})^3}$$

$$\therefore \quad \text{Percentage error in density} = \left(\frac{\delta d}{d} \times 100\right) = \left(\frac{\boldsymbol{\delta m}}{\mathbf{m}} \times \mathbf{100}\right) + \left(\frac{\mathbf{3}\boldsymbol{\delta l}}{\boldsymbol{l}} \times \mathbf{100}\right)$$

$$= (\% \text{ error in mass (given)}) + (3 \times \% \text{ error in length (given)}) = 2\% + (3 \times 3\%)$$

$$\boxed{\text{Percentage error in density} = 11\%}$$

Example 12 : *If the distance covered by a vehicle is measured as 7230 ± 50 m and time required is measured as 720 ± 10 sec, calculate percentage error in measurement of speed.*

Solution : Given : distance (s) **=** 7230 ± 50 m, time (t) = 720 ± 10 sec

We have, $\text{Speed} = \dfrac{\text{distance}}{\text{time}}$

$$v = \frac{s}{t}$$

$$\text{Percentage error in speed} = \left(\frac{\delta v}{v} \times 100\right) = \left(\frac{\delta s}{s} \times 100\right) + \left(\frac{\delta t}{t} \times 100\right) = \left(\frac{50}{7230} \times 100\right) + \left(\frac{10}{720} \times 100\right)$$

$$= (0.692) + (1.388)$$

$$\boxed{\text{Percentage error in speed} = 2.08\ \%}$$

EXAMPLES ON SIGNIFICANT FIGURES

Example 13 : *State the number of significant figures in the following measurements :*

(i) 0.0025, (ii) 0.0250, (iii) 0.002500, (iv) 5.98×10^{24}.

Solution :

	Number of significant figures
(i) 0.0025	2
(ii) 0.0250	3
(iii) 0.002500	4
(iv) 5.98×10^{24}	3

Note : To understand why it is so, please refer to section significant figures in the topic.

Examples 14 : *State the number of significant figures in the following (i)* 3.730×10^5, *(ii)* 1.23×10^4, *(iii) 1234, (iv) 3200, (v) 3.0045, (vi) 0.00345, (vii) 200.034, (viii) 244.000, (ix)* 2.34×10^{-3}, *(x) 0.020.*

Solution :

Given	**Number of significant figures**
(i) 3.730×10^5	4
(ii) 1.23×10^4	3
(iii) 1234	4
(iv) 3200	2
(v) 3.0045	5
(vi) 0.00345	3
(vii) 200.034	6
(viii) 244.000	6
(ix) 2.34×10^{-3}	3
(x) 0.020	2

Example 15 : *Find in significant figures, the surface area and volume of a sphere whose diameter is measured by micrometer as 3.456 cm.*

Solution : Given : diameter = 3.456 cm $\therefore$ r = 1.728 cm

We have, $\text{Area of sphere} = 4\pi r^2 = 4 \times 3.124 \times (1.728)^2 = 37.5278469$

(We have to consider only 3 digits after decimal point)

$$\boxed{\therefore \text{Area} = 37.528\ \text{cm}^2}$$

We have, $\text{Volume of sphere} = \dfrac{4}{3} \times \pi r^3 = \dfrac{4}{3} \times 3.142 \times (1.728)^3 = 21.616039$

$$\boxed{\text{Volume} = 21.616\ \text{cm}^3}$$

Example 16 : *Calculate ($4.56 \times 10^4 - 2.34 \times 10^3$) with respect to significant figures.*

Solution : 45600 – 2340 = 43260

∴ Since we have to consider only 3 significant figures, i.e. rounding to 3 significant numbers. **Ans.** 4.33×10^4

Example 17 : *Considering four significant digits, round off the following given numbers (i) 12.3466, (ii) 12.3466×10^8, (iii) 12.0487*

Solution :

Given number	Rounded off to four significant digits
12.3466	12.35
12.3466×10^8	12.35×10^8
12.0487	12.05

EXAMPLES ON CONVERSIONS (SI PREFIXES)

Example 18 : *Convert the following :*

(i) 2.5 kg to gm

(ii) 4 MW to W

(iii) 5 GHz to Hz

(iv) 15 cm to m

(v) 90 mm to m

(vi) 100 μF to F

(vii) 1200 nF to F

(viii) 1500 pF to F

Solution :

Given value	Converted value
(i) 2.5 kg	$2.5 \times \mathbf{10^3}$ g
(ii) 4 MW	$4 \times \mathbf{10^6}$ W
(iii) 5 GHz	$5 \times \mathbf{10^9}$ Hz
(iv) 15 cm	$15 \times \mathbf{10^{-2}}$ m
(v) 90 mm	$90 \times \mathbf{10^{-3}}$ m
(vi) 100 μF	$100 \times \mathbf{10^{-6}}$ F
(vii) 1200 nF	$1200 \times \mathbf{10^{-9}}$ F
(viii) 1500 pF	$1500 \times \mathbf{10^{-12}}$ F

Practice Questions

1. Define a unit. What are the requirements of a good unit ?
2. Explain need of measurement in engineering and science.
3. What are the different systems of units ?
4. Define fundamental and derived quantities.
5. Define fundamental and derived units.
6. State seven fundamental quantities.
7. What do you understand by S.I. system of units ?
8. Define error.
9. What are significant figures ?
10. State the rules adopted to determine significant figures.
11. Define absolute error, relative error, percentage error.
12. Differentiate between fundamental quantities and derived quantities.
13. State the fundamental units of S.I. system.
14. Differentiate between fundamental units and derived units.

15. Define dimensions and dimensional formula.
16. What is the system of units ? Explain.
17. Give an example of a physical quantity with has no dimension.
18. Write any two uses of dimensional analysis.
19. Write any two limitations of dimensional analysis.
20. State the principle of homogeneity of dimensions.
21. Define accuracy and precision.
22. Differentiate between accuracy and precision.
23. Define : (a) Percentage error, (b) Significant figure.
24. List two supplementary fundamental physical quantities with their unit.
25. List four fundamental physical quantities.
26. List four derived physical quantities.
27. List all fundamental physical quantities. State S.I. unit for any two of them.
28. Correct the formula given below :

    $$\text{Percentage error} = \frac{\text{Absolute error}}{\text{True error}}$$

29. Define system of units. State different types of system of units and name the fundamental quantity in them with their units.
30. Define :
    (a) Absolute error (b) Average absolute error
    (c) Relative error (d) Percentage (%) error
31. Classify the following quantities as fundamental and derived quantities :
    Length, force, temperature, mass, volume, acceleration, area.
32. State dimensions of (i) Area, (ii) Acceleration, (iii) Work, (iv) Momentum.
33. State dimensions of (i) Density, (ii) Force, (iii) Pressure and (iv) Power.
34. Differentiate between absolute error and relative error.

Objective Type Questions

Fill in the blanks with suitable answers from the bracket and rewrite :

(i) .... is not a fundamental quantity (Density, Length, Mass, Time)

(ii) In both MKS and CGS systems, the unit of .... is the same. (length, mass, time, none of these)

(iii) Lack of proper precautions gives rise to .... (errors, mistakes).

(iv) Errors can be .... (completely removed by careful calculations, minimized by careful observations, tolerated in any measurements)

(v) The only fundamental unit of the following is ....(newton, coulomb, radian, candela)

(vi) Out of the following measurements, the only measurement containing significant zero is .... (0.352, 0.0352, 0.03052)

Ans. : (i) density, (ii) time, (iii) mistakes, (iv) minimized by careful observation, (v) candela, (vi) 0.03052.

Unsolved Problems

(1) Diameter of a wire is measured with micrometer screw gauge for five times as 0.421 cm, 0.422 cm, 0.421 cm, 0.423 cm, 0.423 cm. Calculate (i) average corrected reading, (ii) average absolute error and (iii) percentage error in measuring the diameter of the wire if zero error of micrometer is +0.002 cm

Ans. (i) Average corrected reading x_m **=** 0.420 cm
(ii) Average absolute error δx_{avg} = 0.0012 cm (iii) Percentage error = 0.2857%

(2) Diameter of pipe was measured by vernier caliper. The measurements were 3.71 cm, 3.70 cm and 3.67 cm. Calculate average absolute error and percentage error.

Ans. Average absolute error = δx_{avg} = 0.0052 cm, Percentage error = 0.141%

(3) Diameter of ball is measured with micrometer screw gauge for three times as 1.553 cm, 1.554 cm, and 1.554 cm. Calculate percentage error in measuring the diameter of ball if zero error of micrometer is +0.003 cm.

(4) The length of the object measured by vernier caliper is 10 ± 0.02 cm. Calculate the percentage error.

Ans. Percentage error = 0.2%

(5) Which of the following is more accurate reading and why ?

(a) 3.67 ± 0.003 cm (b) 3.85 ± 0.0033 m **Ans :** In the case of

(a) Percentage error is 0.0817% and in the case of (b) Percentage error is 0.0857%

Therefore reading (a) 3.67 ± 0.003 is more accurate

(6) The thickness of the job is 123.4 ± 0.023 mm. Estimate percentage error in the measurement.

(7) Diameter of the wire measured by micrometer screw gauge is 0.123 cm. If the least count of micrometer screw gauge is 0.001 cm, calculate percentage error. **Ans.** Percentage error = 0.813%

(8) Thickness of the object is measured by vernier caliper as 3.45 cm. If the L.C. of the vernier is caliper is 0.01 cm, calculate percentage error. **Ans.** Percentage error = 0.289%

(9) Calculate percentage error in K.E. of a body of mass 12 ± 0.2 g and moving with velocity of 23 ± 0.1 cm/s.

Ans. Percentage error = 2.536 %

(10) Calculate percentage error in the measurement of area of the plate. The length and breadth of the plate are given by

$$l = 14.5 \pm 0.1 \text{ cm}$$

$$b = 7.2 \pm 0.1 \text{ cm}$$

Ans. Percentage error = 2.078 %

(11) Calculate percentage error in the measurement of density of cube, if mass of the cube has 1.5% error and length has 0.3% error. **Ans.** Percentage error = 2.4%

(12) If the distance covered by an object is measured as 37 ± 0.05 m and time required is measured as 11 ± 0.1 sec, calculate percentage error in the measurement of speed. **Ans.** Percentage error = 1.044%

(13) State the number of significant figures in the following measurements :

(i) 0.00378, (ii) 0.037800, (iii) 6.023×10^{11}, (iv) 2500. **Ans.**

Given number	Significant figures
(i) 0.00378	3
(ii) 0.037800	5
(iii) 6.023×10^{11}	4
(iv) 2500	2

(14) State the number of significant figures in the following : (i) 2.0034, (ii) 0.002003, (iii) 500.2, (iv) 123.00, (v) 2.540×10^{11}, (vi) 2400 **Ans.**

Given	Number of significant figures
2.0034	5
0.002003	4
500.2	4
123.00	5
2.540×10^{11}	4
2400	2

(15) Find in significant figures, the area of a circle whose radius is measured as 1.243 cm. **Ans.** Area = 4.854

(16) Calculate $(2.3 \times 10^{-3} - 4.2 \times 10^{-4})$. **Ans.** 1.9

(17) Considering three significant digits, round off the following given numbers :

(i) 47.523 (ii) 47.583×10^6 (iii) 34.023. **Ans.**

Given number	Rounded off to three significant digits
47.523	47.5
47.583×10^6	47.6×10^6
34.023	34.0

(18) Express the result of the following in significant figures :

(i) $325 \times 10^8 \times 0.620$ **Ans.** 2.02×10^{10}

(19) Find in significant figures the area of sphere whose radius is 5.3 cm. **Ans.** 3.5×10^2 cm^2

(20) Find the number of significant figures in the following measurements :

(a) 0.031400 (b) 0.03140

(c) 0.0314 (d) 6.62×10^{-34}

(e) 6.620×10^{-34} (f) 6.6200×10^{-34} **Ans.** (a) 5, (b) 4, (c) 3, (d) 3, (e) 4, (f) 5

(21) Which of the following is more accurate reading and why ?

(a) 1.75 ± 0.012 cm (b) 0.75 ± 0.007 cm

Ans. (a) % error = 0.686 %, (b) % error = 0.933 %

% error in the first case is less therefore more accurate, first reading 1.75 ± 0.012 cm is more accurate.

(22) The length of the object is 18.2 ± 0.01 cm. Estimate the percentage error in this measurement.

(23) Diameter of ball measured by micrometer screw gauge is 0.12 cm. If the L.C. of micrometer screw gauge is 0.001 cm, calculate the percentage error. **Ans.** Percentage error = 0.83 %

(24) The length of the object measured by vernier caliper is 2.34 cm. If the L.C. of vernier caliper is 0.01 cm, calculate the percentage error. **Ans.** % error = 0.427 %.

(25) Convert the following :

(i) 6 kg into … gm (ii) 50 MW into … W (iii) 45 GHz to … Hz

(iv) 30 cm to m (v) 1000 mm to … m (vi) 2.7 μF to … F

(vii) 1500 nF to … F (viii) 2000 pF to … F

MCQs on Units and Measurements

1. *The parameter used for calculating weight of the man is ……*
   (a) *length* (b) *mass* (c) *time* (d) *none of these*
2. *The quantity measured in kelvin is ……*
   (a) *length* (b) *mass* (c) *time* (d) *temperature*
3. *The unit of acceleration in S.I. is ……*
   (a) *m/s* (b) *km/h* (c) *m/s*^{2} (d) *km/h*^{2}
4. *The unit of force in C.G.S. is ……*
   (a) *pound force* (b) *Newton* (c) *kg force* (d) *dyne*
5. *Kilogram metre per second square is the unit of ……*
   (a) *force* (b) *pressure* (c) *work* (d) *velocity*
6. *The unit of work is ……*
   (a) *Newton-metre* (b) *Newton* (c) *Joule/s* (d) *kilogram-metre*
7. *The unit of plane angle is ……*
   (a) *degree Celsius* (b) *radian* (c) *steradian* (d) *degree*
8. *The length of the table is 3 metre, here 3 is the ……*
   (a) *standard* (b) *unit* (c) *magnitude* (d) *quantity*
9. *Out of the following which is not a requirement of standard unit ……*
   (a) *it should be same for all quantities* (b) *it should be universally accepted*
   (c) *it should be well defined* (d) *it should be fixed with time and place*
10. *The …… used for measurement of physical quantity is called unit of that quantity.*
    (a) *quantity* (b) *dimension* (c) *time* (d) *standard*
11. *A quantity which can be measured (computed, quantified or enumerated) is known as ……*
    (a) *fundamental quantity* (b) *derived quantity* (c) *physical quantity* (d) *mechanical quantity*

12. *A physical quantity is a quantity which can ……*
 (a) be defined (b) be measured (c) not quantified (d) not computed
13. *The physical quantity which do not depend on any other physical quantity for their measurement is called ……*
 (a) fundamental quantity (b) derived quantity (c) scalar quantity (d) vector quantity
14. *Which of the following is not a fundamental quantity ?*
 (a) length (b) speed (c) mass (d) time
15. *Out of the following the fundamental quantity is ……*
 (a) density (b) pressure (c) momentum (d) time
16. *Physical quantity which depends on one or more fundamental quantities for their measurement is called as ……*
 (a) fundamental quantity (b) derived quantity (c) MKS quantity (d) CGS quantity
17. *Which of the following is not a fundamental unit ?*
 (a) metre (b) kilogram (c) newton (d) second
18. *Out of the following the derived unit is ……*
 (a) metre (b) kilogram (c) second (d) joule
19. *Pascal is the S.I. unit of ……*
 (a) force (b) pressure (c) density (d) momentum
20. *The system of units which are in use are ……*
 (a) C.G.S., M.K.S., P.S.T. and S.I. (b) M.K.S., C.G.S., V.I.T. and S.I.
 (c) C.G.S., M.K.S., P.S.T. and F.I. (d) C.G.S., M.K.S., F.P.S. and S.I.
21. *MKS means …… .*
 (a) micro-kg-sec (b) milli-kilo-s (c) m-kg-s (d) micro-kilo-s
22. *In M.K.S. system, the units of length, mass and time are ……*
 (a) millisecond, kilohertz and second (b) metre, kilogram and second
 (c) millimetre, kilobyte and second (d) mile, kilogram and second
23. *CGS means …… .*
 (a) calorie-grade-sec (b) cm-g-sec (c) calorie-g-sec (d) cm-grade-sec
24. *The units of length, mass and time are centimetre, gram and second which are used in the …… system.*
 (a) C.G.S. (b) M.K.S. (c) F.P.S. (d) S.I.
25. *FPS means …… .*
 (a) ft-lb-s (b) farad-pico-s (c) femto-pound-s (d) foot pico-s
26. *1 gigahertz means ……*
 (a) 10^6 Hz (b) 10^3 Hz (c) 10^{12} Hz (d) 10^9 Hz
27. *1 millimetre means ……*
 (a) 10^{-2} m (b) 10^{-3} m (c) 10^{-6} m (d) 10^{-9} m
28. *10^{-6} metre means ……*
 (a) 1 mm (b) 1 cm (c) 1 nm (d) 1 μm
29. *1 nanometre equals to ……*
 (a) 10^{-9} m (b) 10^{-6} m (c) 10^{-3} m (d) 10^{-1} m
30. *The SI unit of temperature is …… .*
 (a) °C (b) °K (c) °F (d) calorie
31. *The SI unit of luminous intensity is ________.*
 (a) ampere (b) flux (c) candela (d) weber
32. *The SI unit of amount of substance is ________.*
 (a) gram (b) candela (c) kilogram (d) mole

33. *The SI unit of solid angle is ————.*
   (a) degree (b) radian (c) steradian (d) degree celcius
34. *The SI unit of temperature gradient is ————.*
   (a) ℃/m (b) °K/m (c) m/°K (d) ℃/cm
35. *The unit of area in M.K.S. system is ……*
   (a) hectare (b) metre square (c) guntha (d) square feet
36. *Centimetre per second is the unit of speed in ……*
   (a) S.I. system (b) F.P.S. system (c) M.K.S. system (d) C.G.S. system
37. *The dimensions of physical quantity are the …… to which fundamental units must be …… to obtain the unit of a given physical quantity.*
   (a) scales, caliberated (b) system, scaled (c) powers, raised (d) false
38. *To decide dimensions of a physical quantity, the unit of time is expressed by ……*
   (a) 'S' (b) 'L' (c) 'M' (d) 'T'
39. *Dimensional formula for '**area**' is ……*
   (a) $[L^2 M^0 T^0]$ (b) $[L^2 M^{-1} T^0]$ (c) $[L^0 M^2 T^1]$ (d) $[L^0 M^0 T^2]$
40. *Dimensional formula for '**density**' is ……*
   (a) $[L^1 M^{-3} T^0]$ (b) $[L^{-3} M^1 T^0]$ (c) $[L^1 M^0 T^{-3}]$ (d) $[L^3 M^1 T^0]$
41. *Out of the following which physical quantity has dimensional formula $[L^{-1} M^1 T^{-2}]$?*
   (a) force (b) acceleration (c) velocity (d) density
42. *The dimensional formula for velocity is ————.*
   (a) $[L^1 M^0 T^{-1}]$ (b) $[L^1 M^2 T^1]$ (c) $[L^{-1} M^1 T^0]$ (d) $[L^1 M^1 T^{-1}]$
43. *In the dimensional equation $[L^a, M^b, T^c] \rightarrow [^a, ^b, ^c]$ are called ……*
   (a) dimensional formulae (b) dimensions (c) basic quantities (d) derived quantities
44. *$[L^1 M^0 T^{-1}]$ are the dimensions of the quantity ……*
   (a) acceleration (b) density (c) speed (d) area
45. *Dimensions of …… and …… are same.*
   (a) pressure, stress (b) work, force
   (c) velocity, acceleration (d) length, mass
46. *$[L^2M^1T^{-2}]$ are the dimensions of ……*
   (a) force (b) pressure (c) work (d) power
47. *$[L^2M^1T^{-3}]$ are the dimensions of ……*
   (a) force (b) pressure (c) work (d) power
48. *$[L^{-1}M^1T^{-2}]$ are the dimensions of ……*
   (a) force (b) pressure (c) work (d) power
49. *Dimensions of surface tension are ……*
   $[L^0M^1T^{-2}]$ (b) $[L^1M^1T^{-2}]$ (c) $[L^{-2}M^0T^{-1}]$ (d) $[L^2M^{-1}T^0]$
50. *The dimensions on both sides of physical equation must be same is the ……*
   (a) principle of Archimedes's (b) principle of homogeneity
   (c) law of Newton (d) law of Stoke's
51. *Two or more physical quantities can be added or subtracted if and only if ……*
   (a) they are fundamental (b) they are derived
   (c) their dimensions are same (d) their dimensions are not same
52. *Dimensions [L] + [L] = ……*
   (a) 2 [L] (b) $[L^2]$ (c) $[L] \times 2$ (d) [L]

53. *Out of the following which is not an application of dimensional analysis ?*
   - *(a) to find correctness of physical equation*
   - *(b) to convert physical quantity from one system to other*
   - *(c) to establish relationship between related physical quantities*
   - *(d) to derive formula for trigonometric function*
54. *Using dimensional analysis, one cannot derive a formula containing ……*
   - *(a) trigonometric function* *(b) logarithmic function*
   - *(c) exponential function* *(d) all of the above*
55. *The closeness of values measured by an instrument to the actual value is called as ……*
   - *(a) error* *(b) accuracy* *(c) precision* *(d) mistake*
56. *Precision is defined as ……*
   - *(a) uncertainity* *(b) closeness* *(c) repeatability* *(d) measurability*
57. *More precision means ……*
   - *(a) more scattered reading* *(b) less accurate reading*
   - *(c) more concentrated reading* *(d) less concentrated reading*
58. *Less the least count of instrument means ……*
   - *(a) less accuracy* *(b) more accuracy* *(c) less sensitivity* *(d) none of these*
59. *The deviation of the measured value to the desired value is called as ……*
   - *(a) mistake* *(b) error* *(c) difference* *(d) precision*
60. *The length and breadth of a glass sheet are 1.27 m and 0.86 m respectively. The area of this sheet upto three significant figures is ……*
   - *(a)* $1.092\ m^2$ *(b)* $1.0922\ m^2$ *(c)* $1.09\ m^2$ *(d)* $1.093\ m^2$
61. *Error is …… in a given measurement.*
   - *(a) mistake* *(b) accuracy* *(c) uncertainty* *(d) certainty*
62. *The difference between true value and measured value is known as* ————.
   - *(a) error* *(b) precision* *(c) mistake* *(d) accuracy*
63. ———— *cannot be eliminated but they can be minimised.*
   - *(a) errors* *(b) mistakes* *(c) accuracy* *(d) precision*
64. *An error caused due to faulty instrument is called ……*
   - *(a) systematic error* *(b) personal error* *(c) random error* *(d) constant error*
65. *The error because of defective setting is known as* ————.
   - *(a) systematic error* *(b) instrumental error*
   - *(c) random error* *(d) defective error*
66. *The error because of sudden change in experimental conditions is known as* ————.
   - *(a) systematic error* *(b) random error*
   - *(c) instrumental error* *(d) environmental error*
67. *Out of the following, the error which cannot be controlled is ----------*
   - *(a) instrumental error* *(b) experimental error*
   - *(c) random error* *(d) systematic error*
68. *Error can be minimized by ……*
   - *(a) taking large magnitude of physical quantity which is to be measured*
   - *(b) taking large number of readings*
   - *(c) using smallest least count instrument*
   - *(d) all of the above*

69. *The difference between reading and the mean reading is called as ……*
    (a) corrected reading (b) absolute error (c) average absolute error (d) relative error
70. *The ratio of average absolute error to mean reading is called ……*
    (a) average absolute error (b) absolute error (c) relative error (d) random error
71. *Same person may get different readings because of human limitations, this comes under ……*
    (a) instrumental error (b) constant error (c) random error (d) personal error
72. *Out of the following, the most accurate instrument is ————.*
    (a) measuring tape (b) metre scale
    (c) vernier caliper (d) micrometer screw gauge
73. *A significant figure is defined as a figure in any place which is reasonably ……*
    (a) non considerable (b) meaningless (c) not important (d) meaningful
74. *A figure which has some significance but it does not necessarily denote a certainity is called ————.*
    (a) significant figure (b) basic figure
    (c) numbering figure (d) decimal figure
75. *The digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are ……*
    (a) not significant (b) sometimes significant (c) always significant (d) all of the above
76. *All non-zero digits are ……*
    (a) always significant (b) not significant
    (c) sometimes significant (d) all of the above
77. *If distance between Mumbai to Pune by train is 90.5 km, in this, zero is ……*
    (a) not significant (b) significant (c) may be significant (d) may not be significant
78. *Out of the following the more precise value is ……*
    (a) 1.200×10^3 km (b) 1.20×10^3 km (c) 1.2×10^3 km (d) 1200 km
79. *If readings of thickness of metal plate are 0.626 cm, 0.627 cm, 0.625 cm, 0.627 cm and 0.626 cm then the average reading will be ……*
    (a) 0.6362 cm (b) 0.6226 cm (c) 0.6262 cm (d) 0.6622 cm
80. *If corrected average reading is 0.2525 cm and average absolute error is 0.00031 cm then relative error will be ……*
    (a) 0.001227 (b) 0.01227 (c) 0.002127 (d) 0.02127
81. *In a given measurement, if percentage error is 0.123, then relative error is ……*
    (a) 0.00213 units (b) 0.0123 units (c) 0.0213 units (d) 0.00123 units
82. *Calculate corrected reading, if diameter of rod measured by micrometer screw gauge is 1.234 cm (zero error of micrometer is + 0.002 cm) ……*
    (a) 1.322 cm (b) 1.232 cm (c) 1.223 cm (d) 2.132 cm
83. *The mass of the object is 23.4 ± 0.02 gm. Percentage error in this measurement is ……*
    (a) 0.0585% (b) 0.585% (c) 0.0855% (d) 0.855%
84. *Out of the following the more accurate reading is ……*
    (1) 1.22 ± 0.003 mm (2) 3.22 ± 0.004 mm (3) 4.22 ± 0.005 mm
    (a) case 1 (b) case 2 (c) case 3 (d) case 1 and case 3
85. *Diameter of the metal ball measured by vernier calliper is 2.34 cm. If least count of vernier calliper is 0.01 cm, calculate percentage error.*
    (a) 0.427% (b) 0.247% (c) 0.742% (d) 0.724%
86. *Calculate percentage error in kinetic energy of a body of mass 23 ± 0.1 gm and moving with velocity of 46 ± 0.2 cm/s.*
    (a) 4.013% (b) 3.104% (c) 1.043% (d) 1.304%

87. *The percentage error in the distance 100 ± 4 cm is ————.*

(a) 2% (b) 3% (c) 4% (d) 5%

88. *Calculate percentage error in the measurement of density of cube, if mass of cube has 3% error and length has 2% error.*

(a) 6% (b) 8% (c) 9% (d) 7%

89. *While measuring the density of a cube, the percentage error in the measurement of mass is 0.10% and the percentage error in the measurement of length is 0.20%, what will be the percentage error in measuring its density ?*

(a) 0.50% (b) 0.60% (c) 0.70% (d) 0.80%

90. *State the number of significant figures in the measurement of 0.0045.*

(a) 1 (b) 2 (c) 3 (d) 4

91. *The number of significant figure in the measurement of 0.0560 is ……*

(a) 1 (b) 2 (c) 3 (d) 4

92. *The number of significant figure in the measurement of 0.007800 is ……*

(a) 1 (b) 2 (c) 3 (d) 4

93. *The number of significant figure in the measurement of* 2.34×10^{11} *is ……*

(a) 1 (b) 2 (c) 3 (d) 4

94. *The number of significant figure in the measurement of 203 is ……*

(a) 1 (b) 2 (c) 3 (d) 4

95. *Calculate* $(2.34 \times 10^4 - 1.23 \times 10^3)$ *with respect to significant figures.*

(a) 2.22×10^3 (b) 2.22×10^4 (c) 2.22×10^5 (d) 2.22×10^6

96. *Considering four significant digits, round off the number* 12.3466×10^9.

(a) 12.347×10^9 (b) 12.34×10^9 (c) 12.35×10^9 (d) 12.346×10^9

97. *5.6 kg is equal to ……*

(a) 5.6×10^6 gm (b) 5.6×10^9 gm (c) 5.6×10^{-3} gm (d) 5.6×10^3 gm

98. *23 mm is equal to ……*

(a) 2.3 cm (b) 230 cm (c) 0.23 cm (d) 0.023 cm

99. *200 μF is equal to ……*

(a) 200×10^{-9} F (b) 200×10^6 F (c) 200×10^{-6} F (d) 200×10^9 F

100. *6 MW is equal to ……*

(a) 6×10^9 W (b) 6×10^6 W (c) 6×10^{-9} W (d) 6×10^{-6} W

101. *2 GHz is equal to ……*

(a) 2×10^{-6} Hz (b) 2×10^6 Hz (c) 2×10^{-9} Hz (d) 2×10^9 Hz

102. *1500 nF is equal to ……*

(a) 1500×10^{-6} F (b) 1500×10^6 F (c) 1500×10^{-9} F (d) 1500×10^9 F

103. *2000 pF is equal to ……*

(a) 2000×10^{-9} F (b) 2000×10^9 F (c) 2000×10^{12} F (d) 2000×10^{-12} F

104. *Length of table is 3 m. Convert this into mm.*

(a) 3×10^{-3} mm (b) 3×10^3 mm (c) 3×10^{-2} mm (d) 3×10^2 mm

105. *220 cm is equal to ……*

(a) 220×10^{-2} m (b) 220×10^2 m (c) 220×10^3 m (d) 220×10^{-3} m

Answers and Hints on Unit-I – Units and Measurements

1. (b)	2. (d)	3. (c)	4. (d)	5. (a)	6. (a)	7. (b)	8. (c)	9. (a)	10. (d)
11. (d)	12. (b)	13. (a)	14. (b)	15. (d)	16. (b)	17. (c)	18. (d)	19. (b)	20. (d)
21. (c)	22. (b)	23. (b)	24. (a)	25. (a)	26. (d)	27. (b)	28. (d)	29. (a)	30. (b)
31. (c)	32. (d)	33. (c)	34. (b)	35. (b)	36. (d)	37. (c)	38. (d)	39. (a)	40. (b)
41. (a)	42. (a)	43. (b)	44. (c)	45. (a)	46. (c)	47. (d)	48. (b)	49. (a)	50. (b)
51. (c)	52. (d)	53. (d)	54. (d)	55. (b)	56. (c)	57. (c)	58. (b)	59. (b)	60. (c)
61. (d)	62. (a)	63. (a)	64. (d)	65. (a)	66. (b)	67. (c)	68. (d)	69. (b)	70. (c)
71. (c)	72. (d)	73. (d)	74. (a)	75. (c)	76. (a)	77. (b)	78. (a)	79. (c)	80. (a)
81. (d)	82. (b)	83. (c)	84. (c)	85. (a)	86. (d)	87. (c)	88. (c)	89. (c)	90. (b)
91. (c)	92. (d)	93. (c)	94. (c)	95. (b)	96. (c)	97. (d)	98. (a)	99. (c)	100. (b)
101. (d)	102. (c)	103. (d)	104. (b)	105. (a)					

39. **Hint :** Area = $L \times b = L \times L$.

40. **Hint :** Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{\text{Mass}}{L \times B \times H} = \dfrac{M}{L \times L \times L}$

44. **Hint :** $[L^1 M^0 T^{-1}] = \dfrac{L}{T} = \dfrac{\text{Distance}}{\text{Time}}$ = Speed

46. **Hint :** Force × Displacement = $[L^1M^1T^{-2}] \times [L^1] = [L^2M^1T^{-2}]$.

47. **Hint :** $\dfrac{\text{Work}}{\text{Time}} = \dfrac{[L^2M^1T^{-2}]}{[T^1]} = [L^2M^1T^{-3}]$

48. **Hint :** $P = \dfrac{F}{A} = \dfrac{[L^1M^1T^{-2}]}{[L^2]} = [L^{-1}M^1T^{-2}]$

49. **Hint :** Surface tension $= \dfrac{\text{Force}}{\text{Length}} = \dfrac{[L^1M^1T^{-2}]}{[L^1]} = [L^0M^1T^{-2}]$

64. **Hint :** $\dfrac{(0.626 + 0.627 + 0.625 + 0.627 + 0.626)}{5} = 0.6262$

65. **Hint :** Relative error $= \dfrac{\delta x_{avg}}{x_m} = \dfrac{0.00031}{0.2525} = 0.001227$

66. **Hint :** $R = \dfrac{\%\text{ error}}{100}$

67. **Hint :** $x_c = x - z = (1.234 - 0.002) = 1.232$ cm.

68. **Hint :** % error $= \dfrac{0.02}{23.4} \times 100 = 0.0855$

69. **Hint :** (1) % error is 0.246, (2) % error is 0.124, (3) % error is 0.118. % error of case (3) is less, therefore more accurate.

70. **Hint :** $\dfrac{0.01}{2.34} \times 100 = 0.427\%$

71. **Hint :** $\left(\dfrac{0.1}{23} \times 100\right) + 2 \times \left(\dfrac{0.2}{46} \times 100\right)$

73. **Hint :** $3\% + (3 \times 2\%) = 9\%$

74. **Hint :**

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{\text{mass}}{(\text{length})^3}$$

$$\therefore \quad \%\text{ error in density} = \%\text{ error in mass} + \%\text{ error in } L^3$$

$$= \%\text{ error in mass} + 3 \times \%\text{ error in } L$$

$$= 0.10 + 3 \times 0.20 = 0.7\%$$

❒❒❒

Chapter 2

MECHANICS (LINEAR AND ANGULAR MOTION)

2.1 MOTION ALONG STRAIGHT LINE AND FORCE

2.1.1 Scalar and Vector Quantities

If I say, my house is 2 km from college, this information is enough and person will get idea about distance between college and house.

But if I say, my house is 2 km from college and come to my house tomorrow morning, now this information is not enough for the person. In this case I need to tell direction in addition to distance 2 km.

Thus in the 1st case distance is 2 km, needs only magnitude to describe it completely, i.e., distance-scalar quantity.

In the 2nd case I need to tell 2 km (magnitude) and in east direction i.e. displacement-vector quantity.

Physical quantities are broadly classified into two types i.e. scalar and vector quantities.

Physical quantities
↓
Scalar quantities (scalars) → needs only magnitude to describe the quantity
Vector quantities (vectors) → needs magnitude as well as direction to describe the quantity

1. **Scalar quantities or scalars :**

The physical quantities which have magnitude only are called scalars.

OR

Definition : The physical quantities which are completely described by their magnitude alone are scalar quantities.

Scalars are specified by number (magnitude) and unit.

e.g.
- Distance of house from college is 2 km.
- Mass of sugar is 5 kg.
- Volume of water is 1 litre.
- Speed of the vehicle is 20 m/s etc.

2. **Vector quantities or vectors :**

Definition : The physical quantities which have magnitude as well as direction are called vectors.

OR

The physical quantities which needs magnitude as well as direction to describe it completely are called vector quantities or vectors.

e.g. Displacement 2 km towards east, force of 5 N in the downward direction.

Difference between Scalar and Vector Quantities :

Scalar quantities (Scalars)	Vector quantities (Vectors)
1. Needs only magnitude to describe it.	1. Needs magnitude as well as direction to describe it.
2. Scalar quantity can be changed by changing its magnitude.	2. Vector quantity can be changed by changing its magnitude or direction or both.

… Contd.

3. Two or more (same) scalar quantities can be added or subtracted to find net effect. e.g. 2 kg + 4 kg = 6 kg (sugar)	3. Two or more (same) vector quantities can be simply added or subtracted to find net (effect) quantity if they are in the same or opposite direction e.g. 2 N in east and 5 N in north cannot be added algebraically. (We can add them if they are in the same direction or we can subtract if they are in the opposite direction.)
4. Scalars are represented by simple letters. e.g. length L, mass M, distance d etc.	4. They are represented by letters having arrow over them. e.g. force $\vec{F}$, velocity $\vec{v}$.
5. Scalar quantities e.g. Distance, length, mass, time, volume, density, work, energy, electric potential etc.	5. Vector quantities e.g. Displacement, velocity, acceleration, force, momentum, impulse, electric field, magnetic field etc.

2.1.2 Rectilinear Motion or Motion along a Straight Line and a Force

Introduction

- If a body does not change its position w.r.t. surroundings as the time passes, then it is said to be at rest.
- If a body changes its position with time, then it is said to be moving. A body is said to be in motion, if it changes its position w.r.t. surroundings with the passage of time.
- **Kinematics is a branch of mechanics which deals with motion of the particles (bodies).**
- To understand the rectilinear motion, it is necessary to introduce some prior basic concepts.

Prior basic concepts :

Inertia : Every body at rest has a tendency to remain at rest. Similarly, a body in uniform motion has tendency to remain in that motion. This is known as property of inertia.

Motion : If a body changes its position with the time, then it is said to be in motion. Motion of a body is caused due to force acting on it.

Speed : The rate of change of distance with time is called as speed. It is a scalar quantity (needs only magnitude to explain).

Displacement (s) : The change in position of a given particle in a particular direction is called as displacement. It is a vector quantity (needs magnitude as well as direction for explanation).

Velocity (v) : The rate of change of displacement with time in a particular direction is called as velocity. It is a vector quantity.

Thus, if the change in displacement is ds in time dt, then the velocity will be

$$v = \frac{ds}{dt}$$

Acceleration (a) : The rate of change of velocity with respect to time in a given direction is called as acceleration. It is a vector quantity. Thus, if the velocity changes from u to v in time t, then

$$\text{Acceleration (a)} = \frac{v - u}{t}$$

Retardation : The negative acceleration is called as retardation. If the rate of change of velocity is negative, then it is called as retardation. For example, when brakes are applied, velocity decreases and retardation takes place.

Fig. 2.1

Rectilinear motion (Definition) : *A body moving along a straight line is said to be in rectilinear motion or linear motion.* e.g. Motion of a car along a straight line (Fig. 2.1), apple freely falling due to gravitation.

2.1.3 Equations of Motion (Kinematics)

- Consider a body moving with initial velocity u and let the velocity change to v in time t.
- Let,
  u = Initial velocity
  v = Final velocity
  t = Time taken by a particle to change the velocity from u to v
  s = Distance travelled in time t
  a = Uniform acceleration

- Then, the three equations of rectilinear motion with uniform acceleration are

$$\boxed{v = u + at} \quad \dots (1)$$

$$\boxed{s = ut + \frac{1}{2}at^2} \quad \dots (2)$$

$$\boxed{v^2 = u^2 + 2as} \quad \dots (3)$$

2.1.4 Equations of Motion for Motion under Gravity

- There is a force of attraction between the earth and any body. This force of attraction causes an acceleration called gravitational acceleration (g).
- A freely falling body is always subjected to the constant gravitational acceleration 'g'.
- Thus, in the above equations (1), (2) and (3), substituting a = g for downward motion and a = –g for upward motion, we get the following two cases.

Case 1 : When a body is freely falling under gravity i.e. when a body moves vertically downwards towards the earth, then a = g.

$$\therefore \quad \boxed{v = u + gt} \quad \dots (1)$$

$$\boxed{s = ut + \frac{1}{2}gt^2} \quad \dots (2)$$

$$\boxed{v^2 = u^2 + 2gs} \quad \dots (3)$$

Case 2 : When a body moves vertically upwards away from the earth i.e. when motion takes place against the force of gravity, its velocity goes on decreasing by 'g'. The corresponding equations for upward motion will be as under

$$\therefore \quad \boxed{v = u - gt}$$

$$\boxed{s = ut - \frac{1}{2}gt^2}$$

$$\boxed{v^2 = u^2 - 2gs}$$

i.e. $\mathbf{a = -g}$ ($\because$ against gravity)

2.1.5 Distance Travelled by a Particle in n^{th} Seconds

- The distance travelled by a body in particular n^{th} seconds can be obtained by finding the distance travelled by a body in n seconds and (n – 1) seconds.
- Let, s = Distance travelled in n seconds

  s' = Distance travelled in (n – 1) seconds

  a = Uniform acceleration

$$\therefore \quad s_{n^{th}} = s - s' = \left[un + \frac{1}{2}an^2\right] - \left[un - u + \frac{1}{2}a\,(n^2 - 2n + 1)\right]$$

$\therefore$ Distance travelled in n^{th} seconds,

$$\boxed{s_{n^{th}} = u + \frac{a}{2}(2n - 1)}$$

- In case of a body freely falling under gravity, the distance travelled by the body in n^{th} seconds will be

$$s_{n^{th}} = 0 + \frac{g}{2}(2n - 1) \quad (\because u = 0 \text{ for free fall})$$

$$\boxed{s_{n^{th}} = \frac{g}{2}(2n - 1)}$$

2.1.6 Uniform Velocity

We know that the velocity is the rate of change of displacement w.r.t. time. Its unit is m/s or cm/s.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$. Its unit is m/s or cm/s.

Definition : *If a body covers equal distances in equal intervals of time in a particular direction, then it is said to be having uniform velocity.*

For example, a car moving with same speed (10 m/s) in the same direction.

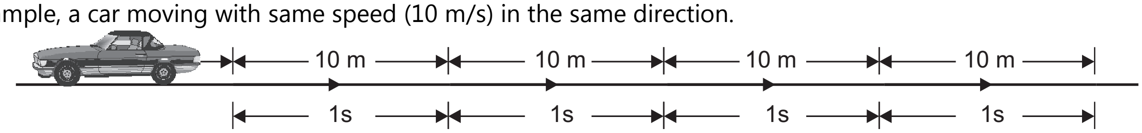


Fig. 2.2 : Illustrating uniform velocity

OR

Uniform velocity (Definition) : *If a body covers equal displacement in equal interval of time, then it is said to be in uniform velocity.*

OR

Uniform velocity (Definition) : *If a body is moving with constant speed in the same direction, then it is said to be in uniform velocity.*

If a body is moving with uniform velocity, then it has zero acceleration because change in velocity is zero.

2.1.7 Uniform Acceleration

We know that acceleration is the rate of change of velocity w.r.t. time.

i.e. $$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}}$$

$$a = \frac{v - u}{t}$$

where,
a – Acceleration
v – Final velocity
u – Initial velocity
t – Time interval

Its unit is $\mathbf{m/s^2}$ or $\mathbf{cm/s^2}$.

Uniform acceleration (Definition) : *If the acceleration of a body is uniform in magnitude and direction w.r.t. time, then it is called as uniform acceleration.*

OR

Uniform acceleration (Definition) : *If a change in velocity of a body is constant in every equal interval of time, then it is called as uniform acceleration.*

For example, a ball falling down. We can use three equations of motion if and only if the body is moving with *uniform acceleration.*

Uniform accelerated motion (Definition) : *A motion in which the acceleration of a body is uniform in magnitude and direction w.r.t. time, then it is called as uniform accelerated motion.*

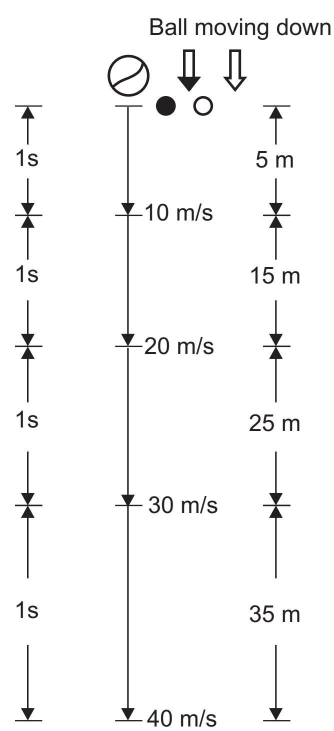


Fig. 2.3 : Illustrating uniform acceleration

2.1.8 Uniform Retardation

We know that, *retardation* is the *negative acceleration*.

i.e. the rate of change of velocity is negative. i.e. the speed of a body goes on decreasing.

Examples : (1) When brakes are applied to a train in motion. (2) Ball thrown up.

$$\text{Retardation} = -a = -\left(\frac{v-u}{t}\right) = \frac{(u-v)}{t}$$

Thus in case of retardation, final velocity is less than initial velocity.

Uniform retardation (Definition) : *If the acceleration of a body is negative and uniform in magnitude and direction w.r.t. time, then it is called as uniform retardation.*

OR

Uniform retardation (Definition) : *If the change in velocity of a body is negative and constant in every equal interval of time, then it is called as uniform retardation.*

Uniform retarded motion (Definition) : *A motion in which the acceleration of a body is negative and uniform in magnitude and direction w.r.t. time, then it is called as uniform retarded motion.*

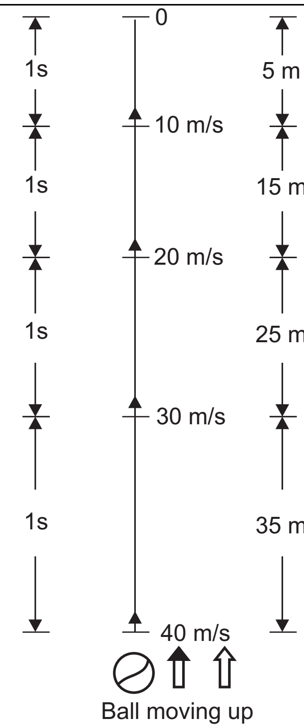


Fig. 2.4 : Illustrating uniform retardation

2.1.9 Kinetics

2.1.9.1 Definitions

1. **Kinetics**
   - **It is the branch of dynamics which deals with effect of forces on the bodies in motion, by considering the forces causing the motion.**
2. **Momentum**
   - **Definition of momentum : It is defined as the quantity of motion possessed by a moving body.** Momentum of a body is equal to the product of mass and velocity of a body.

$$\boxed{\text{Momentum} = \text{Mass} \times \text{Velocity}} = m \times v$$

$$\text{S.I. unit} = \text{kg} \times \frac{\text{m}}{\text{sec}} \text{ or N-sec.}$$

S.I. unit is **N-sec** or **kg-m/s**

   - It is a vector quantity.
3. **Impulse**
   - **Definition of impulse : Impulse is defined as the change in momentum.** This change in momentum takes place in a short duration of time.

$$\boxed{\text{Impulse} = \text{Change in momentum}}$$

$$\boxed{\text{Impulse} = mv - mu}$$

where, m = Mass of a body

u = Initial velocity

v = Final velocity

OR

- **Definition : Impulse is defined as the product of large force on a body and the very small time for which the force acts.**

S.I. unit : Since impulse is the change in momentum, it has the same unit as that of momentum.

- Thus, **S.I.** unit of impulse is **kg-m/s** or **N-sec**.
- It is also a vector quantity.

4. **Impulsive Force**

- **Definition : Impulsive force is the force which acts for a short time and produces considerable change in the momentum of a body.**
- For example, a foot ball hit by a player. Here the kick applied by a foot ball player is impulsive force.
- **Impulsive force is the rate of impulse with respect to time.**

i.e. $$\text{Impulsive force} = \frac{\text{Impulse}}{\text{Time}}$$

- **S.I.** unit of impulsive force is **N**.

2.1.10 Newton's Laws of Motion and Equations

1. **Newton's First Law of Motion (Law of Inertia)**

Statement : It states that every body continues in its state of rest or of uniform motion in a straight line, unless it is acted upon by some external agency.

- Thus, a body at rest tends to remain at rest and a body in motion tends to remain in motion. This property is called as inertia.

Example : (1) When fast moving bus suddenly stops, passengers in it lean forward.

2. **Newton's Second Law of Motion (Law of Momentum)**

Statement : It states that the rate of change of momentum of a body is proportional to the applied force and takes place in the direction of the force.

Thus, $$F \propto \frac{(mv - mu)}{t}$$

where, F = Force
mv = Final momentum
mu = Initial momentum
t = Time
m = Mass
u = Initial velocity
v = Final velocity

$$F \propto \frac{m(v - u)}{t} \qquad \text{but } \frac{(v-u)}{t} = \text{acceleration} = a$$

$$F \propto ma$$

$$F = ma \qquad (\because \text{constant} = 1)$$

3. **Newton's Third Law of Motion (Law of Action-Reaction)**

- **Statement : It states that for every action, there is always an equal and opposite reaction.**
- For example, a swimmer pushes the water backward (i.e. action) and the water pushes him forward with equal force (i.e. reaction).
- **Law of conservation of momentum :** It *states that the total momentum of a system consisting of two or more colliding bodies before impact remains unchanged after impact, provided no external force acts on it.*

2.1.11 Applications of Newton's Laws of Motion

(1) Applications (examples) of Newton's first law of motion (law of inertia)

- **(i) Safety seat belts in a car :** When sudden brakes are applied to a speedy car, persons inside the car leans forward suddenly and may collide on dash board due to inertia. To avoid this impact, seat belts are used for safety.
- **(ii) Use of seat belts in aeroplane** while take off.
- **(iii) Use of seat belt in Merry-Go-Round**.
- **(iv) Dust it off (tool) :** The dust in your bed, clothes, chairs remains intact due to inertia unless you dust it off the objects using any tool.
- **(v) Removing fruits and flowers from a tree :** If you shake a tree, the stationary fruits and flowers will break away and fall down.
- **(vi) To and fro motion of pendulum (used in clock) :** Once pendulum is displaced and released, it moves to and fro. From extreme end to mean position due to gravity and **from mean position to extreme end due to inertia.**
- **(vii) To tighten the head of the hammer onto the wooden handle**, you have to bang the bottom of the handle on hard surface.
- **(viii) Removing honey, ketch up from bottle using** upper technique you have to halt a speed, down moving bottle.
- **(ix)** While riding a skate board, you fly forward off the board when hitting a rock.
- **(x) Technique used in drop the coin experiment :** Place a coin over cardboard and now place a cardboard on glass. You have to quickly pull the cardboard to drop it in glass.

2. Applications (examples) of Newton's second law of motion :

- **(i) Catch the ball :** Professional cricketer swing their hand back (lowers his hand) while catching the ball which provides ball more time to loose its speed and momentum. Due to more time taken to stop the ball, the rate of change of momentum of ball is decreased and hence small force is exerted on the hands.
- **(ii) Case of high jumper :** High jumping athlete needs cushion (soft sand), when high jumper athlete land on cushion, it takes longer time to reduce the rate of change of momentum.
- **(iii) The use of stretchable seat belts :** Slight stretchable seat belts in a car increases the time taken to fall forward and hence reduces the rate of change of momentum.
- **(iv) Shock absorber in a vehicle :** When vehicle bumps over rough road, spring of shock absorber increases the time of jerk, which reduces force (jerk).
- **(v) Bogies of trains are fitted with buffers :** Buffers increase the time interval of jerk and reduce jerk.
- **(vi) Pushing a table :** To push a table with more acceleration, more force is required.
- **(vii)** Pushing a scooter needs less force than pushing a car.
- **(viii)** Pulling a cycle with one person on it needs less force than two persons on a cycle.
- **(ix)** Team of persons is required to lift big box (machine) with more weight.

3. Applications (examples) of Newton's third law of motion :

- **(i) Firing of rocket :** Jet of hot gases downside on earth makes rocket to move up.
- **(ii) Jet aircraft :** Hot gases expelled which makes motion or diversion of jet plane easy.
- **(iii) Lawn spray sprinkler :** Sprinkler rotates because of jet force of water from nozzle.
- **(iv)** Jumping on earth.
- **(v)** Swimmer moves forward by pushing water backside.
- **(vi)** Swimmer pushes against pull wall with his feet to get speed in forward direction.
- **(vii)** Push back chairs in cinema theatre.
- **(viii)** Helicopters create lift by pushing air down.
- **(ix)** Birds fly in air by exerting force on air.
- **(x) Recoil of a Gun**

- If a bullet is fired from a gun, then bullet shoots out with a large velocity, and at the same time gun moves back (jerk) with the little velocity. **This backward movement (jerk) is known as recoil of the gun.**

Fig. 2.5

Recoil velocity of a Gun :

- Let, m_1 = Mass of bullet

  v_1 = Velocity of bullet

  m_2 = Mass of gun

  v_2 = Velocity of gun

Before firing : The gun and bullet are at rest i.e. has zero velocity. Therefore, the total momentum before firing is zero.

After firing : Bullet moves forward and the gun moves backward.

$\therefore$ Momentum of bullet $= m_1v_1$

and Momentum of gun $= m_2(-v_2)$

(–ve sign indicates that gun moves back).

$$\therefore \begin{bmatrix}\text{Total momentum of the} \\ \text{system after firing}\end{bmatrix} = m_1v_1 - m_2v_2$$

$$\therefore \boxed{m_1v_1 = m_2v_2}$$

According to the law of conservation of momentum,

Total momentum before firing = Total momentum after firing

$$\therefore \quad 0 = m_1v_1 - m_2v_2$$

$$\therefore \quad m_2v_2 = m_1v_1$$

$$\therefore \quad \boxed{v_2 = \frac{m_1v_1}{m_2}}$$

This gives the **recoil velocity** of a gun.

(xi) **Book kept on a table :** Book exerts downward force (weight i.e. action) and table exerts equal and opposite reaction on the book.

(xii) While running we press the ground (action) and ground exerts equal and opposite reaction.

(xiii) **Rotating lawn sprinkler :** Water comes out through nozzle with high pressure (action) and sprinkler rotates (reaction).

(xiv) Support is required while hitting nail into wooden block – without support it is difficult to hit rail, because there will not be any reaction. Hence, it is required to rest a block with support.

2.1.12 Conservation of Momentum

The application of Newton's second law of motion and Newton's third law of motion plays important role in conservation of momentum.

Law of conservation of momentum :

It states that, when the external resultant force on the system of interacting bodies is zero (i.e. isolated system), the **total linear momentum** of the system **remains constant**. i.e. total linear momentum before collision (impact) is equal to total linear momentum after collision (impact), if external resultant force is zero.

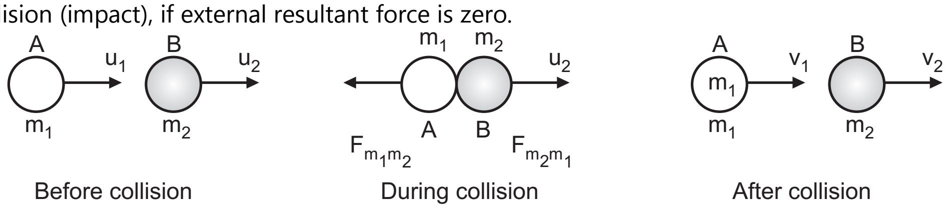


Fig. 2.6

Before collision (impact) :

Consider two bodies of mass m_1 and m_2 moving with initial velocities u_1 and u_2.

$\therefore$ Total linear momentum $(\vec{P}_1)$ before collision,

$$\vec{P}_1 = m_1\vec{u}_1 + m_2\vec{u}_2 \qquad \ldots (1)$$

During collision (impact) :

Let $u_1 > u_2$, when two bodies collide on each other, by Newton's third law,

$$\vec{F}_{AB} = -\vec{F}_{BA} \qquad \ldots (2)$$

(i.e. action and reaction are equal but opposite).

After collision (impact) :

Suppose two bodies move in the same direction (but m_1 with less speed now) ($v_1 < v_2$).

Total linear momentum ($\vec{P}_2$) after collision is given by,

$$\vec{P}_2 = m_1\vec{v}_1 + m_2\vec{v}_2 \quad \text{… (3)}$$

Using Newton's second law of motion,

$$\vec{F}_{AB} = m_1\left(\frac{\vec{v}_1 - \vec{u}_1}{t}\right) \quad \text{and} \quad \vec{F}_{BA} = m_2\left(\frac{\vec{v}_2 - \vec{u}_2}{t}\right)$$

But as per equation (2), $\vec{F}_{AB} = -\vec{F}_{BA}$

$$m_1\left(\frac{\vec{v}_1 - \vec{u}_1}{t}\right) = -m_2\left(\frac{\vec{v}_2 - \vec{u}_2}{t}\right)$$

$$m_1\vec{v}_1 - m_1\vec{u}_1 = m_2\vec{u}_2 - m_2\vec{v}_2$$

$\therefore$ $$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2$$

i.e. $$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

i.e. $$\vec{P}_1 = \vec{P}_2$$

Total linear momentum before collision (impact) is equal to total linear momentum after collision (impact).

2.1.12.1 Examples of Conservation of Linear Momentum

1. Bullet fired from gun, gun recoils :

Total momentum before firing = Total momentum after firing

$$\underbrace{m_1u_1}_{\text{bullet}} + \underbrace{m_2u_2}_{\text{gun}} = \underbrace{m_1v_1}_{\text{bullet}} - \underbrace{m_2v_2}_{\text{gun}}$$

$$0 = m_1v_1 - m_2v_2$$

$\therefore$ $$\boxed{m_1v_1 = m_2v_2}$$

$\therefore$ $$\boxed{v_2 = \frac{m_1v_1}{m_2}}$$ …… recoil velocity of a gun.

2. Firing of rocket :

The hot gases are ejected through a tail of rocket with high pressure (i.e. action), this imparts equal forward momentum (i.e. reaction) on the rocket and rocket moves upward.

Due to escaping gases, mass of rocket goes on decreasing and velocity and acceleration goes on increasing (because momentum = m × v remains constant).

3. Person jumping from a boat (or swimmer pushes water back) :

When person jumps out of stationary boat, the boat gets pushed back. Here linear momentum of the person is equal and opposite to that of boat.

4. Explosion of a bomb :

Before explosion, bomb is at rest. When bomb explodes into number of pieces, they get scattered in different directions with different speed.

Here total momentum before explosion = Total momentum after explosion

$$0 = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots\ldots$$

2.2 ANGULAR MOTION

2.2.1 Introduction

- If the motion of a body takes place along the circumference of a circle, then it is called as circular motion or angular motion.
- When a particle moves along a circular path, then its linear velocity is directed along the tangent to a circle at any given instant. This velocity is called as instantaneous linear velocity of the particle.

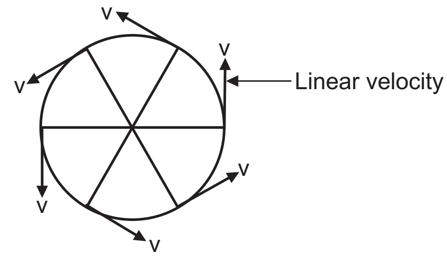


Fig. 2.7

- As the particle moves along a circle, the direction of the instantaneous linear velocity changes continuously.
- However, the magnitude of the instantaneous linear velocity may or may not change.
- When the magnitude of instantaneous linear velocity remains constant, then the motion is called as a uniform circular motion (U.C.M.).
- In this chapter, we are going to study angular motion with constant angular acceleration and its relation with rectilinear motion. Initially, we will go through following terms.

2.2.2 Angular Displacement (θ)

Consider a particle moving along a circle and let it moves from A to B in short time.

A = First position

B = Second position

θ = Angular displacement

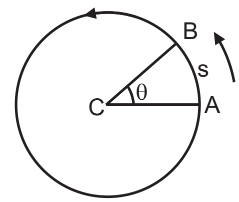
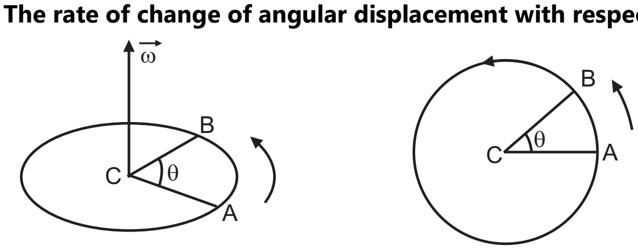


Fig. 2.8

- The line joining the centre and position of a particle is called the radius vector. Thus, CA and CB are the radius vectors.
- Now, let the particle moves from A to B through a small distance 's' along the curve, in time t. Then the radius vector will trace out a small angle ∠ ACB = θ in the same short interval of time 't'.
- **The angle through which the radius vector turns is called as angular displacement.**
- **The angular displacement can also be defined as the angle subtended at the centre of a circle by the path travelled.**
- The S.I. unit of angular displacement is radians (rad).
- The direction of angular displacement is perpendicular to the plane of the circle given by right-handed screw rule.

2.2.3 Angular Velocity (ω)

- **The rate of change of angular displacement with respect to time is called as angular velocity (ω) of a particle.**



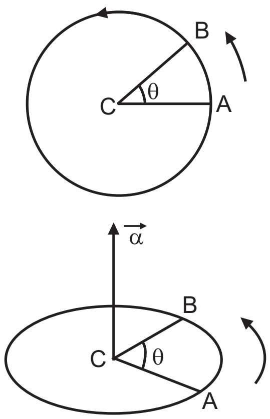
$$\therefore \omega = \frac{dq}{dt}$$

Fig. 2.9

- S.I. unit of angular velocity is rad/s.
- The direction of angular velocity is perpendicular to the plane of a circle given by the right-handed screw rule.

2.2.4 Angular Acceleration (α)

- **The rate of change of angular velocity with respect to time is called as angular acceleration.**



$$\therefore \alpha = \frac{d\omega}{dt}$$

$$\text{or} \quad \alpha = \frac{(\omega_2 - \omega_1)}{t}$$

Fig. 2.10

- S.I. unit of angular acceleration is rad/s^2.
- The direction of angular acceleration is perpendicular to the plane of a circle given by the right-handed screw rule.

2.2.5 Relation between Angular Velocity (ω) and Linear Velocity (v)

- Consider a particle undergoing uniform circular motion. It moves from point A to point B in time 't'.
- Let,
  - s = Linear displacement
  - θ = Angular displacement
  - v = Linear velocity
  - r = Radius of a circle
  - ω = Angular velocity

$$\therefore \quad v = \frac{s}{t}$$

$$\therefore \quad v = \frac{r\theta}{t} \qquad \because s = r\theta$$

$$= r \times \frac{\theta}{t} \qquad \because \frac{\theta}{t} = \omega$$

$$\therefore \quad \boxed{v = r\omega}$$

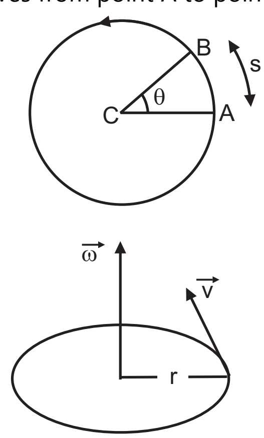


Fig. 2.11

- Thus, linear velocity is radius times the angular velocity.

2.2.6 Three Equations of Circular (Angular) Motion

Consider circular motion of a body starting from A. Let B be its position after t seconds.

Let,
- θ = Total angular displacement in radians
- ω_o = Initial angular velocity in rad/s i.e. velocity at 'A'
- ω = Final angular velocity in rad/s i.e. velocity at 'B'
- t = Time in seconds taken by a particle to change its velocity from ω_o to ω.
- α = Constant angular acceleration in rad/sec^2.

Three equations of circular (angular) motion are as under :

$$\omega = \omega_o + \alpha t \qquad \text{... (1)}$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 \qquad \text{... (2)}$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta \qquad \text{... (3)}$$

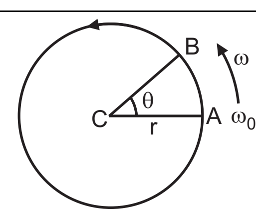


Fig. 2.12

These equations are similar to three equations of rectilinear motion i.e.

$$v = u + at \quad \text{... (a)}$$

$$s = ut + \frac{1}{2}at^2 \quad \text{... (b)}$$

$$v^2 = u^2 + 2as \quad \text{... (c)}$$

2.2.7 Angular Distance Travelled by a Particle in n^{th} Seconds

Let,
ω_o = Initial angular velocity of a body
ω = Final angular velocity of a body
t = Time
θ = Angular distance travelled in n seconds
θ' = Angular distance travelled in (n – 1) seconds
α = Angular acceleration

We have,

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 \quad \text{... (1)}$$

Substituting t = n and then t = (n – 1) and subtracting, we get

$$\begin{bmatrix}\text{Angular distance} \\ \text{travelled in} \\ n^{th}\text{ seconds}\end{bmatrix} = \theta^{n^{th}} = \omega_o + \frac{\alpha}{2}(2n-1)$$

Similarity (Resembleness)

Equations of rectilinear motion	Equations of motion under gravity	Equations of angular motion
1. $v = u + at$ 2. $s = ut + \frac{1}{2}at^2$ 3. $v^2 = u^2 + 2as$ 4. $s^{n^{th}} = u + \frac{a}{2}(2n-1)$ where u = Initial velocity v = Final velocity t = Time taken a = Uniform acceleration s = Distance covered in 't' seconds $s^{n^{th}}$ = Distance covered in particular n^{th} seccond	1. $v = u + gt$ 2. $s = ut + \frac{1}{2}gt^2$ 3. $v^2 = u^2 + 2gs$ 4. $s^{n^{th}} = u + \frac{g}{2}(2n-1)$ where u = Initial velocity v = Final velocity t = Time taken g = Gravitational acceleration s = Distance covered in 't' seconds $s^{n^{th}}$ = Distance covered in particular n^{th} second	1. $\omega = \omega_o + \alpha t$ 2. $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ 3. $\omega^2 = \omega_o^2 + 2\alpha\theta$ 4. $\theta^{n^{th}} = \omega_o + \frac{\alpha}{2}(2n-1)$ where ω_o = Initial angular velocity ω = Final angular velocity t = Time taken α = Angular acceleration θ = Angular displacement in 't' seconds $\theta^{n^{th}}$ = Angular displacement in particular n^{th} second
Hint : Put u = 0 if a body starts from rest. v = 0 if a body comes to rest.	**Hint :** Put u = 0 if a body starts from rest. v = 0 if a body comes to rest. g = 0 if a body is freely falling Replace g by –g if a body moves up (against gravitation)	**Hint :** Put ω_o = 0 if a body starts from rest. ω = 0 if a body comes to rest. θ should be in radians.

2.2.8 Simple Harmonic Motion

Periodic Motion :

- A motion of a body which is repeated again and again after a certain interval of time is called as periodic motion.
- e.g. spinning of Earth about its own axis, motion of moon round the earth, motion of a pendulum.
- Periodic motion is also called as *harmonic motion*.

Simple Harmonic Motion :

- **In general, simple harmonic motion (S.H.M.) is defined as the periodic motion of a body in which the force (or acceleration) is always directed towards the mean position and its magnitude is proportional to its displacement from the mean position.**
- S.H.M. can be (1) linear or (2) angular depending upon its path.
- e.g. Simple pendulum : If the bob of pendulum is displaced and released, then it undergoes *periodic motion* and every time the bob tends to move towards the mean position.
- Linear S.H.M. is defined as the periodic motion along a straight line in which the force (or acceleration) is always directed towards the mean position and its magnitude is proportional to its displacement from the mean position.
- e.g. Motion of pendulum : If very small oscillations are given, then it is along a straight line and periodic.
- Motion of the needle of a sewing machine.

2.2.9 Explanation of Linear S.H.M. or S.H.M. as Projection of Uniform Circular Motion on any One Diameter

- Consider a point 'P' moving along a circle in anticlockwise direction with constant speed. Let 'r' be the radius of the circular path and 'C' be the centre.
- Draw a perpendicular from P on the diameter AA'. It meets at C. Then C is called as projection of P on the diameter. Similarly B_1, B_2, B_3 are projections of particles at positions P_1, P_2, P_3.
- Thus, when a particle moves from P to A, its projection moves from C to A. When a particle completes one rotation along a circle, its projection will complete one oscillation along a straight line (diameter). i.e. the particle is performing circular motion and its projection is performing up and down motion along straight line (S.H.M.).

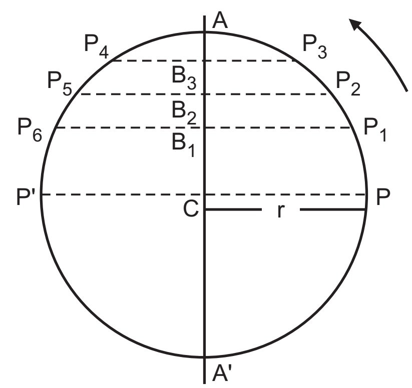


Fig. 2.13

- Here the distance covered by a particle along the circumference is equal in equal intervals of time, but distance covered by the projection decreases from C to A i.e. velocity is decreasing, hence acceleration is negative.
- On the contrary, when particle moves from A to P' covering equal distances, the distance covered by its projection increases from A to C i.e. its velocity increases i.e. acceleration is positive.
- Thus, *projection of the particle moves up and down and always tends to move towards the centre 'C' (mean position) i.e. performs linear SHM.*
- The circle shown above is called the reference circle. C is the mean position. The maximum possible displacement of projection (i.e. CA) is called amplitude 'a' and it is equal to radius 'r'.
- Thus *S.H.M. can also be defined as the projection of a uniform circular motion on a diameter of a circle.*

Characteristics of Linear S.H.M. :

1. Motion is periodic along a straight line.
2. Force (or acceleration) is directed towards the mean position.
3. Force (or acceleration) is directly proportional to its displacement from the mean position.
4. Velocity of a particle is maximum at the centre and minimum at the extreme position.

2.2.10 Equation of S.H.M. or Displacement in S.H.M.

Derivation of Displacement of a Particle Executing S.H.M.

- We consider the S.H.M. as a projection of the movement of a particle performing uniform circular motion taken on diameter of a circle.

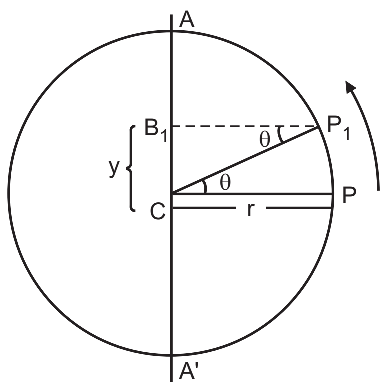


Fig. 2.14

- Consider a particle P performing uniform circular motion in anticlockwise direction.
- Let

  ω - Angular velocity of a particle in circular motion

  r - Radius of circular path called radius vector

  C - Centre of circular path

(I) Equation of S.H.M. from mean position : The particle is initially at position P, and C is its projection.

- Particle moves from P to P_1 in time 't' seconds. Its projection moves (displaces) from C to B_1 i.e. $CB_1 = y$ is the displacement of projection in time 't' seconds.
- 'θ' is the angular displacement of a particle in 't' seconds (angular displacement is the angle through which radius vector turns).

In $\Delta\ CP_1B_1$, $\sin\theta = \frac{CB_1}{CP_1}$

i.e. $\sin\theta = \frac{y}{r}$ $\therefore$ $y = r\sin\theta$ but $\theta = \omega t$

$\therefore$ $\boxed{y = r\sin\omega t}$ This is the equation of S.H.M.

- r is the maximum possible displacement of a particle in S.H.M. and it is called as amplitude 'a'.

Put r = a $\boxed{y = a\sin\omega t}$ where a is the amplitude

(II) General case :

- In the earlier case, we assumed that a particle starts from position 'P' along horizontal. But in general, a particle can start from any position.
- Consider a particle is initially at position P, and the radius makes an angle α with the horizontal as shown in Fig. 2.15.
- Particle moves from P to P_1 in time 't' seconds. Its projection is at position B_1. $CB_1 = y$ is the displacement of projection from the mean position.

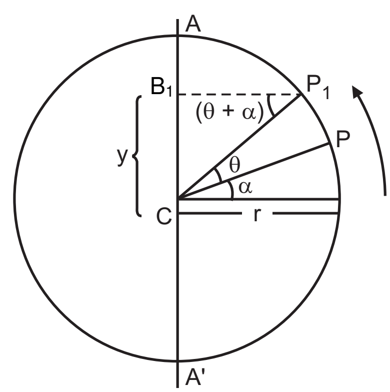


Fig. 2.15

- In ΔCP_1B_1, $\sin\angle CP_1B_1 = \frac{CB_1}{CP_1}$

$$\sin(\theta + \alpha) = \frac{y}{r}$$

$\therefore$ $y = r\sin(\theta + \alpha)$ but $\theta = t$

$\therefore$ $\boxed{y = r\sin(\omega t + \alpha)}$ also we can put r = a = amplitude

$\boxed{y = a\sin(\omega t + \alpha)}$ is the equation of S.H.M. in general.

where, α is initial phase angle, also called epoch.

(III) If particle starts from extreme position, then α = 90°.

$\therefore \quad y = r \sin(\omega t + 90°)$

$y = r \cos \omega t$

or $\quad y = a \cos \omega t$

This is the equation for displacement of a particle in SHM, if the particle is initially at extreme position.

2.2.11 Velocity in S.H.M.

Derivation of Velocity of a Body Executing S.H.M.

To find the velocity of a particle in SHM, a component of velocity of the reference particle P is to be considered which is parallel to the diameter AA'.

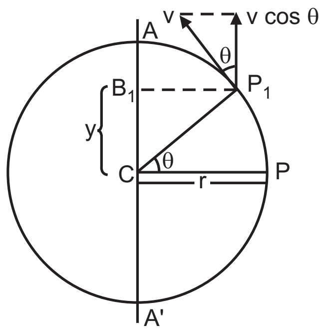


Fig. 2.16

Hence, the component of velocity of P on diameter gives velocity of particle in SHM.

r - Radius of reference circle

v - Linear velocity of reference particle P

θ - Angular displacement

$CB_1 = y$ = Displacement of particle in SHM

ω - Angular velocity

Velocity of a particle in SHM at position B_1

= Component of 'v' parallel to AA'

$= v \cos \theta \qquad$ but $\theta = \omega t$

$= v \cos \omega t \qquad$ but $\cos \omega t = \sqrt{1 - \sin^2 \omega t}$

and $v = r\omega$

i.e. $v = a\omega$

$\therefore \quad = a\omega \sqrt{1 - \sin^2 \omega t}$ but $y = a \sin \omega t$ (equation of SHM)

$\therefore \quad \frac{y}{a} = \sin \omega t$

$= a\omega \sqrt{1 - \left(\frac{y}{a}\right)^2}$

$= a\omega \sqrt{\frac{a^2 - y^2}{a^2}}$

$= a\omega \frac{1}{a} \sqrt{a^2 - y^2} = \omega \sqrt{a^2 - y^2}$

Thus velocity of particle in SHM is

$$\boxed{v = \omega \sqrt{a^2 - y^2}}$$ where a - amplitude, y - displacement of particle

i.e. $\quad v = \omega \sqrt{(\text{amplitude})^2 - (\text{displacement})^2}$

Case - 1 : When particle is at mean position, i.e. $y = 0$

then $\quad v = \omega \sqrt{a^2 - 0}$

$\boxed{v = a\,\omega}$ or $\boxed{v = r\omega}$

Thus velocity v is maximum at mean position.

Case - 2 : When particle is at extreme position, i.e. $y = a$,

then $\quad v = \omega \sqrt{a^2 - a^2}$

$\boxed{v = 0}$

Thus velocity v is minimum at extreme position.

2.2.12 Acceleration in S.H.M.

Derivation of Acceleration of a Body Executing S.H.M.

Reference particle P is performing uniform circular motion. Therefore, it has acceleration always directed towards centre and along radius, called radial acceleration.

Radial acceleration, $a = r\,\omega^2$ [From circular motion topic]

The component of radial acceleration along parallel to AA' gives acceleration of particle in SHM at position B_1.

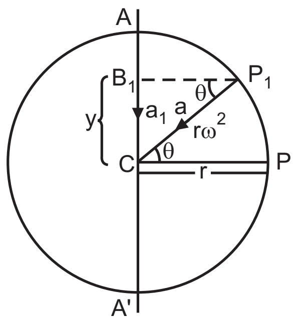


Fig. 2.17

r	-	Radius of reference circle
a	=	$r\omega^2$ - Radial or centripetal acceleration
θ	-	Angular displacement
$CB_1 = y$	=	Displacement of particle in SHM
ω	-	Angular velocity

Acceleration of particle in SHM = Component of radial acceleration along parallel direction of AA'

$= a \sin\theta$ but $\theta = \omega t$, $a = r\,\omega^2$

$= r\,\omega^2 \sin\omega t$

$= \omega^2 r \sin\omega t$ but $r \sin\omega t = y$

∴ Acceleration of a particle in SHM is

$$\boxed{a_1 = \omega^2 y}$$

and this acceleration is directed towards the mean position.

Case - 1 : When particle is at mean position i.e. $y = 0$,

then acceleration $a_1 = \omega^2 y = \omega^2 \times 0 = 0$

$$\boxed{a_1 = 0}$$

i.e. Acceleration of a particle in SHM is minimum at mean position.

Case - 2 : When particle is at extreme position i.e. $y = a$,

then acceleration $a_1 = \omega^2 y$ becomes

$$\boxed{a_1 = \omega^2 a} \quad \text{or} \quad \boxed{a_1 = \omega^2 r} \quad \text{where } y = a = \text{Amplitude}$$

i.e. acceleration of a particle in S.H.M. is maximum at extreme position.

Important Points

- **Inertia :** Every body at rest has tendency to remain at rest. Similarly, a body in uniform motion has tendency to remain in that motion.
- **Motion :** If a body changes its position with time, then it is said to be in motion.
- **Rectilinear motion :** A body moving along a straight line is said to be in rectilinear motion.
- **Speed :** The rate of change of distance with time is called as speed.
- **Displacement (s) :** The change in position of a given particle in a particular direction is called as displacement.
- **Velocity (v) :** The rate of change of displacement with time in a particular direction is called as velocity.
- **Uniform velocity :** If the velocity of a particle is constant in magnitude and direction, then it is called as uniform velocity.
- **Acceleration (a) :** The rate of change of velocity with respect to time in a given direction is called as acceleration.
- **Uniform acceleration (a) :** If acceleration of a particle is uniform in magnitude and direction with respect to time, then it is called as uniform acceleration.
- **Retardation :** The negative acceleration is called as retardation. If rate of change of velocity is negative, then it is called as retardation. e.g. when brakes are applied, velocity decreases and retardation takes place.

- Three equations of motion are

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

- For freely falling body, substitute a = g in the above equation and for body moving vertically upward, substitute a = – g in the above equation.
- The distance travelled by a body in particular n^{th} second is given by, $s_{n^{th}} = u + \frac{a}{2}(2n - 1)$

Important Points

KINETICS

- **Kinetics :** It is the branch of dynamics which deals with effect of forces on bodies in motion, by considering the forces causing the motion.
- **Momentum :** It is the quantity of motion possessed by a moving body. Momentum = Mass × Velocity,

  S.I. unit – kg-m/s or Ns.
- **Impulse :** It is the change in momentum.
- Impulse is also defined as the product of large force on a body and very small time for which force acts.

  SI unit – kg-m/s or Ns.
- **Impulsive force :** Impulsive force is the force which acts for short time and produces considerable change in momentum of a body. S.I. unit – N.
- **Newton's first law :** Every body in the universe continues to be in its state of rest or motion in straight line unless it is acted upon by some external agency.
- **Newton's second law :** The rate of change of momentum of a body is directly proportional to the net force acting on a body and takes place in the direction of force, F = m × a.
- **Newton's third law :** For every actiion there is an equal and opposite reaction.

  In case of two bodies connected to each other by a string passing over a pulley both the bodies have equal acceleration and tension on both sides of the string.
- **Law of conservation of momentum :** It states that the total momentum of system consisting of two or more colliding bodies before impact remains unchanged after impact, provided no external force acts. e.g. recoil of a gun.

ANGULAR MOTION AND SHM

- The angle through which the radius vector turns is called as angular displacement (θ).
- The rate of change of angular displacement with respect to time is called as angular velocity (ω).
- The rate of change of angular velocity is called as angular acceleration.
- Three equations of circular (angular) motion are

$$\omega = \omega_o + \alpha t$$

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$

- S.H.M. is defined as the periodic motion of a body in which the force is always directed towards the mean position and its magnitude is proportional to its displacement from the mean position.

2.2.13 Characteristics of Linear SHM

(1) Motion is periodic along a straight line.

(2) Force (acceleration) is directed towards the mean position.

(3) Force (acceleration) is directly proportional to its displacement from mean position.

(4) Velocity of a particle is maximum at centre and minimum at extreme position.

- Equations of SHM
  $y = r \sin \omega t$
  $y = a \sin \omega t$
  $y = a \sin (\omega t + \alpha)$ … general case
  $y = a \cos \omega t$ … particle starts from extreme position

Formulae

FOR RECTILINEAR MOTION

1. $v = u + at$ where, u = Initial velocity
2. $s = ut + \frac{1}{2}at^2$ v = Final velocity
3. $v^2 = u^2 + 2as$ t = Time

a = Acceleration

s = Distance covered.

FOR FREELY FALLING BODY DUE TO GRAVITY

4. $v = u + gt$
5. $s = ut + \frac{1}{2}gt^2$
6. $v^2 = u^2 + 2gs$

FOR BODY MOVING VERTICALLY UPWARDS AGAINST GRAVITY

7. $v = u - gt$ where, g = Gravitational acceleration
8. $s = ut - \frac{1}{2}gt^2$ $= 9.81\ m/s^2$ (for earth) (**Important :** For simplicity of calculations, take $g = 10\ m/s^2$)
9. $v^2 = u^2 - 2gs$ u, v, s, t ... have usual meanings as given above.

DISTANCE TRAVELLED IN n^{th} SECONDS

10. Distance travelled in n^{th} sec $= u + \frac{a}{2}(2n - 1)$ ... for rectilinear motion.
11. Distance travelled in n^{th} sec $= u + \frac{g}{2}(2n - 1)$ ... for freely falling body.
12. Distance travelled in n^{th} sec $= u - \frac{g}{2}(2n - 1)$ ... for body moving upward.

Formulae

KINETICS

(1) Momentum = Mass × Velocity

$= m \times v$ where m = Mass

v = Velocity

(2) Impulse = Change in momentum

$= mv - mu$ where m = Mass of a body

$= $ Force × Time u = Initial velocity

$= F \times t$ v = Final velocity

F = Force

t = Time

(3) Impulsive force $= \frac{\text{Impulse}}{\text{Time}}$ = Mass × Acceleration

(4) Recoil velocity of a gun $= v_2 = \frac{m_1 v_1}{m_2}$ where m_1 = Mass of bullet

v_1 = Velocity of bullet

m_2 = Mass of gun

ANGULAR MOTION

1. $\omega = \frac{\theta}{t}$ where, ω = Angular velocity

   θ = Angular displacement

   t = Time

2. $T = \frac{2\pi}{\omega}$ T = Period

   $n = \frac{1}{T}$ n = Frequency

   $\therefore \quad n = \frac{\omega}{2\pi}$

   $\therefore \quad \omega = 2\pi n$

3. $v = r\omega = r\left(\frac{2\pi}{T}\right)$ v = Linear velocity, r = Radius vector

   $= r\,(2\pi n)$ α = Angular acceleration

4. $a = r\alpha$

5. $\alpha = \frac{\omega - \omega_o}{t}$ ω_o = Initial angular velocity

6. $\omega = \omega_o + \alpha t$ ω = Final angular velocity

7. $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ t = time

8. $\omega^2 = \omega_o^2 + 2\alpha\theta$ α = Angular acceleration, θ = Angular displacement

   a = Linear acceleration

9. Angular displacement covered in n^{th} second $\theta^{nth} = \omega_o + \frac{\alpha}{2}(2n - 1)$

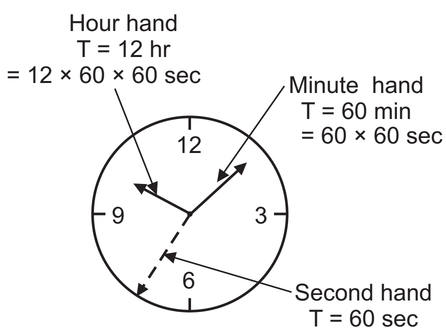


Fig. 2.18

Conversions

1. rpm i.e. rotations per minute; rps i.e. rotations per second;

   $\therefore \quad 1 \text{ rps} = 60 \text{ rpm}$

2. In one rotation, angular displacement $\theta = 360° = 2\pi$ radian

   $\pi \text{ radian} = 180°$

   $\therefore \quad 1° = \frac{\pi}{180}$ radian

3. $\omega = 1 \text{ rpm} = \frac{1 \times 2\pi}{60}$ rad/sec

SHM Formulae

- $y = r \sin \omega t = a \sin \omega t$ — y = Displacement of a particle in S.H.M; r = Radius of a circle; a = Amplitude; ω = Angular velocity
- $y = r \sin (\omega t + \alpha) = a \sin (\omega t + \alpha)$ — t = Time; α = Initial phase angle called epoch
- $v = \omega\sqrt{a^2 - y^2}$ and $v = a\omega$ (max) at mean position
- $a = \omega^2 y$ and $a = \omega^2 a$ (max) at extreme position

SOLVED EXAMPLES ON RECTILINEAR MOTION

Example 1 : *A car starts from rest. It accelerates for 10 seconds at the rate of 0.3 m/s². Determine its final velocity.*

Solution : Given : $u = 0, \quad t = 10 \text{ sec}, \quad a = 0.3 \text{ m/s}^2, \quad v = ?$

We have, $v = u + at = 0 + (0.3)(10)$

$$\boxed{v = 3 \text{ m/s}}$$

Example 2 : *A car starting from rest is moving with uniform acceleration. If it gains a velocity of 54 km/hr in 15 seconds, find its acceleration, total distance covered in 15 seconds and distance travelled in 7th second.*

Solution : Given : $u = 0, \; v = 54 \text{ km/hr} = \frac{54 \times 1000}{(60 \times 60)} = 15 \text{ m/s}, \; t = 15 \text{ seconds}, \; a = ?$

Distance covered in 15 sec, $s = ?$

Distance travelled in 7th sec. $= s_{7^{th}} = ?$

We have, $v = u + at$

$\therefore$ $a = \frac{v - u}{t} = \frac{15 - 0}{15}$

$$\boxed{a = 1 \text{ m/s}^2}$$

Now, $s = ut + \frac{1}{2}at^2 = (0 \times 15) + \frac{1}{2}(1) \times (15)^2 = 0 + \frac{225}{2}$

$$\boxed{s = 112.5 \text{ m}}$$

We have, distance travelled in nth sec

$$s_{n^{th}} = u + \frac{a}{2}(2n - 1)$$

$\therefore$ $s_{7^{th}} = 0 + \frac{(1)}{2}(2(7) - 1)$

$$\boxed{s_{7^{th}} = 6.5 \text{ m}}$$

Example 3 : *An object comes to rest from a velocity of 40 m/s in a distance of 8 m. If the acceleration is constant, calculate its value.*

Solution : Given : $u = 40 \text{ m/s}, \quad v = 0, \quad s = 8 \text{ m}, \quad a = ?$

We have, $v^2 = u^2 + 2as$

$\therefore$ $v^2 - u^2 = 2as$

$$\frac{v^2 - u^2}{2(s)} = a$$

$\therefore$ $a = \frac{0 - (40)^2}{2(8)}$

$$\boxed{a = -100 \text{ m/s}^2}$$

Example 4 : *A car has initial velocity of 3 m/s. It accelerates for 12 seconds at the rate of 3.5 m/s². Determine the final velocity and the distance travelled during this time.*

Solution : Given : $u = 3$ m/s, $t = 12$ sec., $a = 3.5$ m/s², $v = ?$, $s = ?$ in time 12 sec.

We have, $v = u + at = 3 + (3.5)(12)$

$$\boxed{v = 45 \text{ m/s}}$$

Now, $s = ut + \frac{1}{2}at^2 = (3)(12) + \frac{1}{2}(3.5)(12)^2 = 36 + 252$

$$\boxed{s = 288 \text{ m}}$$

Example 5 : *A train crosses a tunnel in 20 sec. At the entry of the tunnel, velocity is 72 km/hr and at the exit of the tunnel, velocity is 36 km/hr. Find the length of the tunnel.*

Solution : Given : $t = 20$ sec., $u = 72 \text{ km/hr} = \frac{72 \times 1000}{(60 \times 60)} = 20$ m/s, $v = 36 \text{ km/hr} = \frac{36 \times 1000}{(60 \times 60)} = 10$ m/s

Length of a tunnel = Distance covered = s = ?

First find acceleration 'a'.

We have, $a = \frac{v - u}{t} = \frac{10 - 20}{20} = -0.5 \text{ m/s}^2$

Now, $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$\therefore \quad \frac{v^2 - u^2}{2a} = s$

$\therefore \quad s = \frac{10^2 - 20^2}{2(-0.5)}$

$\boxed{s = 300 \text{ m}}$ ... length of the tunnel.

Example 6 : *A train man driving the train at 90 km/hr observes the red signal and applies the brakes. If the retardation is 6 m/s², find the distance covered in bringing the train to rest after applying brakes.*

Solution : Given : $u = 90 \text{ km/hr} = \frac{90 \times 1000}{(60 \times 60)} = 25$ m/s

Retardation = 6 m/s²

i.e. $a = -6 \text{ m/s}^2$

$v = 0$

$s = ?$

We have, $v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$\therefore \quad \frac{v^2 - u^2}{2a} = s$

$\therefore \quad s = \frac{(0)^2 - (25)^2}{2(-6)} = \frac{-625}{-12}$

$$\boxed{s = 52.08 \text{ m}}$$

Example 7 : *A vehicle covers 56 m in 4th second and 90 m in 7th second during its motion. Calculate the acceleration and distance travelled in 18th second.*

Solution : Given : $s_{4^{th}} = 56$ m, $s_{7^{th}} = 90$ m, $a = ?$, $s_{18^{th}} = ?$

We have, $s_{n^{th}} = u + \frac{a}{2}(2n - 1)$ ... (1)

Substituting n = 4 in equation (1),

$$s_{4^{th}} = u + \frac{a}{2}(2 \times 4 - 1) \qquad \text{but } s_{4^{th}} = 56 \text{ m}$$

$$\therefore \quad 56 = u + \frac{a}{2}(8 - 1)$$

$$56 = u + \frac{7}{2}a \qquad \text{... (a)}$$

Substituting n = 7 in equation (1),

$$\therefore \quad s_{7^{th}} = u + \frac{a}{2}(2 \times 7 - 1) = u + \frac{a}{2}(13) \qquad \text{but } s_{7^{th}} = 90 \text{ m}$$

$$\therefore \quad 90 = u + \frac{13}{2}a \qquad \text{... (b)}$$

Now, subtracting equation (b) from equation (a) gives,

$$(90 - 56) = 0 + \left(\frac{13}{2}a\right) - \left(\frac{7}{2}a\right)$$

$$34 = a\left(\frac{13}{2} - \frac{7}{2}\right) = a\left(\frac{6}{2}\right)$$

$$34 = 3a$$

$$\therefore \quad a = \frac{34}{3}$$

$$\boxed{a = 11.33 \text{ m/s}^2}$$

Substituting the value of a in equation (a),

$$\therefore \quad 56 = u + \frac{7}{2}(11.33)$$

$$\therefore \quad 56 = u + 39.655$$

$$\therefore \quad u = (56 - 39.655)$$

$$\therefore \quad \boxed{u = 16.345 \text{ m/s}}$$

Now, for $s_{18^{th}}$, substituting n = 18 in equation (1),

$$\therefore \quad s_{18^{th}} = u + \frac{a}{2}(2n - 1) = 16.345 + \frac{11.33}{2}(2 \times 18 - 1) = 16.345 + 198.275$$

$$\boxed{s_{18^{th}} = 214.62 \text{ m}}$$

Example 8 : *Two vehicles A and B are moving in the same direction at a speed of 54 km/hr. But car B is ahead of car A by 300 m. If vehicle A is accelerated by 3 m/s² and vehicle B has same speed as that of earlier, find at what distance vehicles A and B meet each other ?*

Solution : Initial speed of both vehicles = u = 54 km/hr = 15 m/s.

Vehicle B is ahead of A by 300 m. Let them meet at a distance 'x' from B.

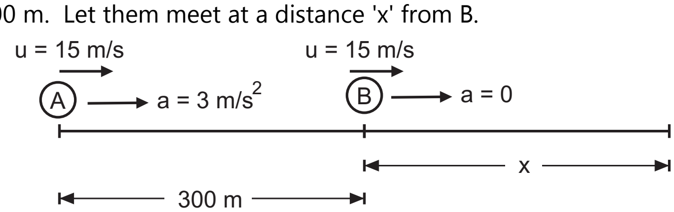


Fig. 2.19

Here both vehicles will meet each other after certain period 't'. Hence, for both vehicles, time is 't'.

Consider vehicle 'B',

Since 'B' is moving with uniform velocity, we can use

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time}}$$

$$15 = \frac{x}{t}$$

$$\therefore \quad t = \frac{x}{15} \quad \ldots (1)$$

Now, consider vehicle 'A', $s = (300 + x)$, $u = 15$ m/s, $t = \text{time} = \frac{x}{15}$, $a = 3$ m/s^2

$$s = ut + \frac{1}{2}at^2 \qquad \text{but} \quad s = (300 + x)$$

$$(300 + x) = (15)\,t + \frac{1}{2}(3)\,t^2$$

Substituting $t = \frac{x}{15}$ from equation (1),

$$\therefore \quad (300 + x) = 15\left(\frac{x}{15}\right) + \frac{1}{2}(3)\left(\frac{x}{15}\right)^2$$

$$\therefore \quad (300 + x) = x + 1.5\frac{x^2}{225}$$

$$\therefore \quad \frac{300 \times 225}{1.5} = x^2$$

$$45000 = x^2$$

$$\therefore \quad \boxed{x = 212.13 \text{ m}}$$

∵ Vehicles A and B will meet each other at a distance of 212.13 m from vehicle 'B'.

Example 9 : *A body is thrown vertically upwards with an initial velocity of 30 m/s. Find the maximum height reached by the body.*

Solution : Given : $u = 30$ m/s, $s = ?$

When a body covers maximum height, then $v = 0$ and the body falls back.

We have, $v^2 = u^2 - 2gs$ (∵ body is thrown up against gravity)

$$\therefore \quad 2gs = u^2 - v^2$$

$$\therefore \quad s = \frac{u^2 - v^2}{2g} = \frac{30^2 - 0^2}{2\,(9.81)}$$

$$\boxed{s = 45.87 \text{ m}} \quad \ldots \text{maximum height covered}$$

Example 10 : *A body is allowed to fall from the terrace of a building 200 m high. After what time will it reach the ground ? What will be its velocity at that time ?*

Solution : Given : $u = 0$ (∵ free falling body)

$v = ?$, $s = 200$ m, $t = ?$

We have, $s = ut + \frac{1}{2}gt^2$ (∵ body is falling down)

$$200 = 0 + \frac{1}{2}(9.81)\,t^2$$

$$\therefore \quad \frac{200 \times 2}{9.81} = t^2$$

$$t^2 = 40.77$$

$$\therefore \quad \boxed{t = 6.385 \text{ sec.}}$$

Now, $v = u + gt$

$v = 0 + (9.81)(6.385)$

$\therefore$ $\boxed{v = 62.64 \text{ m/s}}$

Example 11 : *A ball is thrown vertically up. It falls back to ground (same spot) after 3 sec. Find the height reached by it.*

Solution : Let, Height = Distance = s = ?

Total distance covered by a ball = 2s in time 3 sec.

$\therefore$ Time required to cover distance 's' = 1.5 sec.

Now, consider the downward motion.

$u = 0, \quad t = 1.5 \text{ sec}, \quad s = ?$

$$s = ut + \frac{1}{2} gt^2 \qquad \text{... } (\because \text{ falling down towards gravity})$$

$$s = 0 + \frac{1}{2}(9.81)(1.5)^2$$

$$\boxed{s = 11.04 \text{ m}} \qquad \text{... height reached.}$$

Example 12 : *Vijay is standing on the terrace of the building which is 50 m in height. Jay is standing on the ground near the building. Vijay releases a ball freely from the terrace and at the same instant Jay throws another ball vertically upward with a velocity of 20 m/s. At what height from the ground will these balls cross each other ?*

Solution : Given : Height of the building, $h = 50$ m.

The balls will cross each other after the same time 't' because they are thrown at the same instant.

$h_2 = ?$

We have
$$s = ut + \frac{1}{2} at^2 \qquad \text{... (1)}$$

Case (1) : (Vijay) Equation (1) becomes

$$h_1 = 0 + \frac{1}{2} gt^2 \qquad \text{... (2)}$$

$u = 0$ because ball is freely released.

Case (2) : (Jay) Equation (1) becomes

$$h_2 = ut - \frac{1}{2} gt^2$$

$$h_2 = (20 \times t) - \frac{1}{2} gt^2 \qquad \text{... (3)}$$

Equation (2) + Equation (3) gives

$h_1 + h_2 = 20\,t$

But $h_1 + h_2 = h = 50$ m

$\therefore$ $50 = 20 \times t$

$\therefore$ $t = \frac{50}{20}$

$\boxed{t = 2.5 \text{ sec.}}$

Put this value in Equation (3). $h_2 = (20 \times t) - \left(\frac{1}{2} gt^2\right) = (20 \times 2.5) - \left(\frac{1}{2} \times 9.81 \times (2.5)^2\right)$

$\boxed{h_2 = 19.34 \text{ m}}$

i.e. Balls will cross each other at height 19.34 m from the ground level.

Example 13 : *A car is moving with initial velocity 30 m/s. Then brakes are applied and car receives an acceleration of – 2 m/s². How far will it have gone ?*

(i) When its velocity has decreased to 15 m/s ? (ii) When it comes to rest ?

Solution : Given : $u = 30$ m/s, $a = -2$ m/s², $s = ?$

(i) $v = 15$ m/s

We have, $v^2 = u^2 + 2as$

$$\therefore \quad s = \frac{v^2 - u^2}{2a} = \frac{(15)^2 - (30)^2}{2(-2)} \quad \therefore \quad \boxed{s = 68.75 \text{ m}}$$

(ii) $v = 0$

$$\therefore \quad s = \frac{v^2 - u^2}{2a} = \frac{0 - (30)^2}{2(-2)} \quad \therefore \quad \boxed{s = 225 \text{ m}}$$

SOLVED EXAMPLES ON KINETICS

Example 14 : *A two wheeler vehicle of mass 200 kg has a velocity of 16 m/s. Find the momentum.*

Solution : Given : m = 200 kg, v = 16 m/s

Momentum = ?

We have, Momentum = mass × velocity = (200) × (16)

$$\boxed{\text{Momentum} = 3200 \text{ kg m/s or N sec}}$$

Example 15 : *A car of mass 1200 kg is moving with a velocity of 70 km/hr. Find the momentum of a car.*

Solution : Given : m = 1200 kg

$$v = 70 \text{ km/hr} = \frac{70 \times 1000}{(60 \times 60)} \text{ m/s} = 19.44 \text{ m/s}$$

Momentum = m × v = (1200) (19.44)

$$\boxed{\text{Momentum} = 23328 \text{ kg-m/s or N-sec}}$$

Example 16 : *Find the momentum of the train moving at 100 km/hr, if its weight is 3000 kN.*

Solution : Given :

$$v = 100 \text{ km/hr} = 100 \times \frac{1000}{60 \times 60} \text{ m/s} = 27.78 \text{ m/s}$$

Weight = W = 3000 kN = 3000×10^3 N

$$\therefore \quad \text{Mass} = \frac{W}{g} = \frac{3000 \times 10^3}{9.81} = 305.81 \times 10^3 \text{ kg}$$

We have, momentum = mass × velocity = m × v = (305.81×10^3) (27.78)

$$\boxed{\text{Momentum} = 8495.4 \times 10^3 \text{ kg-m/s or N-sec}}$$

EXAMPLES ON IMPULSE AND IMPULSIVE FORCE

Example 17 : *If a body of mass 100 kg changes velocity of 20 m/s to 5 m/s, calculate the impulse acting on the body.*

Solution : Given : m = 100 kg, u = 20 m/s, v = 5 m/s

Impulse = ?

Impulse = mv – mu = m (v – u) = 100 (5 – 20)

$$\boxed{\text{Impulse} = -1500 \text{ kg-m/s or N-sec}}$$

(–ve sign indicates that impulse is in the opposite direction of motion.)

Example 18 : *A ball of mass 250 gm rolls with a velocity of 150 cm/s. It is hitted with a bat in the direction of its motion. The velocity changes to 18 m/s. If the bat is in contact with a ball for 0.015 sec, find the impulse and the impulsive force.*

Solution : Given : m = 250 gm = 0.25 kg, u = 150 cm/s = 1.5 m/s, v = 18 m/s, t = 0.015 sec.

We have, impulse $= mv - mu = m(v - u) = 0.25\ (18 - 1.5)$

$$\boxed{\text{Impulse} = 4.125 \text{ kg m/s or N sec}}$$

$$\text{Impulsive force} = \frac{\text{Impulse}}{\text{Time}} = \frac{4.125}{0.015}$$

$$\boxed{\text{Impulsive force} = 275 \text{ N}}$$

Example 19 : *A ball of mass 200 gm has initial velocity of 30 m/s. After hitting a bat, its velocity becomes 45 m/s in the opposite direction. If a ball remains in contact with a bat for 0.006 second, find the impulse and the impulsive force.*

Solution : Given : m = 200 gm = 0.2 kg, u = 30 m/s, v = –45 m/s, t = 0.006 sec, Impulse = ?, Impulsive force = ?

$$\text{Impulse} = mv - mu = m(v - u) = 0.2\ (-45 - 30)$$

$$\boxed{\text{Impulse} = -15 \text{ kg-m/s or N-sec}}$$

(–ve sign indicates that the impulse acting on a body is in the opposite direction of motion.)

$$\text{Impulsive force} = \frac{\text{Impulse}}{\text{Time}} = \frac{15}{0.006}$$

$$\boxed{\text{Impulsive force} = 2500 \text{ N}}$$

Example 20 : *A bullet of mass 450 gm leaves the barrel of the gun with a muzzle velocity of 700 m/s. If the length of the barrel is 80 cm, find the impulse and the impulsive force.*

Solution : Given : m = 450 gm = 0.45 kg, u = 0, v = 700 m/s

Length of the barrel i.e. distance, s = 80 cm = 0.80 m.

We have, Impulse $= m(v - u) = 0.45\ (700 - 0)$

$$\boxed{\text{Impulse} = 315 \text{ kg-m/s or N-sec}}$$

We have,

$$v^2 = u^2 + 2as$$

$$700^2 = 0 + 2\,(a)\,(0.80)$$

$$\frac{700^2}{2 \times 0.80} = a$$

$$\therefore \quad a = 306250 \text{ m/s}^2$$

$$\text{Impulsive force} = \text{mass} \times \text{acceleration} = (0.45)\,(306250)$$

$$\boxed{\text{Impulsive force} = 137812.5 \text{ N}}$$

Example 21 : *If a body of mass 200 kg changes its velocity from 40 m/s to 10 m/s, calculate impulse acting on a body.*

Solution : Impulse is defined as the change in momentum.

Given :

$$m = 200 \text{ kg}$$

$$u = 40 \text{ m/s}$$

$$v = 10 \text{ m/s}$$

$$\text{Impulse} = mv - mu = m(v - u) = 200\ (10 - 40)$$

$$\boxed{\text{Impulse} = -6000 \text{ N-s}}$$

–ve sign indicates that impulse acting on a body is in opposite direction of motion.

Example 22 : *A bullet of weight 1 N is fired with a velocity of 500 m/s horizontally in a wooden block weighing 50 N resting on horizontal surface. If the bullet remains embedded in the block, calculate the velocity of the block after impact.*

Solution :

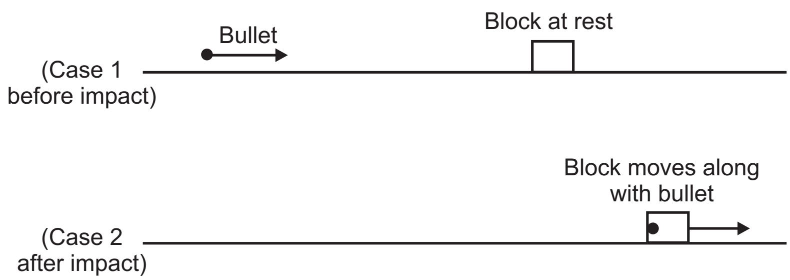


Fig. 2.20

Given :

	Bullet	**Block**
	$W_1 = 1$ N	$W_2 = 50$ N
$\therefore$	$m_1 = \frac{W_1}{9.8} = \frac{1}{9.8}$ kg	$\therefore$ $m_2 = \frac{W_2}{9.8} = \frac{50}{9.8}$ kg
$\therefore$	$m_1 = 0.102$ kg	$\therefore$ $m_2 = 5.1$ kg
Initial velocity,	$u_1 = 500$ m/s	Initial velocity, $u_2 = 0$ ($\because$ block at rest)
Final velocity,	$v_1 = ?$	$v_2 = ?$
But	$v_1 = v_2$	

Because bullet remains embedded in the block, it moves along with bullet with velocity v_2.

We have, $\begin{bmatrix} \text{Total momentum} \\ \text{before impact} \end{bmatrix}$ = Total momentum after impact

$$(m_1u_1 + m_2u_2) = (m_1v_1 + m_2v_2) \quad \text{but } v_1 = v_2$$

$$(m_1u_1 + m_2u_2) = (m_1v_2 + m_2v_2)$$

$$(m_1u_1 + m_2u_2) = v_2 (m_1 + m_2)$$

$$\frac{(m_1u_1 + m_2u_2)}{(m_1 + m_2)} = v_2$$

$$\therefore \quad v_2 = \frac{[(0.102 \times 500) + (5.1 \times 0)]}{(0.102 + 5.1)}$$

$$\therefore \quad v_2 = \frac{51}{5.202} \quad \therefore \quad \boxed{v_2 = 9.8 \text{ m/s}}$$

SOLVED EXAMPLES ON RECOIL VELOCITY OF A GUN

Example 23 : *A bullet of mass 100 gm is fired with a velocity of 400 m/s from a gun of mass 10 kg. Find the velocity with which the gun will recoil (react).*

Solution : Given :

Mass of bullet, m_1 = 100 gm = 0.1 kg

Velocity of bullet, v_1 = 400 m/s

Mass of a gun, m_2 = 10 kg

Velocity of a gun, v_2 = ?

Forward momentum of bullet = Backward momentum of gun

$$m_1v_1 = m_2v_2$$

$$\therefore \quad \frac{m_1v_1}{m_2} = v_2$$

$$\therefore \quad v_2 = \frac{m_1v_1}{m_2} = \frac{(0.1)\ (400)}{(10)}$$

$$\boxed{v_2 = 4 \text{ m/s}}$$ ... recoil velocity of a gun

Example 24 : *The car A of mass 1200 kg travelling at 25 m/s collides with another car B of mass 1000 kg travelling at a speed of 15 m/s in the same direction. After collision the velocity of car A becomes 20 m/s. Calculate the velocity of car B after collision.*

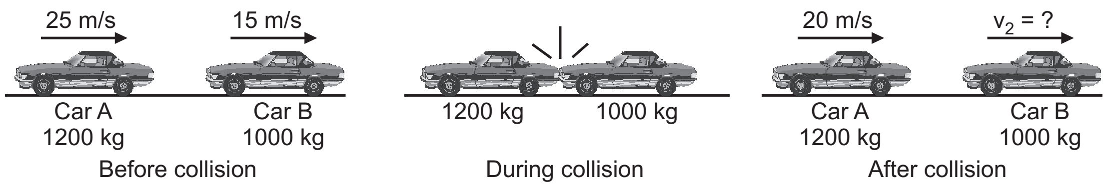


Fig. 2.21

Solution : Total momentum before collision = Total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$(1200 \times 25) + (1000 \times 15) = (1200 \times 20) + (1000 \times v_2)$$

$$\therefore \quad (30000) + (15000) = (24000) + (1000 \times v_2)$$

$$30000 + 15000 - 24000 = 1000\, v_2$$

$$21000 = 1000\, v_2$$

$$\therefore \quad \boxed{v_2 = 21 \text{ m/s}}$$

Example 25 : *A bullet of mass 20 gm moving with velocity of 200 m/s gets embedded in a wooden block of mass 800 gm, which was at rest. What is the velocity of acquired block ?*

Solution :

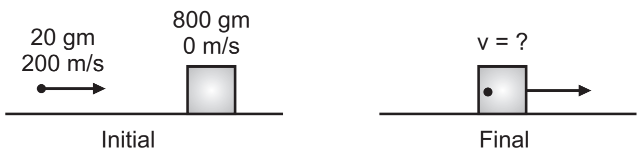


Fig. 2.22

Initial total momentum :

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{but } v_1 = v_2 = \text{say v because bullet remains embedded}$$

$$m_1u_1 + m_2u_2 = m_1v + m_2v$$

$$m_1u_1 + m_2u_2 = (m_1 + m_2) \times v$$

$$\therefore \quad \frac{[(m_1u_1) + (m_2u_2)]}{(m_1 + m_2)} = v$$

$$\frac{[(0.02 \times 200) + (0.8 \times 0)]}{(0.02 + 0.8)} = v$$

$$\therefore \quad v = \frac{4 + 0}{0.82}$$

$$\boxed{v = 4.88 \text{ m/s}} \quad \text{final speed of block along with bullet}$$

SOLVED EXAMPLES ON ANGULAR MOTION

Example 26 : *A fly wheel is rotating at 90 r.p.m. Calculate its angular velocity in radian/sec and degree/sec.*

Solution : Given : $90 \text{ r.p.m.} = \frac{90}{60} \text{ r.p.s.} = 1.5 \text{ r.p.s.}$

We know that in 1 rotation, it covers angle of 2π radians.

$\therefore$ In 1.5 rotations,

$$\text{angle covered} = 1.5 \times 2\pi = 3\pi$$

i.e. Angular velocity = 3π radians/sec

$$\therefore \quad \boxed{\text{Angular velocity} = 9.43 \text{ radians/s}}$$

and $\therefore$ Angular velocity = 3Hp radians/sec = 3×180

$$\boxed{\text{Angular velocity} = 540 \text{ degree/sec}}$$

Example 27 : *A fan rotates with a constant angular acceleration of 60 degree/sec^2. Calculate angular acceleration in rpm^2 and in radian/sec^2.*

Solution : Given : $\alpha = 60$ degree/sec^2

We have, $1° = \dfrac{\pi}{180}$ radians

$\therefore$ $60° = 60 \times \dfrac{\pi}{180} = \boxed{1.047 \text{ rad/sec}^2}$

and 1 revolution = 2π radians = 360°

i.e. 360° = 1 rotation

$1° = \dfrac{1}{360}$ rotations

Given : $60° = \dfrac{60}{360}$ rotations per sec^2

$$= \frac{60}{360} \cdot \frac{\text{rotations}}{\text{sec}^2} \qquad \left(\because 1 \text{ sec} = \frac{1}{60} \text{ min}\right)$$

$$= \frac{60}{360} \frac{\text{rotations}}{\left(\frac{1}{60}\right)^2 \text{ minutes}^2} = \frac{60 \times (60)^2}{360}$$

$$\boxed{\alpha = 600 \text{ rotations/min}^2}$$

Example 28 : *In case of uniform circular motion, if radius vector subtends an angle of $\pi/3$ radians in 3 seconds, calculate the angular velocity.*

Solution : Given : $\theta = \dfrac{\pi}{3}$ radians, $t = 3$ seconds, $\omega = ?$

We have, $\omega = \dfrac{\theta}{t} = \dfrac{\pi/3}{3}$

$$\boxed{\omega = \frac{\pi}{9} \text{ rad/s}}$$

Example 29 : *Calculate the angular velocity with which earth spins about its own axis.*

Solution : We have, T = Period = Time required for earth to complete one spin about it

$= 24 \times 60 \times 60$ sec = 24 hrs

Also, we have, $T = \dfrac{2\pi}{\omega}$

$\therefore$ $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{24 \times 60 \times 60}$

$$\boxed{\omega = \frac{\pi}{43200} \text{ rad/sec}}$$

Example 30 : *The second hand of a clock is 6 cm long. Calculate the linear speed of ant sitting at its tip.*

Solution : Given : $r = 6$ cm $= 0.06$ m, $T = 60$ sec ($\because$ Second hand completes one rotation in 60 sec.)

$v = ?$

We have, $v = r\omega \quad \left(\text{but } \omega = \dfrac{2\pi}{T}\right)$

$$= r\left(\frac{2\pi}{T}\right) = 0.06 \times \left(\frac{2 \times 3.142}{60}\right)$$

$$\boxed{v = 6.284 \times 10^{-3} \text{ m/s}}$$

Example 31 : *The frequency of rotation of a fan changes from 3 rev/sec to 5 rev/sec in 5 seconds. Find the angular acceleration.*

Solution : Given : $n_o = 3$ rev/sec, $n_1 = 5$ rev/sec, $t = 5$ sec, $\alpha = ?$

We have, $$\alpha = \frac{\omega - \omega_o}{t} \quad (\text{but } \omega = 2\pi n)$$

$$\therefore \quad \alpha = \frac{2\pi n - 2\pi n_o}{t} = \frac{[(2 \times \pi \times 5) - (2 \times \pi \times 3)]}{5}$$

$$\boxed{\alpha = 2.513 \text{ rad/sec}^2}$$

Example 32 : *A fly wheel starting from rest attains a speed of 1500 rpm in 1 minute. Calculate the angular acceleration.*

Solution : Given : $\omega_o = 0$, $\omega = 1500 \text{ rpm} = \frac{1500 \times 2\pi}{60} = 157.1$ rad/sec, $t = 1$ min $= 60$ sec, $\alpha = ?$

We have, $$\omega = \omega_o + \alpha t$$

$$\therefore \quad \omega - \omega_o = \alpha t$$

$$\therefore \quad \frac{(\omega - \omega_o)}{t} = \alpha$$

$$\therefore \quad \alpha = \frac{157.1 - 0}{60}$$

$$\boxed{\alpha = 2.62 \text{ rad/sec}^2}$$

Example 33 : *A fly wheel is rotating at 1200 r.p.m. It is brought to rest in 50 revolutions. Calculate the uniform retardation.*

Solution : Given : $\omega_o = 1200 \text{ r.p.m.} = \frac{1200 \times 2\pi}{60} = 125.66$ rad/sec

$\omega = 0$ ($\because$ brought to rest)

Flywheel is brought to rest in 50 revolutions.

$$\theta = 50 \text{ rev.}$$

$$\therefore \quad \theta = 50 \times 2\pi = 314.16 \text{ rad}$$

Now, using the relation, $$\omega^2 = \omega_o^2 + 2\,\alpha\,\theta$$

$$\therefore \quad \omega^2 - \omega_o^2 = 2\alpha\theta$$

$$\therefore \quad \frac{\omega^2 - \omega_o^2}{2\theta} = \alpha$$

$$\therefore \quad \alpha = \frac{0 - (125.66)^2}{2\,(314.16)}$$

$$\boxed{\alpha = -25.13 \text{ rad/sec}^2} \quad \text{... (acceleration)}$$

$$\therefore \quad \boxed{\text{Retardation} = 25.13 \text{ rad/sec}^2}$$

Example 34 : *A fly wheel rotating at 800 r.p.m. accelerates to 2000 r.p.m. in 10 minutes. Calculate the uniform acceleration and the angular displacement within the given period.*

Solution : Given : $\omega_o = 800 \text{ r.p.m.} = 800 \times \frac{2\pi}{60} \text{ rad/sec} = 83.78$ rad/sec

$$\omega = 2000 \text{ r.p.m.} = 2000 \times \frac{2\pi}{60} \text{ rad/sec} = 209.44 \text{ rad/sec}$$

$$t = 10 \text{ min} = 10 \times 60 = 600 \text{ sec}$$

We have, $$\omega = \omega_o + \alpha t$$

$$\therefore \quad \alpha = \frac{(\omega - \omega_o)}{t} = \frac{(209.44 - 83.78)}{600}$$

$$\boxed{\alpha = 0.209 \text{ rad/sec}^2}$$

Now, angular displacement in time t = 600 sec.

$$\theta = \omega_o t + \frac{1}{2}\alpha t^2 = (83.78 \times 600) + \frac{1}{2}(0.209)(600)^2$$

$$\boxed{\theta = 87888 \text{ radians}}$$

Example 35 : *A fly wheel starting from rest is subjected to an acceleration of 0.5 rad/sec^2. Calculate its angular displacement in 6th second.*

Solution : Given : $\omega_o = 0, \quad \alpha = 0.5 \text{ rad/sec}^2, \quad \theta_{6^{th}} = ?$

We have, $$\theta_{n^{th}} = \omega_o + \frac{\alpha}{2}(2n - 1) \quad \text{put } n = 6$$

$\therefore$ $$\theta_{6^{th}} = \omega_o + \frac{\alpha}{2}(2 \times 6 - 1) = 0 + \frac{0.5}{2}(11)$$

$$\boxed{\theta_{6^{th}} = 2.75 \text{ radians}}$$

Example 36 : *A motor cycle with 60 cm wheel diameter has angular velocity of 30 rad/sec. Calculate its linear velocity.*

Solution : Diameter = 60 cm, r = 30 cm = 0.30 m, ω = 30 rad/s, v = ?

$$v = r\omega = (0.30)(30)$$

$$\boxed{v = 9 \text{ m/s}}$$

Example 37 : *Angular acceleration of a cycle is 5 rad/sec^2, where its diameter of wheel is 70 cm. Calculate its linear acceleration.*

Solution : Given : α = 5 rad/sec^2, Diameter = 70 cm, r = 35 cm = 0.35 m, a = ?

We have, $$a = r \times \alpha = (0.35) \times (5)$$

$$\boxed{a = 1.75 \text{ m/s}^2}$$

Example 38 : *A wheel 80 cm in diameter turns at 120 r.p.m.*

(i) What is angular velocity in rad/s ? (ii) What is linear velocity of a point on the rim of a wheel ?

Solution : Given : Diameter = 80 cm, r = 40 cm = 0.4 m, ω = 120 r.p.m.

(i) $$\omega = 120 \text{ r.p.m.} = \frac{(120) \times 2\pi}{60} \text{ rad/s}$$

$$\boxed{\omega = 12.57 \text{ rad/s}}$$

(ii) We have, $$v = r\omega = (0.4) \times (12.57)$$

$$\boxed{v = 5.028 \text{ m/s}}$$

Example 39 : *A body moves along a circular path of radius 60 cm at 3 revolutions per second. Calculate the linear speed.*

Solution : r = 60 cm = 0.6 m, n = 3 i.e. revolutions per sec., v = ?, a = ?

We have, $$v = r(\omega) = 2(2\pi n) = 0.6(2 \times 3.142 \times 3)$$

$$\boxed{v = 11.31 \text{ m/s}}$$

Example 40 : *A flywheel rotating at 300 r.p.m. is brought to rest in 40 revolutions. Calculate uniform retardation.*

Solution : Given : $$\omega_o = 300 \text{ r.p.m.} = \frac{300 \times 2\pi}{60} = 31.4 \text{ rad/s}, \ \omega = 0$$

Flywheel brought to rest in 40 revolutions.

$\therefore$ $$\theta = 40 \text{ rev} = 40 \times 2\pi = 251.2 \text{ rad}$$

We have, $\omega^2 = \omega_o^2 + 2\alpha\theta$

$$\omega^2 - \omega_o^2 = 2\alpha\theta$$

$$\therefore \quad \alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{0 - (31.4)^2}{2\ (251.2)}$$

$$\boxed{\alpha = -1.9625 \text{ rad/s}^2}$$

Example 41 : *A motor cycle with 60 cm wheel diameter has an angular velocity of 30 rad/sec. Calculate its linear velocity.*

Solution : Diameter = 60 cm, r = 30 cm = 0.30 m, ω = 30 rad/s, v = ?

$$v = r\omega = (0.30)\ (30)$$

$$\boxed{v = 9 \text{ m/s}}$$

Example 42 : *The angular acceleration of a cycle is 5 rad/sec^2, where the diameter of the wheel is 70 cm. Calculate its linear acceleration.*

Solution : Given : α = 5 rad/sec^2, Diameter = 70 cm, r = 35 cm = 0.35 m, a = ?

We have, $a = r \times \alpha = (0.35) \times (5)$

$$\boxed{a = 1.75 \text{ m/s}^2}$$

Example 43 : *A particle executing S.H.M. has a period of 2 sec. and an amplitude of 2 cm. Determine its displacement at the end of 1 sec.*

Solution : Given : T = 2 sec, a = 2 cm, t = 1 sec

$$y = a \sin \omega t \qquad \text{but } \omega = \frac{2\pi}{T}$$

$$\therefore \quad y = a \sin \left(\frac{2\pi}{T}\right) t = a \sin \left(\frac{2\pi}{2}\right) \times 1 = 2 \sin \pi$$

$\boxed{y = 0}$ i.e. particle is at mean position.

Example 44 : *A particle executing S.H.M. has a frequency of 30 cycles/min and amplitude of 5 cm. If the initial phase angle is zero, find the displacement after 4 sec.*

Solution : Given : 30 cycles/min.

$$\therefore \quad \text{Frequency} = n = \frac{30}{60} = 0.5 \text{ cycles/sec}$$

$$a = 5 \text{ cm}, \quad \alpha = 0$$

$$y = ? \text{ at } t = 4 \text{ sec.}$$

$$y = a \sin \omega t = a \sin (2\pi n)\ t$$

$$y = 5 \sin (2\pi \times 0.5)\ (4) = 5 \sin 4\pi$$

$\boxed{y = 0}$ i.e. mean position.

Example 45 : *A wheel 80 cm in diameter turns at 120 r.p.m.*

(i) What is the angular velocity in rad/s ? (ii) What is the linear velocity of a point on the rim of a wheel ?

Solution : Given : Diameter = 80 cm, r = 40 cm = 0.4 m, ω = 120 r.p.m.

(i) $$\omega = 120 \text{ r.p.m.} = \frac{(120) \times 2\pi}{60}$$

$$\boxed{\omega = 12.57 \text{ rad/s}}$$

(ii) We have, $v = r\omega = (0.4) \times (12.57)$

$$\boxed{v = 5.028 \text{ m/s}}$$

Example 46 : *A body moves along a circular path of radius 60 cm at 3 revolutions per second. Calculate the linear speed and acceleration of the body.*

Solution : r = 60 cm = 0.6 m, n = 3 i.e. revolutions per sec., v = ?, a = ?

We have, $v = r(\omega) = 2(2\pi n) = 0.6(2 \times 3.142 \times 3)$

$$\boxed{v = 11.31 \text{ m/s}}$$

$$a = \omega^2 r = (2\pi n)^2 \times r = (2 \times 3.142 \times 3)^2 \times 0.6$$

$$\boxed{a = 213.18 \text{ m/s}^2}$$

Example 47 : *Equation of S.H.M. is* $Y = 4 \sin 2\pi\left(t + \frac{\pi}{6}\right)$. *Calculate the acceleration at a distance of 2 m.*

Solution : We have, $$Y = a \sin(\omega t + \alpha) \quad \text{... (1)}$$

Given : $$Y = 4 \sin 2\pi\left(t + \frac{\pi}{6}\right) = 4 \sin\left(2\pi t + 2\pi \times \frac{\pi}{6}\right)$$

$$Y = 4 \sin\left(2\pi t + \frac{\pi^2}{3}\right) \quad \text{... (2)}$$

Comparing equations (1) and (2), we get

$$\omega = 2\pi$$

Acceleration at a distance x is given by $\omega^2 x$.

∴ Acceleration at a distance of 2 m = $(\omega)^2 \times (2) = (2\pi)^2 \times (2)$

$$\boxed{\text{Acceleration at 2 m} = 78.95 \text{ m/s}^2}$$

Example 48 : *A particle performs S.H.M. Its acceleration is 4.5 cm/sec^2, when it is at 8 cm from its mean position. Calculate its time period.*

Solution : Given : a = 4.5 cm/sec^2, α = 8 cm, T = ?

∴ $$a = \omega^2 x$$

$$\omega^2 = \frac{a}{x} = \frac{4.5}{8} = 36$$

∴ $$\boxed{\omega = 6 \text{ rad/sec}}$$

$$T = \frac{2\pi}{\omega} = \frac{2 \times 3.14}{6}$$

$$\boxed{T = 1.046}$$

Example 49 : *Equation of SHM is : Y = 0.3 sin (2πt + π/3). State*

(i) amplitude, (ii) period, (iii) frequency, and (iv) epoch of SHM.

Solution : Given : $$Y = 0.3 \sin\left(2\pi t + \frac{\pi}{3}\right) \quad \text{... (1)}$$

We have equation of SHM $$Y = a \sin(\omega t + \alpha) \quad \text{... (2)}$$

Comparing equations (1) and (2), we get

(i) Amplitude $\boxed{a = 0.3 \text{ units}}$

(ii) Period = $T = \frac{2\pi}{\omega} = \frac{2\pi}{(2\pi)}$

$$\boxed{T = 1 \text{ sec}}$$

(iii) Frequency, $n = \frac{\omega}{2\pi} = \frac{(2\pi)}{2\pi}$

$$\boxed{n = 1 \text{ Hz}}$$

(iv) Epoch of SHM = $\alpha = \frac{\pi}{3}$.

Example 50 : *A wheel of diameter 3 m increases its speed uniformly from 150 r.p.m. to 300 r.p.m. in 30 seconds. Calculate : (i) Angular acceleration. (ii) Linear acceleration.*

Solution : dia = 3 m, r = 1.5 m, ω_1 = 150 r.p.m., $n_1 = \frac{150}{60}$ rps = 2.5 Hz, ω_2 = 300 rpm = $\frac{300}{60}$ rps = 5 Hz, t = 30 sec

(i) Angular acceleration (α) : $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{(2\pi n_2 - 2\pi n_1)}{t} = \frac{2\pi (n_2 - n_1)}{t} = \frac{2 \times 3.142 (5 - 2.5)}{30}$

$$\boxed{\alpha = 0.524 \text{ rad/s}^2}$$

(ii) Linear acceleration (a) : $a = r\alpha = (1.5)(0.524)$

$$\boxed{a = 0.786 \text{ m/s}^2}$$

Questions

1. Define angular displacement, angular velocity, angular acceleration. State the S.I. unit of each.
2. What is the relation between angular velocity and linear velocity ?
3. Define S.H.M. State the equation of S.H.M. giving the symbol meaning.
4. Represent displacement, velocity and acceleration of a particle in S.H.M. for a particle starting from the mean position.
5. Represent displacement, velocity and acceleration of a particle in S.H.M. for a particle starting from the extreme position.
6. Define angular displacement, angular velocity, angular acceleration. State S.I. unit of each.
7. What is the relation between angular velocity and linear velocity ?
8. State three equations of angular motion. Give meaning of each symbol.
9. State the equation for angular distance travelled by a particle in n^{th} seconds.

Practice Questions

1. Define kinetics and momentum. State unit of momentum.
2. Define impulse. State the S.I. unit of impulse.
3. Define impulsive force. State its S.I. unit.
4. State Newton's laws of motion.
5. State Newton's third law of motion. What is the recoil velocity of a gun ? State the formula for recoil velocity of a gun.
6. Define S.H.M. State the equation of S.H.M. giving the symbol meaning.
7. Represent displacement, velocity and acceleration of a particle in S.H.M. for a particle starting from the mean position.
8. Represent displacement, velocity and acceleration of a particle in S.H.M. for a particle starting from the extreme position.

BTE Questions

1. Define angular displacement and state its S.I. unit.
2. Show the graphical representation of velocity of S.H.M.
   (i) starting from mean position. (ii) starting from extreme position.
3. Define S.H.M. State the equations of circular motion. Give the meaning of each symbol.
4. State three equations of circular motion. Give the meaning of each symbol.
5. Derive the relation between angular velocity and linear velocity.
6. Show using graphs the displacement of particle in S.H.M. for
   (a) starting from mean position.
   (b) starting from extreme position.
7. Give graphical representation of displacement and velocity of S.H.M. starting from mean position.
8. Calculate the angular velocity of a second hand of a clock.
9. Define S.H.M. Obtain the equation of S.H.M. from the mean position.

Unsolved Problems

1. A vehicle starting from rest accelerates for 15 seconds at the rate of 0.6 m/s^2. Determine the final velocity.
(**Ans.** v = 9 m/s)
2. A body starting from rest is moving with uniform acceleration. If it gains a velocity of 72 km/hr in 10 seconds, find its acceleration and total distance covered in 10 seconds and the distance travelled in the 6^{th} second.
(**Ans.** a = 2 m/s^2, s = 100 m, $s_{6^{th}}$ = 11 m)
3. A car is moving with a velocity of 50 m/s. Then brakes are applied and brought it to rest. It covers 10 m distance during that period. Find the retardation value (it is constant). (**Ans.** a = –125 m/s^2, i.e. retardation = 125 m/s^2)
4. A train crosses a tunnel in 25 seconds. At the entry of the tunnel, its velocity is 54 km/hr and at the exit of the tunnel, the velocity is 72 km/hr. Find the length of the tunnel. (**Ans.** Length of the tunnel = s = 437.5 m)
5. A vehicle covers 60 m in 3^{rd} second and 100 m in 7^{th} second during its motion. Calculate the acceleration and distance travelled in 11^{th} second. (**Ans.** a = 10 m/s^2 and $s_{11^{th}}$ = 140 m)
6. Two cars A and B are 500 m apart from each other. Car B is ahead of car A. They are moving in the same direction and the same speed of 10 m/s. If vehicle 'A' is accelerated by 4 m/s^2 and vehicle 'B' is having same speed as earlier, find at what distance vehicles A and B will meet.
(**Ans.** s = 158.112 m i.e. they will meet at a distance of 158.112 m from car B)
7. A ball is thrown vertically up with an initial velocity of 25 m/s. Find the maximum height attained by the ball.
(**Ans.** Maximum height = s = 31.86 m)
8. A ball thrown vertically upward falls back to the ground (same spot) after 2 sec. Find the height attained by the ball.
(**Ans.** Height = s = 4.9 m)
9. Vijay is standing on the terrace of a building which is 30 m in height. Jay is standing on the ground near the same building. Vijay releases a ball freely from the terrace of a building and at the same instant Jay throws another ball vertically up from the ground with a velocity of 15 m/s. At what height from the ground will these balls cross each other ? (**Ans.** 10.38 m from the ground level)

Practice Problems

10. A body of mass 2.3 kg is moving with a velocity of 600 cm/s. Calculate the momentum possessed by it.
(**Ans.** Momentum = 13.8 kg-m/s or N-sec)
11. A car of mass 1500 kg is moving with a speed of 90 km/hr. Find the momentum of a car.
(**Ans.** Momentum = 37500 kg-m/s or N-sec)
12. Find the momentum of the train moving at 110 km/hr, if its weight is 3800 kN.
(**Ans.** Momentum = 11836×10^3 kg m/s or N sec)
13. If a body of mass 160 kg changes its velocity from 18 m/s to 9 m/s, calculate the impulse acting on a body.
(**Ans.** Impulse = –1440 kg m/s or N sec)
14. If a ball of mass 180 gm rolls with a velocity of 2 m/s, it is hitted with a bat in the direction of its motion. The velocity changes to 20 m/s. If a bat is in contact with a ball for 0.01 sec, find the impulse and the impulsive force.
(**Ans.** Impulse = 3.24 kg m/s or N sec, Impulsive force = 324 N)
15. A bullet of mass 30 gm leaves the barrel of a gun with muzzle velocity of 800 m/s. If the length of the barrel is 1 m, find the impulse and the impulsive force. (**Ans.** Impulse = 24 N sec, Impulsive force = 9600 N)
16. A bullet of mass 70 gm is fired with a muzzle velocity of 300 m/s from a gun of mass 7 kg. Calculate the recoil velocity of a gun. (**Ans.** Recoil velocity = 3 m/s)
17. The minute hand of a clock is 9 cm long. Calculate the linear speed of end tip of the minute hand.
(**Ans.** 1.571×10^{-4} m/s)
18. Convert the following :

	Ans.
(a) 150 rpm to rps	(2.5 rps)
(b) 2.4 revolutions to degree	(864°)
(c) 1.8 revolutions to radian	(11.31^c)
(d) 6 radians to degree	(343.77°)
(e) 70 degrees to radian	(1.22^c)

19. The frequency of rotation of fan changes from 2 rev/sec to 7 rev/sec in 10 seconds. Find the angular acceleration.
(**Ans.** α = 3.14 rad/sec^2)
20. A flywheel starting from rest attains a velocity of 2100 rpm in 1 minute. Calculate the angular acceleration.
(**Ans.** 3.67 rad/sec^2)
21. A fan is rotating at a speed of 900 rpm. If it is comes to rest in 70 revolutions, calculate the uniform retardation.
(**Ans.** Retardation = 10.099 rad/sec^2)
22. A disc rotating at 300 rpm accelerates to 1200 rpm in 5 minutes. Calculate the uniform acceleration and the angular displacement within the given period. (**Ans.** α = 0.314 rad/sec^2 and θ = 23554.78 radians)
23. A flywheel starting from rest is subjected to an acceleration of 0.7 rad/sec^2. Calculate its angular displacement in 5th second. (**Ans.** $\theta_{5^{th}}$ = 3.15 radians)
24. A scooter with 30 cm wheel diameter has angular velocity of 40 rad/sec. Calculate its linear velocity. (**Ans**. v = 6 m/s)
25. Angular acceleration of a vehicle is 9 rad/sec^2 where its diameter of wheel is 50 cm. Calculate its linear acceleration.
(**Ans.** a = 2.25 m/s^2)

MCQs on Rectilinear Motion

1. *A body is said to be in motion, if it …… its position w.r.t. …… with the passage of ……*
   (a) *keeps, surrounding, time* (b) *does not change, place, time*
   (c) *changes, surroundings, time* (d) *none of these*
2. *Every body at rest has tendency to remain at rest and a body in motion has tendency to remain in that motion is known as ……*
   (a) *law of inertia* (b) *Newton's 2nd law of motion*
   (c) *Newton's 3rd law of motion* (d) *none of these*
3. *Speed is a …… quantity and velocity is a …… quantity.*
   (a) *vector, scalar* (b) *scalar, vector* (c) *scalar, scalar* (d) *vector, vector*
4. *Velocity is given by ……*
   (a) *displacement $\times$ time* (b) *displacement + time* (c) *time/displacement* (d) *displacement/time*
5. *The rate of change of velocity w.r.t. time in a given direction is called as ……*
   (a) *acceleration* (b) *displacement* (c) *speed* (d) *velocity*
6. *The negative acceleration is called as ……*
   (a) *slow acceleration* (b) *retardation* (c) *uniform acceleration* (d) *gravitational acceleration*
7. *Acceleration is given by ……*
   (a) *time/change in velocity* (b) *change in velocity $\times$ time* (c) *change in velocity/time* (d) *change in velocity + time*
8. *With usual symbol meaning, the first equation of motion (kinematics) is given by ……*
   (a) $a = v + ut$ (b) $a = u + vt$ (c) $u = v + at$ (d) $v = u + at$
9. *With usual symbol meaning, second equation of motion (kinematics) is given by ……*
   (a) $s = ut + at^2$ (b) $s = ut + \frac{1}{2}at$ (c) $s = ut + \frac{1}{2}at^2$ (d) $s = ut + 2at^2$
10. *With usual symbol meaning, third equation of motion (kinematics) is given by ……*
   (a) $v^2 = u^2 + 2as$ (b) $u^2 = v^2 + as$ (c) $v^2 = u^2 + \frac{1}{2}as^2$ (d) $v^2 = u^2 + 2as^2$
11. *Which of the following is not a equation of motion of body freely falling due to gravity ……*
   (a) $v = u + gt$ (b) $s = ut + \frac{1}{2}gt^2$ (c) $v^2 = u^2 + 2gs$ (d) $v = u - gt$
12. *Which of the following is not a equation of motion of body moving vertically upward against gravity ?*
   (a) $v = u - gt$ (b) $s = ut - \frac{1}{2}gt^2$ (c) $s = ut + \frac{1}{2}gt^2$ (d) $v^2 = u^2 - 2gs$

13. *If body covers equal displacement in equal interval of time, then it is said to be in ……*
    (a) uniform displacement (b) uniform velocity (c) uniform acceleration (d) retardation
14. *If change in velocity of a body is constant in every equal interval of time then it is called ……*
    (a) uniform displacement (b) uniform velocity (c) uniform acceleration (d) retardation
15. *A ball is released from a height and it is freely falling down is the example of ……*
    (a) uniform displacement (b) uniform velocity (c) uniform acceleration (d) retardation
16. *A ball is thrown up is the best example of ……*
    (a) uniform displacement (b) uniform velocity (c) uniform acceleration (d) uniform retardation
17. *If a car starts from rest and accelerated for 10 seconds at the time of 0.5 m/s^2, its final velocity will be ……*
    (a) 0.05 m/s (b) 5 m/s (c) 50 m/s (d) 1.5 m/s
18. *54 km/hr is equal to ……*
    (a) 15 m/s (b) 30 m/s (c) 45 m/s (d) 60 m/s
19. *A car starting from a rest is moving with uniform acceleration. If it gains a velocity of 54 km/hr in 10 seconds, then its acceleration will be ……*
    (a) 0.5 m/s^2 (b) 1 m/s^2 (c) 1.5 m/s^2 (d) 2 m/s^2
20. *A car moving with constant speed of 72 km/hr, total distance covered in 10 sec will be ……*
    (a) 720 m (b) 7.2 m (c) 100 m (d) 200 m
21. *A car starting from rest gains a velocity of 54 km/hr in 15 seconds, total distance covered in 10 seconds will be ……*
    (a) 5.4 m (b) 50 m (c) 540 m (d) 100 m
22. *An object comes to rest from a velocity of 20 m/s in a distance of 10 m. Acceleration will be ……*
    (a) 10 m/s^2 (b) 30 m/s^2 (c) -20 m/s^2 (d) -30 m/s^2
23. *A car has initial velocity of 5 m/s. It is accelerated for 10 sec at the rate of 4 m/s^2. Its final velocity will be ……*
    (a) 200 m/s (b) 205 m/s (c) 20 m/s (d) 25 m/s
24. *If a ball is released freely from a certain height, the approximate distance covered by it in 1 sec will be ……*
    (a) 15 m (b) 10 m (c) 5 m (d) 1 m
25. *If a ball is released freely from a certain height, the approximate distance covered by it in 2 sec will be ……*
    (a) 5 m (b) 10 m (c) 15 m (d) 20 m
26. *If a ball is thrown vertically upward and it attains a maximum height in 1 sec, then the maximum height will be ……*
    (a) 5 m (b) 10 m (c) 15 m (d) 20 m
27. *If a ball is thrown vertically upward and it attains a maximum height in 2 sec, then the maximum height will be ……*
    (a) 5 m (b) 10 m (c) 15 m (d) 20 m
28. *A train crosses a tunnel in 10 sec. At the entry of the tunnel, velocity is 20 m/s and at the exit its velocity is 10 m/s. The length of the tunnel will be ……*
    (a) 50 m (b) 100 m (c) 150 m (d) 200 m
29. *A train man driving the train at 25 m/s observes red signal and applies brakes. If the retardation is 12.5 m/s^2, then the distance covered in brining the train to rest after applying brakes will be ……*
    (a) 15 m (b) 25 m (c) 50 m (d) 75 m
30. *Two vehicles A and B are moving in the same direction at a speed of 15 m/s. Car B is ahead of car A by 300 m. If vehicle A is accelerated by 2 m/s^2 and vehicle B has same speed as earlier, the distance at which vehicles A and B meet each other will be ……*
    (a) 150 m (b) 200 m (c) 260 m (d) 310 m
31. *A ball is thrown vertically upward with initial velocity of 20 m/s. The maximum height attained by the ball will be ……*
    (a) 10 m (b) 20 m (c) 30 m (d) 40 m
32. *A ball is released from terrace of building 80 m height. The time it will take to reach the ground will be ……*
    (a) 1 sec (b) 2 sec (c) 3 sec (d) 4 sec

33. *A ball is thrown vertically up. It falls back to ground (same spot) after 4 sec. The maximum height reached by it will be ……*
    (a) 10 m (b) 15 m (c) 20 m (d) 30 m
34. *A ball is thrown vertically up. It falls back to ground (same spot) after '2' sec. The maximum height reached by it will be ……*
    (a) 1 m (b) 5 m (c) 10 m (d) 15 m
35. *A car is moving with initial velocity of 20 m/s then suddenly brakes are applied and it is brought to rest with retardation of 10 m/s^2. The distance covered by a car will be ……*
    (a) 10 m (b) 20 m (c) 30 m (d) 40 m

MCQs on Kinetics and Work, Power, Energy

36. *Quantity of motion possessed by a moving body is called as ……*
    (a) impulse (b) impulsive force (c) momentum (d) quantum
37. *Impulse is defined as the change in ……*
    (a) mass (b) momentum (c) velocity (d) acceleration
38. *SI unit of momentum is ……*
    (a) ms/kg (b) kg m/s (c) kg s/m (d) kg m/s^2
39. *The force which acts for a short time and produce considerable change in momentum of a body is called as ……*
    (a) impulse (b) impulsive force (c) momentum (d) quantum
40. *Impulsive force is the rate of impulse w.r.t. time and it has unit ……*
    (a) 'N' (b) Ns (c) N/s (d) kg m/s
41. *Every body continues in its state of rest or of uniform motion, unless it is acted upon by some external agency, is known as ……*
    (a) Newton's 1st law of motion (b) Newton's 2nd law of motion
    (c) Newton's 3rd law of motion (d) none of these
42. *Newton's 2nd law of motion states that the rate of change of momentum of a body is proportional to …… and takes place in the direction of ……*
    (a) velocity, force (b) force, velocity (c) displacement, velocity (d) applied force, force
43. *For every action, there is equal and opposite reaction is known as ……*
    (a) Newton's 1st law of motion (b) Newton's 2nd law of motion
    (c) Newton's 3rd law of motion (d) Newton's law of gravitation
44. *As per law of conservation of momentum when there is no force then total momentum of a system ……*
    (a) before impact = after impact (b) before impact > after impact
    (c) before impact < after impact (d) none of these
45. *Which of the following is not an example of Newton's 1st law of motion ……*
    (a) use of seat belt in a car
    (b) removal of flowers from tree by shaking the tree
    (c) banging of bottom of handle on hard surface to tighten head of hammer
    (d) firing a rocket
46. *Which of the following is the application of Newton's 2nd law of motion ?*
    (a) to and fro motion of pendulum (b) while catching the ball cricketer swings his hands back
    (c) jumping on earth (d) birds can fly
47. *Which of the following is not an application of Newton's 3rd law of motion ?*
    (a) removing ketch up from bottle by shaking (b) firing a rocket
    (c) rotation of lawn spray sprinkler (d) recoil of gun
48. *As per law of conservation of momentum ……*
    (a) $m_1m_2 = v_1v_2$ (b) $m_1v_2 = m_2v_1$ (c) $m_1v_1 = m_2v_2$ (d) $m_1v_1 + m_2v_2 = 0$

49. *A two wheeler vehicle of mass 150 kg has a velocity of 6 m/s. The momentum of the vehicle is ……*
    (a) 125 kg m/s (b) 900 kg m/s (c) 90 kg m/s (d) 250 kg m/s
50. *The momentum of train weighing 3000 kN moving with speed 90 km/hr will be ……*
    (a) 10.5×10^6 Ns (b) 25×10^6 Ns (c) 2.5×10^6 Ns (d) 7.65×10^6 Ns
51. *If a body of mass 50 kg changes its velocity of 5 m/s to 35 m/s, impulse acting on the body will be ……*
    (a) 1500 Ns (b) 2000 Ns (c) 200 Ns (d) 150 Ns
52. *A ball of mass 200 gm rolls with velocity of 10 m/s. It is hit with a bat in the direction of its motion. The velocity changes to 20 m/s. If the bat is in contact with ball for 0.02 sec, the impulsive force on it will be ……*
    (a) 10 N (b) 100 N (c) 200 N (d) 20 N
53. *A bullet of mass 0.1 kg is fired with a velocity of 500 m/s horizontally in a wooden block of mass 5 kg resting on horizontal surface. If bullet remains embedded in the block, the velocity of the block after impact will be ……*
    (a) 4.9 m/s (b) 9.8 m/s (c) 19.6 m/s (d) 25 m/s
54. *A bullet of mass 50 gm is fired with a velocity of 800 m/s from a gun of mass 5 kg. The velocity with which the gun will recoil is ……*
    (a) 4 m/s (b) 6 m/s (c) 8 m/s (d) 10 m/s
55. *A bullet of mass 100 gm is fired with a velocity of 400 m/s from a gun which produces recoil velocity of 4 m/s. The mass of the gun is ……*
    (a) 2.5 kg (b) 5 kg (c) 7.5 kg (d) 10 kg
56. *The tendency of a body to remain in the state of rest or uniform motion until and unless an external force is applied to it is called ……*
    (a) inertia (b) momentum (c) impulse (d) reaction
57. *Scalar quantities need ……*
    (a) only distance to describe
    (b) magnitude and direction both to describe it
    (c) only magnitude to describe
    (d) distance and direction both to describe it
58. *Vector quantities need ……*
    (a) only distance to describe it
    (b) magnitude and direction both to describe it
    (c) only magnitude to describe it
    (d) distance and direction both to describe it
59. *Out of the following, the scalar quantity is ……*
    (a) displacement (b) force (c) weight (d) mass
60. *Out of the following the quantity which is not a vector is ……*
    (a) momentum (b) impulse (c) length (d) acceleration
61. *Out of the following, the vector quantity is ……*
    (a) electric field (b) distance (c) volume (d) time
62. *Out of the following the quantity which is not scalar is ……*
    (a) work (b) energy (c) length (d) magnetic field
63. *Out the following, the example of conservation of momentum is ……*
    (a) explosion of a bomb
    (b) safety seat belts in a car
    (c) to and fro motion of a pendulum
    (d) to tighten the head of the hammer
64. *Out of the following, which is not an example of conservation of momentum ……*
    (a) bullet fired from a gun
    (b) firing of rocket
    (c) person jumping from a boat
    (d) use of seat belt in a car

MCQs on Angular Motion

65. *If the motion of a body takes place along the circumference of a circle, then it is called ……*
    (a) linear motion (b) gravitational motion (c) angular motion (d) none of these

66. *Angle subtended by radius vector when a particle in circular motion moves from one position to other is called as ……*
   (a) displacement (b) angular displacement (c) angular velocity (d) angular acceleration
67. *S.I. unit of angular displacement is ……*
   (a) radian (b) steradian (c) degree (d) none of these
68. *The rate of change of angular displacement with respect to time is called as ……*
   (a) angular displacement (b) angular velocity (c) angular acceleration (d) velocity
69. *The rate of change of angular velocity is called as ……*
   (a) angular displacement (b) angular velocity (c) angular acceleration (d) acceleration
70. *The relation between angular velocity (ω) and linear velocity (v) is given by ……*
   (a) $r = v\omega$ (b) $v = r\omega$ (c) $\omega = vr$ (d) $v = r + \omega$
71. *Which of the following is not a equation of circular (angular) motion ?*
   (a) $\omega = \omega_o + \alpha t$ (b) $\theta = \omega_o t + \frac{1}{2}\alpha t^2$ (c) $\omega^2 = \omega_o^2 + 2\alpha\theta$ (d) $\omega = \omega_o + 2\alpha\theta$
72. *The relation between linear acceleration and angular acceleration is given by ……*
   (a) $a = r\alpha$ (b) $r = a\alpha$ (c) $\alpha = ar$ (d) $a = r + \alpha$
73. *1 r.p.s. is equivalent to ……*
   (a) 1/60 rpm (b) 60 rpm (c) 1/3600 rpm (d) 3600 rpm
74. *π radian (π^c) is equal to ……*
   (a) 60° (b) 120° (c) 180° (d) 360°
75. *A fly wheel is rotating at 120 rpm. Its angular velocity will be ……*
   (a) 2π rad/s (b) 4π rad/s (c) $\pi/2$ rad/s (d) $\pi/4$ rad/s
76. *If radius vector subtends an angle of $\pi/2$ radians in 2 seconds, its angular velocity will be ……*
   (a) 2π rad/s (b) 4π rad/s (c) $\pi/2$ rad/s (d) $\pi/4$ rad/s
77. *The second hand of a clock is 5 cm long. The linear speed of ant sitting on its tip will be ……*
   (a) $\pi/2$ m/s (b) $\pi/4$ m/s (c) $\pi/6$ m/s (d) 2π m/s
78. *The frequency of rotation of fan changes from 2 rev/s to 4 rev/s in 2 sec. Its angular acceleration is ……*
   (a) 2π rad/s^2 (b) 4π rad/s^2 (c) π rad/s^2 (d) $\pi/2$ rad/s^2
79. *A fly wheel starting from rest attains a speed of 1200 rpm in 1 minute. Its angular acceleration will be ……*
   (a) $\pi/3$ rad/s^2 (b) $2\pi/3$ rad/s^2 (c) π rad/s^2 (d) 2π rad/s^2
80. *A fly wheel is rotating at 1800 r.p.m. It is brought to rest in 60 revolutions. Its uniform retardation will be ……*
   (a) 5π rad/s^2 (b) 10π rad/s^2 (c) 15π rad/s^2 (d) 20π rad/s^2
81. *A motor cycle with 90 cm wheel diameter has angular velocity of 50 rad/s. Its linear velocity will be ………*
   (a) 30 m/s (b) 45 m/s (c) 60 m/s (d) 90 m/s
82. *Angular acceleration of cycle is 4 rad/s^2, where its wheel diameter is 60 cm. Its linear acceleration will be ……*
   (a) 2.4 m/s^2 (b) 1.2 m/s^2 (c) 3.6 m/s^2 (d) 4.8 m/s^2
83. *Periodic time of angular motion is 3 sec. Its frequency will be ……*
   (a) 2/3 Hz (b) 6 Hz (c) 3 Hz (d) 1/3 Hz
84. *Frequency of rotation of fan is 4 Hz. Its periodic time will be ……*
   (a) 4 sec (b) 2/4 sec (c) 1/4 sec (d) 2 sec

MCQs on Angular Motion and S.H.M.

85. *The angular displacement in circular motion is ……*
   (a) dimensional quantity (b) dimensionless quantity
   (c) unitless and dimensionless quantities (d) unitless quantity

86. *Angular displacement is measured in ……*
    (a) metre (b) second (c) radian (d) steradian
87. *A flywheel rotates a constant speed of 3000 r.p.m. The angle described by the shaft in one second is ……*
    (a) 20π rad (b) 30π radian (c) 100π rad (d) 300π radian
88. *What is the angular speed of the second's hand of a watch ?*
    (a) 40 rad/s (b) π rad/s (c) $\pi/30$ rad/s (d) 2 rad/s
89. *The angular velocity of a particle rotating in a circular orbit 100 times per minute is ……*
    (a) 2.5 rad/s (b) 10.47 rad/s (c) 10.47 deg/s (d) 2.5 deg/s
90. *A body of mass 100 g is revolving in a horizontal circle. If its frequency of rotation is 3.5 r.p.s. and radius of circular path is 0.5 m, the angular speed of the body is ……*
    (a) 10 rad/s (b) 15 rad/s (c) 22 rad/s (d) 28 rad/s
91. *What is the angular velocity of the earth ?*
    (a) $\frac{2\pi}{86400}$ rad/s (b) $\frac{2\pi}{360}$ rad/s (c) $\frac{2\pi}{24}$ rad/s (d) $\frac{2\pi}{640}$ rad/s
92. *The ratio of angular speeds of second's hand and hour hand of a watch is ……*
    (a) 1 : 720 (b) 60 : 1 (c) 1 : 60 (d) 720 : 1
93. *A body moves with constant angular velocity on a circle. Magnitude of angular acceleration is ……*
    (a) 2ω (b) r (c) ω (d) zero
94. *A wheel having a diameter of 3 m starts from rest and acceleration uniformly to an angular velocity of 210 r.p.m. in 5 seconds. Angular acceleration of the wheel is ……*
    (a) 4.4 rad/s^2 (b) 6.6 rad/s^2 (c) 8.8 rad/s^2 (d) 1.1 rad/s^2
95. *The vector relation between linear velocity $\vec{v}$, angular velocity $\vec{\omega}$ and radius vector $\vec{r}$ is given by ……*
    (a) $\vec{v} = \vec{r} \times \vec{\omega}$ (b) $\vec{v} = \vec{r} + \vec{\omega}$ (c) $\vec{v} = \vec{r} / \vec{\omega}$ (d) $\vec{v} = \vec{r} - \vec{\omega}$
96. *A wheel has circumference c. If it makes f r.p.s., the linear speed of a point on the circumference is ……*
    (a) fc (b) $fc\pi$ (c) $fc/60\pi$ (d) fc/2
97. *A body is whirled in a horizontal circle of radius 20 cm. It has angular velocity of 10 rad/s. What is its linear velocity at any point on the circular path ?*
    (a) 40 m/s (b) 2 m/s (c) 30 m/s (d) 10 m/s
98. *A particle moves in a circular path, 0.4 m in radius with constant speed. If a particle makes 5 revolutions in each second of its motion, the speed of the particle is ……*
    (a) 16.6 m/s (b) 14.2 m/s (c) 12.6 m/s (d) 18.6 m/s
99. *Angular displacement is measured in ……*
    (a) degrees (b) radians (c) rev/s (d) degree/s
100. *An angle of $\frac{3\pi}{4}$ radians is equivalent to ……*
    (a) 280° (b) 87.5° (c) 135° (d) 20.5°
101. *An angle of 120° is equivalent to ……*
    (a) $\frac{2\pi}{3}$ rad (b) $\frac{\pi}{4}$ rad (c) $\frac{8\pi}{4}$ rad (d) $\frac{1}{8}$ rad
102. *An angle of 2 rad at the centre of a circle subtends an arc length of 40 mm at the circumference of the circle. The radius of the circle is ……*
    (a) 40 mm (b) 80 mm (c) 20 mm (d) $(40/\pi)$ mm
103. *A point on a wheel has a constant angular velocity of 3 rad/s. The angular turned through in 15 seconds is ……*
    (a) 45 rad (b) 10π rad (c) 5 rad (d) 90π rad
104. *An angular velocity of 60 revolutions per minute is the same as ……*
    (a) $(1/2\pi)$ rad/s (b) 120π rad/s (c) $(30/\pi)$ rad/s (d) 2π rad/s

105. *A wheel of radius 15 mm has an angular velocity of 10 rad/s. A point on the rim of the wheel has a linear velocity of ……*
(a) 300 mm/s (b) 2/3 mm/s (c) 150 mm/s (d) 1.5 mm/s

106. *The shaft of an electric motor is rotating at 20 rad/s and its speed is increased uniformly to 40 rad/s in 5 s. The angular acceleration of the shaft is ……*
(a) 4000 rad/s^2 (b) 4 rad/s^2 (c) 160 rad/s^2 (d) 12 rad/s^2

107. *A point on a flywheel of radius 0.5 m has a uniform linear acceleration of 2 m/s^2. Its angular acceleration is ……*
(a) 2.5 rad/s^2 (b) 0.25 rad/s^2 (c) 1 rad/s^2 (d) 4 rad/s^2

108. *A mass on a spring undergoes S.H.M. The maximum displacement from the equilibrium is called ?*
(a) period (b) frequency (c) amplitude (d) wavelength

109. *In a periodic process, the number of cycles per unit of time is called ……*
(a) period (b) frequency (c) amplitude (d) wavelength

110. *In a periodic process, the time required to complete one cycle is called ……*
(a) period (b) frequency (c) amplitude (d) wavelength

111. *When the mass in S.H.M. reaches point x = amplitude then its instantaneous velocity is ……*
(a) maximum and positive (b) maximum and negative
(c) zero (d) less than maximum and positive

112. *When the mass in S.H.M. reaches point x = 0 then its instantaneous velocity is ……*
(a) maximum and can be positive or negative (b) constant and does not depend on the location
(c) zero (d) slightly less than maximum and positive

113. *When the mass in S.H.M. reaches point x = +A then its instantaneous acceleration is ……*
(a) maximum and positive (b) maximum and negative
(c) zero (d) slightly less than maximum and positive

114. *When the mass reaches point x = 0 then its instantaneous acceleration is ……*
(a) maximum and positive (b) maximum and negative
(c) zero (d) slightly less than maximum and positive

115. *The periodic time (T) is given by ……*
(a) $\omega/2\pi$ (b) $2\pi/\omega$ (c) $2\pi \times \omega$ (d) π/ω

116. *The velocity of a particle moving with simple harmonic motion is …… at the mean position.*
(a) zero (b) minimum (c) maximum (d) none of the mentioned

117. *The velocity of a particle (v) moving with simple harmonic motion, at any instant is given by ……*
(a) $\omega\sqrt{r^2 - x^2}$ (b) $\omega\sqrt{x^2 - r^2}$ (c) $\omega^2\sqrt{r^2 - x^2}$ (d) $\omega^2\sqrt{x^2 - r^2}$

118. *The maximum acceleration of a particle moving with simple harmonic motion is ……*
(a) ω (b) $\omega \cdot r$ (c) $\omega^2 \cdot r$ (d) ω^2/r

MCQ Answers and Hints on Mechanics

1. (c)	2. (a)	3. (b)	4. (d)	5. (a)	6. (b)	7. (c)	8. (d)	9. (c)	10. (a)	11. (d)	12. (c)
13. (b)	14. (c)	15. (c)	16. (d)	17. (b)	18. (a)	19. (c)	20. (d)	21. (b)	22. (c)	23. (b)	24. (c)
25. (d)	26. (a)	27. (d)	28. (c)	29. (b)	30. (c)	31. (b)	32. (d)	33. (c)	34. (b)	35. (b)	36. (c)
37. (b)	38. (b)	39. (b)	40. (a)	41. (a)	42. (d)	43. (c)	44. (a)	45. (d)	46. (b)	47. (a)	48. (c)
49. (b)	50. (d)	51. (a)	52. (b)	53. (b)	54. (c)	55. (d)	56. (a)	57. (c)	58. (b)	59. (d)	60. (c)
61. (a)	62. (d)	63. (a)	64. (d)	65. (c)	66. (b)	67. (a)	68. (b)	69. (c)	70. (b)	71. (d)	72. (a)
73. (b)	74. (c)	75. (b)	76. (d)	77. (c)	78. (a)	79. (b)	80. (c)	81. (b)	82. (a)	83. (d)	84. (c)
85. (a)	86. (c)	87. (c)	88. (c)	89. (b)	90. (c)	91. (a)	92. (d)	93. (d)	94. (a)	95. (a)	96. (a)
97. (b)	98. (c)	99. (b)	100. (c)	101. (a)	102. (c)	103. (a)	104. (d)	105. (c)	106. (b)	107. (d)	108. (c)
109. (b)	110 (a)	111. (c)	112. (a)	113. (c)	114. (a)	115. (b)	116. (c)	117. (a)	118. (c)		

General Hint : For simplicity of calculations, we can take $g = 10\ m/s^2$ (approximate value)

17. **Hint :** $v = u + at$, $v = 0 + (0.5)(10)$, $v = 5\ m/s^2$

18. **Hint :** $54\ km/hr = 54 \times \frac{5}{18}\ m/s = 15\ m/s$

19. **Hint :** $54\ km/hr \rightarrow 15\ m/s$, $a = \frac{v-u}{t} = \frac{15-0}{10} = \mathbf{1.5\ m/s^2}$

20. **Hint :** $72\ km/hr \rightarrow 20\ m/s$, distance = speed × time $\therefore$ distance = $20 \times 10 =$ **200 m**

21. **Hint :** $54\ km/hr \rightarrow 15\ m/s$, $a = \frac{v-u}{t} = \frac{15-0}{15} = 1$, $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(1)(10)^2 = 50\ m$

22. **Hint :** $v^2 = u^2 + 2as$, $\therefore$ $a = \frac{v^2-u^2}{2(s)} = \frac{0-(20)^2}{2(10)} = -20\ m/s^2$

23. **Hint :** $v = u + at$, $v = 5 + \frac{1}{2}(4)(10)^2 = 205\ m/s$

24. **Hint :** $s = ut + \frac{1}{2}gt^2$, $s = 0 + \frac{1}{2}(9.8)(1)^2 = \mathbf{4.9\ m}$

(We can take $g \fallingdotseq 10\ m/s^2$ for easy calculation)

25. **Hint :** $s = ut + \frac{1}{2}gt^2$, $s = 0 + \frac{1}{2}(9.8)(2)^2 = \mathbf{19.6\ m}$

26. **Hint :** Maximum height means $v = 0$, $v = u - gt$ $\therefore$ $u = v + gt = 0 + (10)(1)$, $u = 10$

Now, $s = ut - \frac{1}{2}gt^2 = 10(1) - \frac{1}{2}(10)(1)^2 = \mathbf{5\ m}$

27. **Hint :** Maximum height $\therefore$ $v = 0$, $v = u - gt$ $\therefore$ $u = v + gt = 0 + (10)(2) = 20\ m/s$

Now, $s = ut - \frac{1}{2}gt^2 = 20(2) - \frac{1}{2}(10)(2)^2 = 20\ m$

28. **Hint :** $a = \frac{v-u}{t} = \frac{10-20}{10} = -1\ m/s^2$, $s = \frac{v^2-u^2}{2a} = \frac{10^2-20^2}{2(-1)} = \mathbf{150\ m}$

29. **Hint :** $s = \frac{v^2-u^2}{2a} = \frac{0-(25)^2}{2(-12.5)} = \frac{-625}{-25} = \mathbf{25\ m}$

30. **Hint :** Vehicle 'B' has constant speed throughout $\therefore$ Speed = $\frac{\text{distance}}{t} = \frac{x}{t}$, $15 = \frac{x}{t}$ $\therefore$ $t = \frac{x}{15}$

Now for vehicle 'A', $s = ut + \frac{1}{2}at^2$ i.e. $(300 + x) = 15\left(\frac{x}{15}\right) + \frac{1}{12}(2)\left(\frac{x}{15}\right)^2$ $\therefore$ $(300 + x) = x + \frac{x^2}{225}$

$\therefore$ $x^2 = \mathbf{300 \times 225} = 67500$ $\therefore$ $x = \mathbf{259.8\ m}$

31. **Hint :** Maximum height, $\therefore$ $v = 0$, $v^2 = u^2 - 2gs$ $\therefore$ $2gs = u^2 - v^2$ $\therefore$ $s = \frac{u^2-v^2}{2g} = \frac{20^2-0}{2 \times (10)} = \mathbf{20\ m}$

32. **Hint :** $s = ut + \frac{1}{2}gt^2$ $\therefore$ $80 = 0 + \frac{1}{2}(10)t^2$ $\therefore$ $t^2 = \frac{80}{5} = 16$ $\therefore$ $\mathbf{t = 4\ sec}$

33. **Hint :** Ball goes up and down in 4 sec $\therefore$ Ball attains maximum height in 2 sec.

If downward motion is considered then $s = ut + \frac{1}{2}gt^2$, $s = 0 + \frac{1}{2}(10)(2)^2 = \mathbf{20\ m}$

34. **Hint :** Same as earlier, ball attains maximum height in 1 sec.

$\therefore$ While coming down $s = ut + \frac{1}{2}gt^2$, $s = 0 + \frac{1}{2}(10)(1)^2 = \mathbf{5\ m}$

35. **Hint :** $u = 20$, $v = 0$, $a = -10\ m/s^2$, $s = ?$ $v^2 = u^2 + 2as$ $\therefore$ $s = \frac{v^2-u^2}{2a} = \frac{0-20^2}{2(-10)} = \mathbf{20\ m}$

49. **Hint :** $m \cdot m = m \times v = 150 \times 6 = 900\ kg\ m/s.$

50. **Hint :** $m = \frac{3000 \times 10^3}{9.8} = 306 \times 10^3\ kg$, $v = 90\ km/hr = 90 \times \frac{5}{18} = 25\ m/s$

Momentum = $m \times v = (306 \times 10^3) \times 25 = \mathbf{7.65 \times 10^6}$ Ns

51. **Hint :** Impulse = m (v – u) = 50 × (35 – 5) = **1500 Ns**

52. Impulse = m (v – u) = 0.2 (20 – 10) = 2 Ns, Impulsive force = $\frac{\text{Impulse}}{\text{Time}} = \frac{2}{0.02}$ = **100 N**

53. **Hint :** $v_2 = \frac{(m_1u_1 + m_2u_2)}{(m_1 + m_2)} = \frac{[(0.1 \times 500) + (5 \times 0)]}{(5 + 0.1)}$ = **9.8 m/s**

54. **Hint :** $v_2 = \frac{m_1v_1}{m_2} = \frac{0.05 \times 800}{5}$ = **8 m/s**

55. **Hint :** $m_2 = \frac{m_1v_1}{v_2} = \frac{(0.1 \times 400)}{4}$ = 10 kg

75. **Hint :** 120 rpm $\rightarrow \frac{120}{60}$ rps = 2 Hz = n, $\omega = 2\pi n = 2\pi \times 2$ = **4π rad/sec**

76. **Hint :** $\omega = \frac{\theta}{t} = \frac{\pi/2}{2}$ = **π/4 rad/sec**

77. **Hint :** Second hand ∴ T = 60 sec. We have $v = r\omega = r\left(\frac{2\pi}{T}\right) = 5 \times \frac{2 \times \pi}{60} = \frac{\pi}{6}$ **m/s**

78. **Hint :** $\alpha = \frac{\omega - \omega_o}{t} = \frac{2\pi n - 2\pi n_o}{t} = 2\pi\left(\frac{n - n_o}{t}\right) = 2\pi\left(\frac{4-2}{2}\right) = \mathbf{2\pi}$

79. **Hint :** $\alpha = \frac{\omega - \omega_o}{t} = 2\pi\frac{(n - n_o)}{t} = 2\pi\frac{\left(\frac{1200}{60} - 0\right)}{60} = \frac{2\pi}{3}$ rad/s^2

80. **Hint :** 1800 rpm, $\omega \rightarrow \frac{1800 \times 2\pi}{60} = 60\pi$ rad/s, brought to rest in 60 rev. ∴ $\theta = (60 \times 2\pi)$ rad

$\omega^2 = \omega_o^2 + 2\alpha\theta$ ∴ $\alpha = \frac{(\omega^2 - \omega_o^2)}{2\theta}$ ∴ $\alpha = \frac{[0 - (60\pi)^2]}{2\,(120\pi)} = 15\pi$ rad/s^2

81. **Hint :** v = rω = 0.9 × 50 = **45 m/s**

53. **Hint :** a = rα = 0.60 × 4 = **2.4 m/s^2**

82. **Hint :** $n = \frac{1}{T} = \frac{1}{3}$

55. **Hint :** $T = \frac{1}{n} = \frac{1}{4}$

87. **Hint :** 1 rotation = 2π radians

88. **Hint :** $\omega = \frac{2\pi}{T} = \frac{2\pi}{60}$

89. **Hint :** $\omega = 2\pi n = \frac{2\pi \times 100}{60}$

90. **Hint :** ω = 2πn = 2π × 3.5

91. **Hint :** $\frac{\omega_1}{\omega_2} = \frac{2\pi/60}{2o/12 \times 60 \times 60}$

94. **Hint :** $\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{2\pi n_2 - 2\pi n_1}{t} = \frac{\frac{2\pi \times 210}{60} - 0}{5}$

96. **Hint :** v = rω = r × 2πf = c × f

97. **Hint :** v = rω = 0.2 × 10

102. arc s = rθ ∴ $r = \frac{s}{\theta} = \frac{40}{2}$

104. **Hint :** $\omega = 2\pi n = 2\pi\frac{60}{60}$

105. **Hint :** v = rω = 15 × 10

106. **Hint :** $\alpha = \frac{\omega_w - \omega_1}{t} = \frac{(40 - 20)}{5}$

107. **Hint :** a = rα ∴ α = a/r

❑❑❑

Chapter 3

GRAVITATION

3.1 INTRODUCTION

It is our general experience that ball released from a certain height falls towards the earth. Also ball thrown upwards reaches a certain height and then returns downward towards earth. We know that moon revolves around the earth. We also know that earth revolves around the sun. Any body performing circular motion experiences two forces on it (i) one is along the tangent, away from the centre, (ii) other is along the radius, towards the centre. e.g. A stone tied at one end of the thread and rotated (whirled). In this example, two forces are acting on the stone, (i) one is along the tangent (because of its speed) away from the centre called centrifugal force which tries to move stone along the tangent, but stone does not move along the tangent because there is other force, (ii) i.e. force along radius towards the centre called centripetal force. In this example, tension in the string plays the role of centripetal force.

Similarly when moon revolves around the earth there are two forces acting on the moon : (i) one force is along tangent (because of speed) away from centre which tries to move moon along the tangent but moon does not move along the tangent it means that there must be some force, (ii) force which is along the radius towards centre of earth (centripetal force), this is realised by the scientist Isaac Newton and that force is called gravitational force of attraction.

3.2 GRAVITATION

Ball released from certain height fall towards earth, moon revolves around earth. Isaac Newton studied that same type of force is responsible in both the cases.

Ball is attracted towards the centre of earth. Motion of the moon around the earth is because of centripetal force i.e. moon gets attracted towards the centre of earth.

3.2.1 Newton's Law of Gravitation (Universal Law of Gravitation)

It states that every body in the universe attracts every other body with a force (F) which is

(i) directional proportional to the product of their masses and

(ii) inversely proportional to the square of the distance between them.

This force is along the line joining centres of two bodies.

Let, $m_1 \rightarrow$ mass of one body in 'kg'

$m_2 \rightarrow$ mass of other body in 'kg'

$d \rightarrow$ distance between two bodies in 'm'

$F \rightarrow$ gravitational force of attraction in 'N'

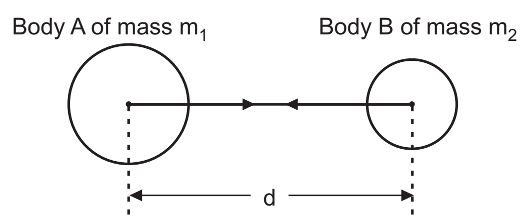


Fig. 3.1

As per Newton's universal law of gravitation,

$$F \propto m_1 m_2$$

$$F \propto \frac{1}{d^2}$$

Combining these two equations (because left hand side is same),

$$\therefore \quad F \propto \frac{m_1 m_2}{d^2}$$

$$\therefore \quad F = \text{constant} \times \frac{m_1 m_2}{d^2}$$

$$\therefore \quad F = G \times \frac{m_1 m_2}{d^2}$$

where, G $\rightarrow$ constant of proportionality called Newton's (universal) gravitational constant

G $\rightarrow$ carries the value 6.67×10^{-11} Nm^2/kg^2

i.e.
$$F = G\frac{m_1 \times m_2}{d^2}$$

i.e.
$$F = 6.67 \times 10^{-11}\frac{m_1 \times m_2}{d^2}$$

3.2.2 Newton's Gravitational Constant (G)

It is constant in the universe.

$$\text{Value of G} = 6.67 \times 10^{-11}\ Nm^2/kg^2$$

$$\text{S.I. unit of G} \rightarrow Nm^2/kg^2 \quad \text{or} \quad Nm^2\ kg^{-2}$$

If we consider body on earth

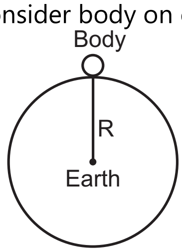


Fig. 3.2

Let M = mass of earth

m = mass of body

R = radius of earth i.e. distance between two centres of body and centre of earth

$$F = G\frac{Mm}{R^2}$$

Acceleration due to gravity (g) and its relation with (G) :

Consider a body of mass 'm' on the surface of earth of mass 'M'.

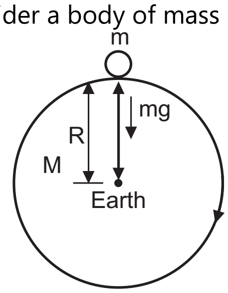


Fig. 3.3

Let M $\rightarrow$ mass of the earth (6×10^{24} kg)

m $\rightarrow$ mass of the body

R $\rightarrow$ radius of the earth

$\rightarrow$ 6400 km = 6400×10^3 m

F $\rightarrow$ gravitational force of attraction

As per Newton's law of gravitation,

$$F = G\frac{Mm}{R^2} \quad \dots (3.1)$$

As per Newton's second law of motion, force is the product of mass and acceleration.

$$F = m \times a$$

We know that acceleration with which body is falling towards centre of earth is called gravitational acceleration 'g'.

Now, $F = m \times g$ $\rightarrow$ weight of the body of mass 'm'

$$F = mg \quad \dots (3.2)$$

Equating equations (3.1) and (3.2), we get

$$mg = \frac{G\,Mm}{R^2} \quad \therefore \quad g = \frac{GM}{R^2}$$

Calculation of gravitational acceleration 'g' due to earth :

$$g = \frac{GM}{R^2}$$

put, $G = 6.7 \times 10^{-11}\ Nm^2/kg^2$

$M = 6 \times 10^{24}$ kg

$R = 6400$ km $= 6400 \times 10^3$ m

$$\therefore \quad g = \frac{(6.7 \times 10^{-11} \times 6 \times 10^{24})}{(6400 \times 10^3)^2}$$

$$g = 9.81\ m/s^2$$

This is the value of gravitational acceleration due to earth.

$$g = 9.81 \text{ m/s}^2 \quad \text{or} \quad g = 981 \text{ cm/s}^2$$

$$g = \frac{GM}{R^2}$$

It means that gravitational acceleration for a earth is constant and it does not depend on mass of the body.

e.g. A stone of 1 kg and 10 kg dropped from a certain height, falls down with same gravitational acceleration.

$$g = 9.81 \text{ m/s}^2$$

3.3 VARIATION OF 'g' WITH ALTITUDE AND LATITUDE

3.3.1 Variation of 'g' with Altitude

Altitude is the distance measured in feet or meter above sea (mean) level.

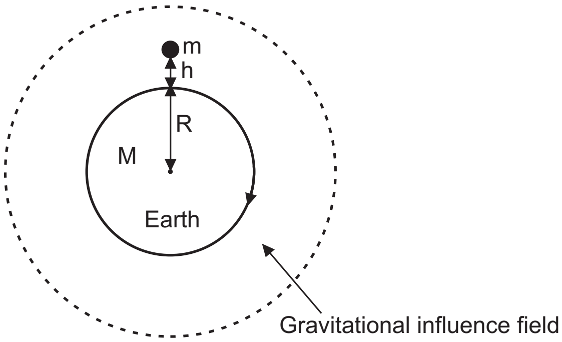


Fig. 3.4

Let M – mass of the earth

r – radius of the earth

m – mass of the body

h – height of the body above earth's surface (sea level) i.e. altitude

g – earth's gravitational acceleration at earth surface

g_h – earth's gravitational acceleration at height 'h' from (seal level) earth's surface

When body is on the earth's surface, we have

$$mg = \frac{G\,Mm}{R^2}$$

$$\therefore \quad g = \frac{GM}{R^2} \quad \ldots (3.3)$$

When body is at a height 'h' from earth's surface

$$mg_h = \frac{G\,Mm}{(R+h)^2}$$

$$\therefore \quad \boxed{g_h = \frac{GM}{(R+h)^2}} \quad \ldots (3.4)$$

Dividing equation (3.4) by equation (3.3), we get

$$\frac{g_h}{g} = \frac{GM/(R+h)^2}{GM/R^2}$$

$$\therefore \quad \frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2\left(1+\frac{h}{R}\right)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

If h << R then $\frac{h}{R}$ << 1.

Hence higher power of $\frac{h}{R}$ can be neglected.

$$\therefore \quad \frac{g_h}{g} = 1 - \frac{2h}{R}$$

$$\therefore \quad \boxed{g_h = g\left(1 - \frac{2h}{R}\right)}$$

3.3.2 Variation of 'g' with Depth

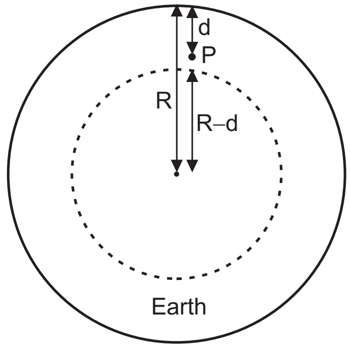


Fig. 3.5

Let g_d be the acceleration due to gravity at point 'P' which is below the surface of the earth at a depth 'd' e.g. mines.

The body experiences gravitational force due to inner shaded sphere of radius (R – d).

$$\frac{g_d}{g} = \frac{R-d}{R} = 1 - \frac{d}{R}$$

$$\therefore \qquad \boxed{g_d = g\left(1 - \frac{d}{R}\right)}$$

3.3.3 Variation of 'g' with Latitude

Latitude is the distance measured in degrees towards north and south from equator. Equator is the plane passing through the centre of earth and perpendicular to its axis of rotation. **The latitude of a place is the angle made by line joining place and centre of the earth with the equator**. The shape of the earth is not perfect sphere. It is bulged out at equator. Therefore radius from centre to north pole is 6350 km and radius along equator is 6372 km.

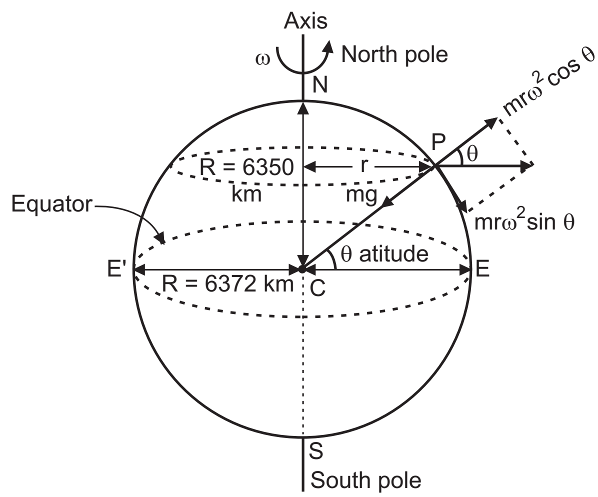


Fig. 3.6

EE' – equator

NS – axis of spinning

ω – angular velocity

P – place on earth surface

mg = weight of body at 'P'

$mr\omega^2 \cos\theta$ = component which opposes 'mg'

$mr\omega^2 \sin\theta$ = component which has no effect on weight of body

Here, P is the place which makes angle 'θ' (latitude) with equator.

Thus at 'P', $\angle$ PCE = θ

at north or south pole $\angle$ NCE = θ = 90°

at equator, $\angle$ θ = 0°.

Earth is spinning about its own axis, therefore, every particle lying on its surface rotates around the axis in horizontal plane with same angular velocity ω.

Thus effective weight of a body of mass 'm'

(i) at P = $mg_p = mg - mr\omega^2 \cos\theta$ (ii) at N = $mg_N = mg - mr\omega^2 \cos 90 = mg$

(iii) at E = $mg_E = mg - mr\omega^2 \cos\theta = mg - mr\omega^2$

Thus effective gravitational acceleration

(i) at P = $g_p = g - r\omega^2 \cos\theta$ (ii) at N = $g_N = g - r\omega^2 \cos 90 = g$ (iii) at E = $g_E = g - r\omega^2 \cos 0 = g - r\omega^2$

Thus g at equator is least and at pole it is highest.

This is one reason and other reason is shape of a earth. Earth is not exact sphere. It is bulged out at equator and flattened at poles as shown. Therefore **radius at equator is highest and radius at poles is least.**

Because of two reasons (i) spinning of earth, (ii) shape of earth; **gravitational acceleration at equator is least and at poles it is highest.** Thus **gravitational acceleration** 'g' is **0.5% more at poles than equator.**

i.e. weight of a body is 0.5% more at poles than at equator.

i.e. $W_{pole} = W_{equator} + 0.5\% \; W_{equator}$

$$W_p = W_E (1 + 0.5\%)$$

$$\boxed{W_p = W_E (1.005)}$$

Variation of 'g' with latitude :

'g' at pole is 9.83 m/s^2.

'g' at equator is 9.78 m/s^2.

average value of g = 9.81 m/s^2.

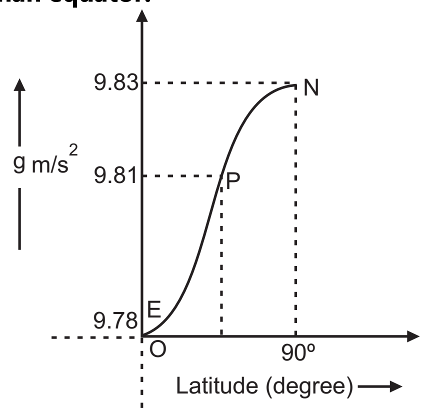


Fig. 3.7

Important Points

- Newton's (universal) law of gravitation states that every body in the universe attracts other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F = G\frac{m_1 m_2}{d^2}$$

'G' is universal gravitational constant and carries the value 6.67×10^{-11} Nm^2/kg^2.

- Gravitational acceleration 'g' due to earth is given by

$$g = \frac{GM}{R^2}$$

$$g = 9.81 \; m/s^2$$

Gravitational acceleration does not depend on mass of the body. Every body falls down with same gravitational acceleration (e.g. 1 kg stone and 10 kg stone fall down with same gravitational acceleration).

- As altitude (height) of body from earth's surface increases, gravitational acceleration decreases.

It is given by the formula $g_h = \frac{GM}{(R + h)^2}$

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

- Gravitational acceleration decreases with increase in depth 'd',

$$g_h = g\left(1 - \frac{d}{R}\right)$$

- Gravitational acceleration at equator is least and it is maximum at pole.
- Gravitational acceleration 'g' is 0.5% more at poles than at equator.

Formulae :

(1) $F = G\frac{m_1 m_2}{d^2}$ where, F = Gravitational force of attraction

$= 6.67 \times 10^{-11} \frac{m_1 m_2}{d^2}$ G = Universal (Newton) gravitational constant

m_1 = mass of one body

m_2 = mass of other body

d = distance between two bodies

(2) Force of gravitation acting due to earth

$F = G\frac{Mm}{R^2}$ where M – mass of earth

m – mass of body on earth's surface

R – radius of earth

(3) $g = \frac{GM}{R^2}$ where g – gravitational acceleration due to earth

$g = 9.81 \text{ m/s}^2$ M – mass of earth, R – radius of earth

(4) $g_h = \frac{GM}{(R + h)^2}$ where g_h = gravitational acceleration due to earth at height 'h' from earth's surface

h = height of body from earth's surface

(5) $g_h = g\left(1 - \frac{2h}{R}\right)$

(6) $g_d = g\left(1 - \frac{d}{R}\right)$

(7) $g_p = g - r\omega^2 \cos\theta$ where g_p – gravitational acceleration at 'P' making angle 'θ' (latitude) with the equator

$g_{pole} = 9.83 \text{ m/s}^2$

$g_{equator} = 9.78 \text{ m/s}^2$

$g_{avg} = 9.81 \text{ m/s}^2$

(8) $W_{pole} = W_{equator} (1 + 0.5\%)$

$W_{pole} = W_{equator} (1.005)$

(9) Equations of motion : $v = u + at$ where, u = initial velocity

$s = ut + \frac{1}{2} at^2$ v = final velocity

$v^2 = u^2 + 2as$ a = acceleration, t = time taken

s = distance covered in time t seconds

⇒ If a body is freely falling due to gravity (earth) put $a = g = 9.81 \text{ m/s}^2$.

⇒ If a body is released, put u = -.

⇒ If a body is thrown up (against gravity) put $a = -g = -9.81 \text{ m/s}^2$.

If a body attains maximum height put v = 0.

Ball is moving vertically up against gravity :

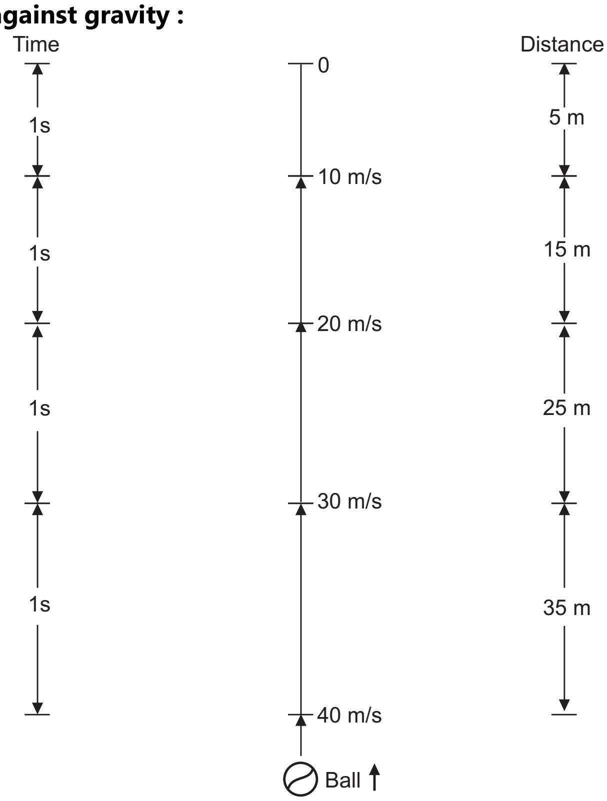


Fig. 3.8

Ball is moving down (freely) : For simplifying assume g = 10 m/s^2 instead of 9.81 m/s^2.

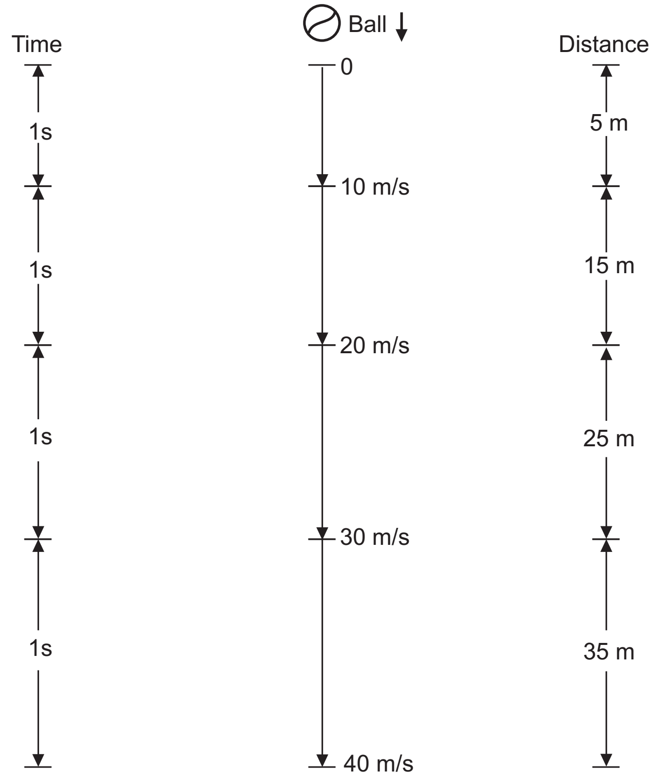


Fig. 3.9

SOLVED EXAMPLES

Example 1 : *The mass of the earth is 6×10^{24} kg, mass of the moon is 0.074×10^{24} kg. Distance between earth and moon is 3.8×10^5 km. Calculate gravitational force of attraction between earth and moon.*

Solution : $m_1 = 6 \times 10^{24}$ kg, $m_2 = 0.074 \times 10^{24}$ kg, $d = 3.8 \times 10^5$ km $= 3.8 \times 10^8$ m, $G = 6.67 \times 10^{-11}$ Nm^2/kg^2, F = ?

We have,
$$F = G\frac{m_1 m_2}{d^2}$$
$$= \frac{(6.67 \times 10^{-11} \times 6 \times 10^{24} \times 0.074 \times 10^{24})}{(3.8 \times 10^8)^2}$$
$$\boxed{F = 2.05 \times 10^{20}\ N}$$

Example 2 : *A ball is released from certain height and it hits the ground in 1 s.*

(i) Calculate its final speed on hitting the ground.

(ii) Calculate height from which it is released.

Solution : Given : u = 0, v = ?, a = g = 9.81 m/s^2, t = 1 s, h = s = ?

(i)
$$v = u + at$$
$$v = 0 + 9.81 \times 1$$
$$\boxed{v = 9.81\ m/s}$$

(ii)
$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(9.81)(1)^2$$
$$\boxed{s = h = 4.905\ m}$$

Example 3 : *A ball is thrown vertically upwards and rises to a height of 20 m and reaches to ground.*

Calculate : (i) initial velocity, (ii) time required to reach maximum height, (iii) time required to reach the ground.

Solution : $u = ?$, $s = h = 20$ m, $v = 0$ (since maximum height), $a = g = -9.81$ m/s^2

(i)
$$v^2 = u^2 + 2as$$
$$0 = u^2 + 2 \times (-9.81) \times 20$$
$$u^2 = 392.4$$
$$\boxed{u = 19.81 \text{ m/s}}$$

(ii)
$$v = u + at$$
$$\therefore \quad \frac{v - u}{a} = t$$
$$\frac{0 - 19.81}{(-9.81)} = t$$

Time required to reach maximum height,
$$\boxed{t = 2.02 \text{ sec}}$$

(iii) Time required to reach the ground = $2 \times t = 2 \times 2.02 = \boxed{4.04 \text{ sec}}$

Example 4 : *Gravitational force on the surface of the moon is $\left(\frac{1}{6}\right)^{th}$ that of earth. Calculate weight of 20 kg object on the moon.*

Solution :
$$\text{Weight on earth} = m \times g = (20 \times 9.81) \text{ N}$$
$$\therefore \quad \text{Weight on moon} = \frac{(20 \times 9.81)}{6} \text{ N} = \boxed{32.7 \text{ N}}$$

Example 5 : *Calculate gravitational force of attraction between earth and sun. Mass of earth is 6×10^{24} kg and mass of sun is 2×10^{30} kg. Average distance between earth and sun is 1.5×10^{11} m.*

Solution : $m_1 = 6 \times 10^{24}$ kg, $m_2 = 2 \times 10^{30}$ kg, $d = 1.5 \times 10^{11}$ m, $F = ?$

$$F = G\frac{m_1 m_2}{d^2} = \frac{(6.67 \times 10^{-11} \times 6 \times 10^{24} \times 2 \times 10^{30})}{(1.5 \times 10^{11})^2}$$
$$\boxed{F = 3.56 \times 10^{22} \text{ N}}$$

Example 6 : *Vijay is standing on the terrace of the building which is 50 m in height. Jay is standing on the ground near the building. Vijay releases a ball freely from the terrace and at the same instant Jay throws another ball vertically upward with velocity 20 m/s. At what height from the ground will these balls cross each other ?*

Solution : Height of the building, h = 50 m.

The balls will cross each other after same time 't' because they are thrown at the same instant.

We have,
$$s = ut + \frac{1}{2}at^2 \quad \text{... (1)}$$

Case 1 : (Vijay) equation (1) becomes $h_1 = 0 + \frac{1}{2}gt^2$ ... (2)

Case 2 : (Jay) equation (1) becomes $h_2 = ut - \frac{1}{2}gt^2 = 20t - \frac{1}{2}gt^2$ ... (3)

Equation (2) and (3) gives, $h_1 + h_2 = 20t$ but $h_1 + h_2 = 50$
$$50 = 20t$$
$$\therefore \quad \boxed{t = 2.5 \text{ sec}}$$

Put this value in equation (3),
$$h_2 = (20t) - \left(\frac{1}{2}gt^2\right) = (20 \times 2.5) - \left(\frac{1}{2} \times 9.81 \times (2.5)^2\right)$$
$$\boxed{h_2 = 19.34 \text{ m}}$$

Thus balls will cross each other at 19.34 m from the ground level.

Example 7 : *What will be the force between two objects if mass of both the objects is doubled and distance between them is tripled ?*

Solution :

$$F_1 = G\frac{m_1m_2}{d_1^2}$$

$$F_2 = G\frac{2m_1 \times 2m_2}{(3d_1)^2} = \frac{4}{9}G\frac{m_1m_2}{d_1^2}$$

$$\therefore \quad \frac{F_2}{F_1} = \frac{4/9}{1}$$

$$\therefore \quad \boxed{F_2 = \frac{4}{9}F_1}$$

i.e. new force will be $\frac{4}{9}$ times the earlier.

Example 8 : *A body is brought to a depth of 10 km from the surface of the earth. What is the value of 'g' at that point ? What is the percentage decrease in weight ? (Take R = 6400 km)*

Solution : Given : $d = 10 \text{ km} = 10 \times 10^3$ m

We have,

$$g_d = g\left(1 - \frac{d}{R}\right)$$

$$= 9.81\left(1 - \frac{10 \times 10^3}{6400 \times 10^3}\right)$$

$$\boxed{g_d = 9.79 \text{ m/s}^2}$$

Now decrease in g is 9.81 – 9.79 = 0.02. For 9.81 decrease is 0.02.

For 100 decreases

$$\therefore \quad \% \text{ decrease} = \frac{100 \times 0.02}{9.81} = \boxed{0.2\%}$$

Percentage decrease in 'g' and percentage decrease in weight is same because mass remains same.

Example 9 : *Calculate the value of 'g' at height 25 km above the earth's surface. Hence calculate percentage decrease in weight. Take radius 6400 km.*

Solution : We can use following equation if h << R.

$$g_h = g\left(1 - \frac{2h}{R}\right) = 9.81\left(1 - \frac{2 \times 25 \times 10^3}{6400 \times 10^3}\right)$$

$$\boxed{g_h = 9.73 \text{ m/s}^2}$$

Now decrease in 'g' = 9.81 – 9.73 = 0.08

For 9.81 decrease is 0.08.

$\therefore$ For 100 decrease = ? $\quad ? = \frac{0.08 \times 100}{9.81}$

$$\boxed{\% \text{ decrease in weight} = 0.82\%}$$

Note : Percentage decrease in 'g' and percentage decrease in weight will be same because mass remains the same.

Example 10 : *Calculate altitude at which acceleration due to gravity of earth falls to 50% of its value at earth's surface. Take R = 6400 km.*

Solution : Since 50% means h is large, therefore we cannot use $g_h = g\left(1 - \frac{2h}{R}\right)$.

But we will have to use

$$g_h = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$0.5g = \frac{9}{\left(1 + \frac{h}{R}\right)^2}$$

$$\therefore \quad 0.5\left(1+\frac{h}{R}\right)^2 = 1$$

$$\left(1+\frac{h}{R}\right)^2 = 2$$

Take square root on both sides.

$$1+\frac{h}{R} = \sqrt{2}$$

$$\frac{h}{R} = \sqrt{2}-1 = 1.414-1$$

$$\therefore \quad h = 0.414 \times R = 0.414 \times 6400 \times 10^3$$

$$\boxed{h = 2649.6 \times 10^3 \text{ m}}$$

OR $$\boxed{h = 2649.6 \text{ km}}$$

Practice Questions

1. State Newton's law of gravitation. What is the value of universal gravitational constant ?
2. State the formula for force of attraction between two bodies of masses m_1 and m_2 and having distance 'd' between them.
3. State formula for gravitational acceleration 'g' and state the value of 'g' due to earth.
4. What is variation of 'g' with altitude ?
5. What is variation of 'g' with latitude ?
6. Gravitational acceleration at pole and at equator, which is more and by what percent ?
7. Stone of 1 kg and 5 kg are dropped from same height at same instant. State whether both will reach the ground at same instant or not and why ?
8. There is a force of attraction between earth and moon. Why does not moon fall on earth ?
9. What is acceleration of free fall ?

Examples for Practice

1. A ball is thrown vertically upward and falls back to ground (same spot) after 2 sec. Find the height attained by the ball.

   (**Ans.** Height = h = s = 4.9 m)
2. A person is standing on the terrace of the building which is 30 m in height. Other person is standing on the ground near the same building. First person releases a ball freely from the terrace of the building and at the same instant second person throws another ball vertically up from the ground with a velocity of 15 m/s. At what height from the ground will these balls cross each other ?
   (**Ans.** 10.38 m from the ground level)
3. Calculate the value of acceleration due to gravity at the surface of the moon.
   Given : mass of the moon = 7.6×10^{22} kg, radius of the moon = 1.7×10^6 m.

   (**Hint :** $g = \frac{GM}{R^2}$)

   (**Ans.** $g_{moon} = 1.75 \text{ m/s}^2$)
4. Calculate mass of the earth if its radius is 6400 km.

   (**Hint :** $g = \frac{GM}{R^2} \quad \therefore \; M = \frac{gR^2}{G}$)

   (**Ans.** $M = 6.02 \times 10^{24}$ kg)
5. A ball is released from certain height and it hits the ground in 2s.
   (i) Calculate its final speed on hitting the ground.
   (ii) Calculate height from which it is released.

   (**Ans.** v = 19.62 m/s, h = s = 19.62 m)

6. A ball is thrown vertically upwards and rise to a height of 30 m and reaches to ground. Calculate :
   (i) initial velocity
   (ii) time required to reach more height
   (iii) time required to reach the ground.
   (**Ans.** u = 24.26 m/s, t = 2.47 sec to reach maximum height, t = 4.94 sec to reach ground)
7. Gravitational force on the surface of the moon is $\left(\frac{1}{6}\right)^{th}$ that of earth. Calculate weight of 100 kg object on the moon.
   (**Ans.** F = 163.5 N)
8. What will be force between two objects if mass of both the objects is halved and distance between them is doubled ?
   (**Ans.** $F_2 = \frac{F}{16}$ i.e. $\frac{1}{16}$ times earlier)
9. A body is brought to a depth of 20 km from the surface of earth. What is the value of 'g' at that point ? What is the percentage decrease in weight ?
   (**Ans.** $g_d = 9.78$ m/s^2, % decrease = 0.31%)
10. Calculate value of 'g' at height 40 km above the earth's surface. Hence calculate percentage decrease in weight.
   (**Ans.** g = 9.69 m/s^2, % decrease in weight = 1.25%)

Multiple Choice Questions

1. *As per Newton's universal law of gravitation, force of attraction between two bodies is ……*
   (a) *directly proportional to square of the distance*
   (b) *directly proportional to product of their masses*
   (c) *inversely proportional to product of their masses*
   (d) *inversely proportional to distance*
2. *As per Newton's universal law of gravitation, force of attraction between two bodies is ……*
   (a) *directly proportional to square of the distance*
   (b) *inversely proportional to product of their masses*
   (c) *inversely proportional to square of distance between them*
   (d) *directly proportional to square of the distance.*
3. *Out of the following, the perfect relation is ……*
   (a) $F = G + \frac{m_1m_2}{d^2}$
   (b) $F = G\frac{m_1m_2}{d^2}$
   (c) $F = G\frac{d^2}{m_1m_2}$
   (d) $F = G\frac{m_1 + m_2}{d^2}$
4. *The value of universal gravitational constant 'G' is ……*
   (a) $G = 6.67 \times 10^{11}$ Nm^2/kg^2
   (b) $G = 9.81$ Nm^2/kg^2
   (c) $G = 981$ Nm^2/kg^2
   (d) $G = 6.67 \times 10^{-11}$ Nm^2/kg^2
5. *Acceleration due to gravity 'g' is given by ……*
   (a) $g = GM + R^2$
   (b) $g = \frac{GM}{R^2}$
   (c) $g = GM - R^2$
   (d) $g = G + \frac{M}{R^2}$
6. *The value of gravitational acceleration 'g' is ……*
   (a) $g = 9.81$ m/s^2
   (b) $g = 981$ cm/s^2
   (c) $g = 9810$ mm/s^2
   (d) *all of the above*
7. *The variation of 'g' with altitude is given by ……*
   (a) $g_h = \frac{(R + h)^2}{GM}$
   (b) $g_h = \frac{G + M}{(Rh)^2}$
   (c) $g_h = \frac{GM}{(R + h)^2}$
   (d) $g_h = GM + (R + h)^2$
8. *If h << R then the gravitational acceleration at height i.e. g_h is given by ……*
   (a) $g_h = g\left(1 - \frac{2h}{R}\right)$
   (b) $g_h = g\left(1 + \frac{2h}{R}\right)$
   (c) $g_h = g\left(1 - \frac{R}{2h}\right)$
   (d) $g_h = g\left(1 + \frac{R}{2h}\right)$
9. *The variation of 'g' with depth is given by ……*
   (a) $g_d = g\left(1 - \frac{d}{R}\right)$
   (b) $g_d = g\left(1 + \frac{d}{R}\right)$
   (c) $g_d = g\left(1 - \frac{R}{d}\right)$
   (d) $g_d = g\left(1 + \frac{R}{d}\right)$

10. *The variation of 'g' with latitude states that ……*
    (a) 'g' at equator is highest and 'g' at poles is least (b) 'g' at equator is least and at poles it is highest
    (c) 'g' at poles and equator is highest (d) 'g' at poles and equator is lowest
11. *Gravitational acceleration 'g' at poles and equator is given by ……*
    (a) 'g' at equator is 5% more than at poles (b) 'g' at poles is 5% more than at equator
    (c) 'g' at equator is 0.5% more than poles (d) 'g' at poles is 0.5% more than at equator
12. *A body of mass 5 kg is attracted by the earth with a force which is equal to ……*
    (a) 49 N (b) 6.67×10^{11} N (c) 5 N (d) 9.8 m/s
13. *The nature of gravitational force between two objects ……*
    (a) is of repulsion (b) is of attraction
    (c) is of repulsion if distance is large (d) is of attraction if distance is large
14. *The value of 'g' ……*
    (a) increases as we move away from earth's surface (b) increases from pole to equator
    (c) remains constant (d) decreases as we go to the centre of the earth
15. *When the ball is thrown vertically up, the value of 'g' will be ……*
    (a) very large (b) negative (c) positive (d) zero
16. *The gravitational force causes ……*
    (a) motion of moon (b) sea tides (c) both (a) and (b) (d) none of these
17. *The mass of the body on moon is 100 kg. What is the weight on the earth ?*
    (a) 600 kg (b) 981 N (c) 5886 N (d) 16 kg
18. *Newton's law of gravitation applies to ……*
    (a) only large bodies (b) only small bodies (c) only planets (d) all bodies
19. *The gravitational force between two objects is F. If masses of both the objects are doubled and the distance between them is halved, then the gravitational force would become ……*
    (a) F/4 (b) F/2 (c) 16 F (d) 8 F
20. *The distance between two bodies becomes 3 times more than the usual distance. Then force becomes ……*
    (a) 3 times (b) 1/3 times (c) 9 times (d) 1/9 times
21. *The period of artificial geostationary satellite is ……*
    (a) 6 hours (b) 12 hours (c) 24 hours (d) 48 hours
22. *If the mass of the earth is not changed and its radius is increased by 1% then the acceleration due to gravity on the surface of the earth would ……*
    (a) increase by 2% (b) decrease by 2% (c) increase by 4% (d) remain unchanged
23. *The value of universal gravitational constant G is ……*
    (a) 6.673×10^{11} Nm^2/kg^2 (b) 6.673×10^{-11} Nm^2/kg^2 (c) 9.81 m/s^2 (d) not certain
24. *The unit of magnitude of the weight is ……*
    (a) kg (b) grams (c) newton (d) meter
25. *Acceleration due to gravity decreases with ……*
    (a) mass of the body (b) weight of the body (c) density of the body (d) altitude
26. *An object revolving around a planet is called a ……*
    (a) moon (b) earth (c) sun (d) satellite
27. *To complete one revolution around earth, communication satellites takes ……*
    (a) 24 hours (b) 36 hours (c) 48 hours (d) 72 hours
28. *Velocity of geostationary satellite with respect to earth is ……*
    (a) 10 ms^{-1} (b) 15 ms^{-1} (c) zero (d) 1 ms^{-1}
29. *The value of gravitational acceleration 'g' at the surface of the earth at sea level is ……*
    (a) 981 m/s^2 (b) 6.67×10^{-11} Nm^2/kg^2 (c) 6.67×10^{11} Nm^2/kg^2 (d) 9.81 m/s^2
30. *When the ball is thrown vertically up and reaches highest point, the value of velocity will be ……*
    (a) zero (b) maximum (c) same throughout (d) none of these

31. *When the ball is released from certain height then its initial velocity is ……*
    (a) zero (b) maximum (c) same (d) none of these
32. *If a stone of 1 kg and 5 kg are released from certain height at the same instant then ……*
    (a) 5 kg stone reach earlier (b) both will reach ground at the same time
    (c) 1 kg stone reach earlier (d) any one will reach earlier
33. *If mass of the body is 10 kg on earth and then mass on moon will be ……*
    (a) 10 kg (b) 60 kg (c) 1.66 kg (d) 98 kg
34. *Weight of the object on the surface of moon is ……*
    (a) half of that on earth (b) $1/6^{th}$ of that on earth
    (c) 6 times that on earth (d) double that on earth
35. *If the distance between objects increases, then gravitational force between them ……*
    (a) increases (b) decreases (c) remains same (d) increases and then decreases
36. *For a freely falling body ……*
    (a) $g = 9.81$ m/s^2 (b) $g = -9.81$ m/s^2 (c) $g = 0$ (d) g = infinite
37. *For a body moving vertically up ……*
    (a) $g = 9.81$ m/s^2 (b) $g = -9.81$ m/s^2 (c) $g = 0$ (d) g = infinite
38. *Mass of the object ……*
    (a) changes from place to place (b) decreases from pole to equator
    (c) increases from pole to equator (d) remains constant at any place
39. *Weight of the body ……*
    (a) remains same everywhere
    (b) decreases from equator to pole
    (c) increases from equator to pole
    (d) sometimes more at pole and sometimes more at equator
40. *The value of 'g' is ……*
    (a) more at equator (b) more at pole
    (c) more at any point other than pole and equator (d) same everywhere
41. *Newton's' law of gravitation is applicable for ……*
    (a) only large bodies (b) only small bodies
    (c) only spherical bodies (d) all bodies of different size and shape
42. *When the ball is thrown vertically up then the value of 'g' is ……*
    (a) positive (b) negative (c) zero (d) infinite
43. *SI unit of gravitational constant 'G' is ……*
    (a) Nm^2/kg^2 (b) $Nm^2\ kg^2$ (c) kg^2/Nm^2 (d) $N/m^2\ kg^2$
44. *If mass of two bodies is doubled, then the force of attraction between them will be ……*
    (a) F (b) 2F (c) 4F (d) F/2
45. *If distance between two bodies is doubled then the force of attraction between them will be ……*
    (a) F (b) 2F (c) 4F (d) F/4
46. *The force of gravitation between two bodies does not depend on ……*
    (a) product of their masses (b) ratio of their masses
    (c) distance between them (d) gravitational constant

47. *A stone is dropped from tower. Its speed after covering distance 50 m will be ……*
    (a) 50 m/s (b) 40.24 m/s (c) 31.32 m/s (d) 9.81 m/s
48. *A ball is thrown vertically up and attains maximum height of 10 m. Its initial speed must be ……*
    (a) 5 m/s (b) 7 m/s (c) 10 m/s (d) 14 m/s
49. *The acceleration due to gravity is zero at ……*
    (a) pole (b) equator (c) sea level (d) centre of earth
50. *The weight of an object of mass 100 kg at the centre of earth is ……*
    (a) zero (b) 100 N (c) 981 N (d) infinite
51. *Two bodies of 10 kg and 20 kg masses falling freely from certain height on earth would have ……*
    (a) same velocity at any instant (b) different velocity at any instant
    (c) different acceleration at any instant (d) double velocity of 20 kg than 10 kg at any instant
52. *The atmosphere is held by earth due to ……*
    (a) earth's magnetic field (b) hills and buildings on earth
    (c) gravity (d) none of these
53. *The force of attraction between two bodies each of mass 1 kg and separated by 1 m is ……*
    (a) 6.67×10^{11} N (b) 6.67×10^{-11} N (c) 9.81 N (d) 1 N
64. *SI unit of weight of the body is ……*
    (a) kg (b) gm (c) N (d) pound
65. *When we stand near huge building, we does not get pulled towards building because ……*
    (a) gravitational attraction due to building is negligible in comparison with earth
    (b) we are small as compared to building
    (c) building is large
    (d) none of these
56. *If earth attracts moon with a force of 'F' then moon attracts earth with a force of ……*
    (a) less than F (b) more than F (c) same F (d) F/2
57. *Weight of the body decreases with ……*
    (a) height of the body from earth's surface (b) if it is kept horizontally on the surface
    (c) if it is kept vertically on the surface (d) none of these

Answers and Hints : Acoustics, Sound

1. (b)	2. (c)	3. (b)	4. (d)	5. (b)	6. (a)	7. (c)	8. (a)	9. (a)	10. (b)
11. (d)	12. (a)	13. (b)	14. (d)	15. (b)	16. (c)	17. (b)	18. (d)	19. (c)	20. (d)
21. (c)	22. (b)	23. (b)	24. (c)	25. (d)	26. (d)	27. (a)	28. (c)	29. (d)	30. (a)
31. (a)	32. (b)	33. (a)	34. (b)	35. (b)	36. (a)	37. (b)	38. (d)	39. (c)	40. (b)
41. (d)	42. (b)	43. (a)	44. (c)	45. (d)	46. (b)	47. (c)	48. (d)	49. (d)	50. (a)
51. (a)	52. (c)	53. (b)	54. (c)	55. (a)	56. (c)	57. (a)			

❐❐❐

Chapter 4

WORK, ENERGY AND POWER

4.1 INTRODUCTION

- A truck is moving with a speed of 40 km/hr and a motor cycle is moving with a speed of 100 km/hr. Which case possess more energy ?
- A weight lifter, lifts a mass of 200 kg over his head and a cricket groundman pulls the roller of mass 500 kg over a pitch. In which case, more energy is required ?
- To find answers to such interesting questions, we need to know work, power and energy.

4.2 WORK

- Work is said to be done by external force if it moves a body through some distance in the direction of force.
- **Definition : Work is defined as the product of force acting on the body and the displacement produced.**
- **Work is a scalar quantity and thus it is representated by only magnitude.**
- When force acts on a body and if a body undergoes some displacement, then work is said to be done.
- But if force acting on a body does not produce any displacement, then work done by it is said to be zero.

Case 1 : Displacement of a body takes place in the direction of force :

- **In this case, the work done is the product of the force and the displacement in the direction of force.**

Let, F = Force on a body

s = Displacement produced

W = Work done

Work done = Force × Displacement

$$\boxed{W = F \times s}$$

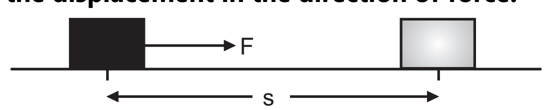


Fig. 4.1

Case 2 : Displacement of a body does not take place in the direction of force :

- Consider a motion of hand-roller on the ground.
- Here, force and displacement are not in the same direction. Hence, the magnitude of the component of force which is in the direction of displacement is considered.

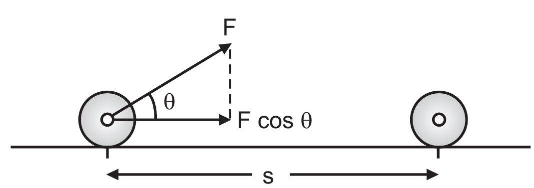


Fig. 4.2

- **Thus, work done is the product of the component of the force in the direction of motion and the displacement.**
- Let, F = Force on a body

  θ = Angle between the force and the motion produced

  ∴ F cos θ = Component of the force in the direction of motion

  s = Displacement

  Then, Work done = $F \cos\theta \times s$

  ∴ $$\boxed{W = F \cos\theta \times s}$$

  [Thus, if θ = 0, then W = F × s. If θ = 90° , then W = 0.]

Unit : Since

$$\text{Work} = \text{Force} \times \text{Displacement} = \text{Newton} \times \text{meter} = \text{Nm}$$

- Thus, S.I. unit of work is newton meter or joule.

$$1 \text{ Nm} = 1 \text{ joule}$$

- **Thus, one joule is the work done by a force of one newton when it displaces a body through one meter**.
- The C.G.S. unit of work is erg.
- The **M.K.S.** unit of work is joule.
- Work done can also be represented in units of power × time. Thus, kilowatt-hour is also the unit of work. One kilowatt hour is the work done in 1 hr if an agent is working at the rate of 1 kW.

4.3 POWER

- **Power is defined as the rate of doing work**. It is measured by the ratio of work done to the time taken.

$$\therefore \quad \text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

- **Power is also a scalar quantity.**
- **Unit of power :**

Since

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Nm}}{\text{sec}} = \frac{\text{joule}}{\text{sec}} = \text{watt}$$

- Thus, **S.I.** unit of power, $1 \text{ watt} = \dfrac{1 \text{ joule}}{\text{sec}} = 1 \text{ Nm/sec}$
- **Thus, if one joule of work is done in one second, then power is said to be one watt**.

4.4 ENERGY

- **Definition : The energy of a body is defined as its capacity to do work. Energy is a scalar quantity.**
- Energy has the same **unit** as work i.e. **joule** or **erg**.
- Energy has other units – calorie, electron volt, kilowatt hour.
- Energy exists in nature in different forms like (1) mechanical energy, (2) heat energy, (3) light energy, (4) sound energy, (5) chemical energy, (6) electrical energy, (7) magnetic energy, (8) nuclear energy, (9) mass energy.
- All forms of energy are transferable.
- For example, light energy can be converted into electrical energy i.e. photo-electric effect.
- The two common forms of energy are (1) potential energy, (2) kinetic energy.

4.5 EQUATIONS FOR P.E. AND K.E.

The mechanical energy of a body exists in the form of kinetic energy and gravitational potential energy.

1. Potential Energy

- **The energy possessed by a body due to its position is called as potential energy.**
- The earth attracts any body placed on it or close to it. Consider a body of mass 'm', lifted to height 'h' from the surface of the ground. If this body is now released, it will fall to ground and work done will be equal to m × g × h.

$$\therefore \quad \text{P.E.} = \text{weight} \times h$$

$$\therefore \quad \boxed{\text{P.E.} = mgh}$$

S.I. unit of P.E. is **Nm** or **joule**.

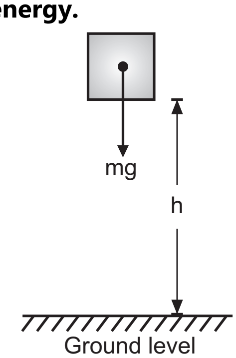


Fig. 4.3

2. Kinetic Energy

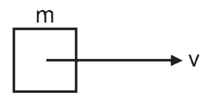


Fig. 4.4

- **Definition : The energy possessed by a body by virtue of its motion is called as kinetic energy.**
- Consider a body of mass 'm' moving with a velocity 'v'.
- Then its kinetic energy is given by

$$\boxed{K.E. = \frac{1}{2} mv^2}$$

- The **S.I.** unit of kinetic energy is **Nm** or **joule**.

4.6 WORK-ENERGY PRINCIPLE

- **It states that the work done by a system of forces acting on a body between any two points is equal to the change in kinetic energy of a body between these same two points.**
- Consider a body of mass 'm'.
- Let, F be the force on it.

  v_1 be the velocity at position P_1.

  v_2 be the velocity at position P_2.

  d be the displacement of a body.

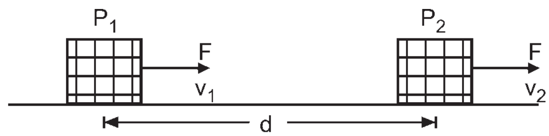


Fig. 4.5

$$\text{K.E. at } P_1 = \frac{1}{2} mv_1^2$$

$$\text{K.E. at } P_2 = \frac{1}{2} mv_2^2$$

$$\therefore \quad \text{Change in K.E.} = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2 = \frac{1}{2} m (v_2^2 - v_1^2) \quad \text{... (1)}$$

$$\text{Work done} = \text{Force} \times \text{Displacement}$$

$$= (F) \times d = (m \times a) \times d$$

Using equations of motion, it can be proved that

$$\boxed{\text{Work done = Change in K.E.}}$$

Law of Conservation of Energy :

- It states that, **energy can neither be created nor be destroyed but can be transformed from one form to another form**. The sum of the energy in its various forms remains constant in the universe.

4.7 REPRESENTATION OF WORK BY USING A GRAPH

- Work can be representated by a graph of force against displacement. The values of force are taken on the Y-axis and displacement on the X-axis.
- Since work done is equal to the product of the force and displacement, the area under the graph represents the work done. Such diagrams are known as force-displacement diagrams.

Case 1 : Work done by a constant force : Since the force is constant, the graph is horizontal. Area under the graph is rectangular.

$$\text{Area under the graph} = OA \times OC = F \times s$$

$$\text{Area under the graph} = \text{Work done}$$

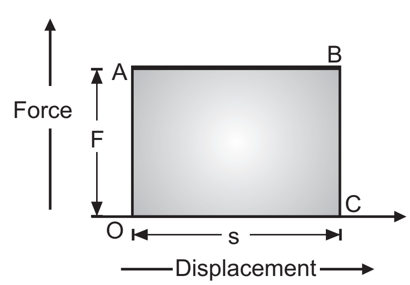


Fig. 4.6

Case 2 : Work done by gradually increasing force : In this case, the graph is a straight line and inclined with a positive slope. Area under the graph is triangular.

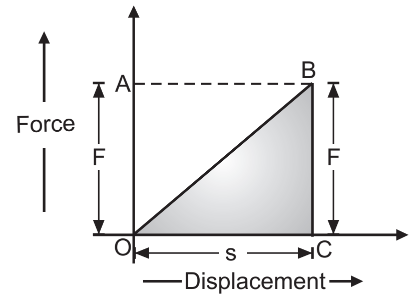


Fig. 4.7

$$\text{Area under the graph} = \frac{1}{2}(OC \times BC)$$
$$= \frac{1}{2}(s \times F)$$
$$= \text{Work done}$$

Case 3 : Work done by gradually decreasing force : In this case, graph is a straight line and inclined. Area under the graph is triangular.

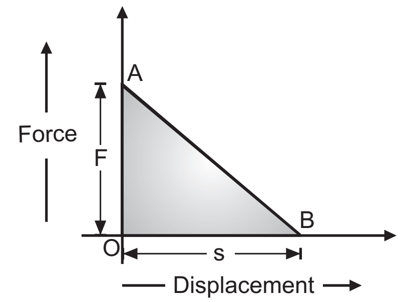


Fig. 4.8

$$\text{Area under the graph} = \frac{1}{2}(OB \times OA)$$
$$= \frac{1}{2}(s \times F)$$
$$= \text{Work done}$$

Case 4 : Work done by variable force : In this case, the force-displacement graph is a trapezium.

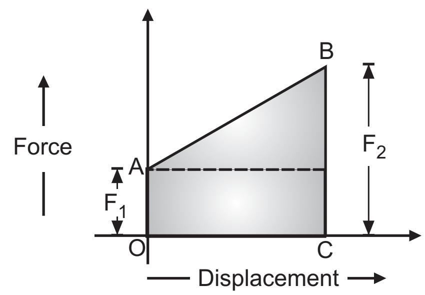


Fig. 4.9

$$\text{Area under the graph} = \frac{1}{2}(OA + CB) \times OC$$
$$= \frac{1}{2}(F_1 + F_2) \times s$$
$$= \text{Work done}$$

Thus, area under the graph represents the work done.

4.8 WORK DONE BY A TORQUE

- Two equal but opposite forces acting on different parts of a body which cause the body to rotate is called as torque.

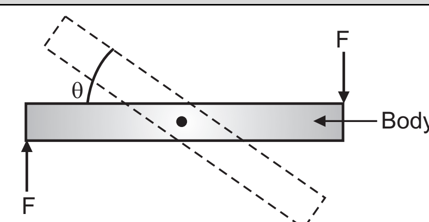


Fig. 4.10

When torque rotates a body then work is said to be done.

$$\text{Work done} = \text{Torque} \times \text{Angular displacement}$$
$$= \tau \times \theta$$

or $$\text{Work done} = \tau(\theta_f - \theta_i)$$

where, τ = Torque

θ_i = Initial angular position before work is done

θ_f = Final angular position after work is done

Important Points

- If force applied on a body produces displacement, then work is said to be done. Work is a scalar quantity and given by, work = force × displacement. Its unit is **joule**.
- Power is defined as rate of doing work. Unit of power = watt = $\frac{\text{joule}}{\text{second}}$ N-m/s
- Energy is the capacity to do work. Unit is **joule**.
- Potential energy and kinetic energy are two common forms of energy.
- Work-energy principle is work done = change in kinetic energy.

- Law of conservation of energy states that, energy can neither be created nor destroyed but can be transformed from one form to another form. Sum of the energy in various forms remains constant.
- In case of force-displacement graph, the area under the graph gives the work done.
- If a graph of force and displacement is plotted, force in N on Y-axis and displacement in m on X-axis is plotted then area under the graph gives work done by the system.

Formulae

$$\text{Work} = \text{Force} \times \text{Displacement}$$
$$= F \times s$$
$$= F \cos\theta \times s \quad \text{... if force is not in the direction of displacement}$$
$$\text{Work} = mgh \text{ i.e. potential energy}$$
$$\text{Work} = \text{Area under the graph}$$
$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$
$$= \frac{\text{Force} \times \text{Displacement}}{\text{Time}}$$
$$= \text{Force} \times \text{Velocity}$$
$$W = \tau \times \theta$$
$$= \tau \times (\theta_f - \theta_i)$$
$$\text{Efficiency of a pump} = \frac{\text{Output power}}{\text{Input power (i.e. power of pump)}}$$

Conversions : For water, 1 litre = 1 kg

Volume of 1 m^3 = 1000 litres = 1000 kg water

SOLVED EXAMPLES

Example 1 : *The load is pulled 50 m along the horizontal by a force of 500 N at 60° to the horizontal. Calculate the work done.*

Solution : Displacement, s = 50 m

Force, F = 500 N

θ = 60°

Work = ?

Work = Component of force along displacement × Displacement

$= (F \cos\theta) \times s$

$= (500 \times \cos 60°) \times 50$

W = 12500 J

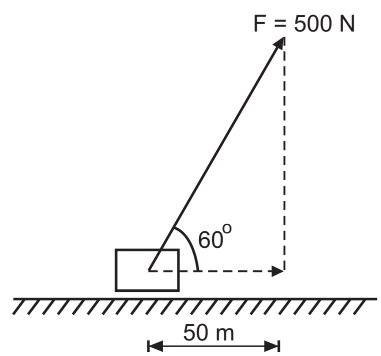


Fig. 4.11

Example 2 : *100 litres of water is pumped to a height of 40 m. Calculate the work done by the pump.*

Solution : Given : Volume = 100 litres

$\therefore$ m = 100 kg ($\because$ water)

Height, h = 40 m

Work = ?

Take density of water = 1000 kg/m^3

Work done = $mgh = 100 \times 9.8 \times 40$

W = 39200 J

Example 3 : *A man pulls a hand roller on a cricket pitch with a force of 150 N inclined at an angle of 45° to the horizontal. Find the work done in pulling the roller over a pitch of 20 m long.*

Solution : Given :

$$F = 150 \text{ N}$$
$$\theta = 45°$$
$$s = 20 \text{ m}$$
$$W = ?$$
$$\text{Work done} = (F \cos \theta) \times s$$
$$= (150 \times \cos 45°) \times 20$$
$$\boxed{W = 2121.3 \text{ J}}$$

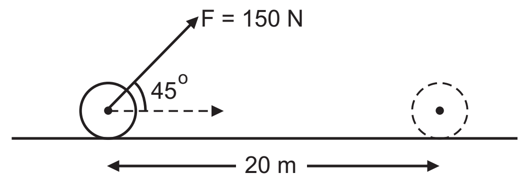


Fig. 4.12

Example 4 : *A constant force of 200 N moves a body through a distance of 70 m. Find the work done.*

Solution : Given : F = 200 N, s = 70 m

Area of □ OABC gives work done.

$$\text{Work done} = \text{Area of } \square \text{ OABC}$$
$$= 200 \times 70$$
$$\boxed{\text{Work} = 14000 \text{ J}}$$

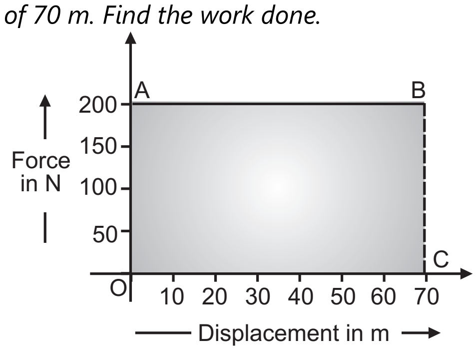


Fig. 4.13

Example 5 : *The springs of bull worker are 40 cm long when unstretched. It is stretched to 110 cm by applying a force which varies at the rate of 6 N per cm stretch, find the work done in stretching the spring.*

Solution : The **distance** for which force acts (net displacement) = Stretched length – Unstretched length

$$= (110 \text{ cm} - 40 \text{ cm})$$
$$\boxed{\text{Displacement} = 70 \text{ cm}}$$

Force is increasing at the rate of 6 N per cm.

$$\therefore \quad \text{Final force} = 6 \times 70$$
$$\boxed{\text{Final force} = 420 \text{ N}}$$
$$\text{Work done} = \text{Area of } \Delta \text{ OBC}$$
$$= \frac{1}{2} \times OC \times BC$$
$$= \frac{1}{2} \times 70 \times 420$$
$$= 14700 \text{ N-cm}$$
$$\text{Work} = 147 \text{ Nm}$$
$$\boxed{\text{Work} = 147 \text{ J}}$$

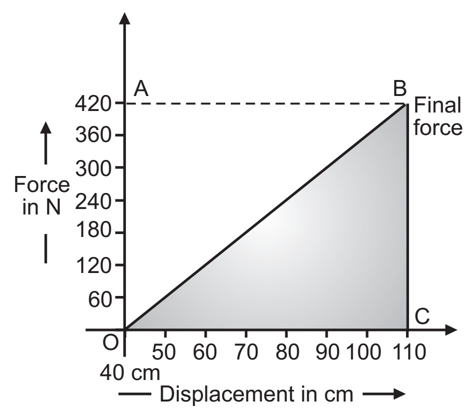


Fig. 4.14

Example 6 : *A 30 m long cable having weight of 12 N per meter hangs vertically from a drum. How much work is done in winding the cable completely ?*

Solution : In this case, the initial force required is maximum and then **gradually it goes on decreasing** to zero because the hanging length of the cable decreases gradually.

$$\text{Initially at start} \rightarrow \text{Weight of the cable}$$
$$= 12 \times 30$$
$$= 360 \text{ N}$$
$$\text{Final weight} = 0$$
$$\text{and} \quad \text{Total displacement} = 30 \text{ m}$$

Hence, the force-displacement graph will be as shown in Fig. 4.15.

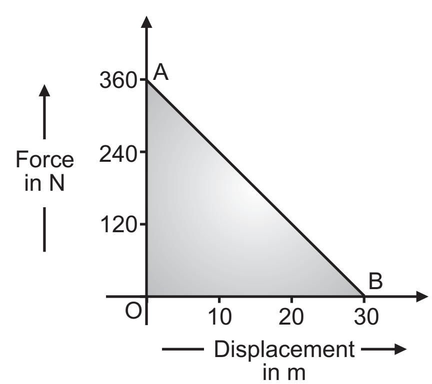


Fig. 4.15

Work done in winding the cable completely = Area of Δ OAB

$$= \frac{1}{2} \times OB \times OA = \frac{1}{2} \times (30) \times (360)$$

Work = 5400 Nm

Work = 5400 J

Example 7 : *A chain weighing 40 N/m has a length of 50 m. It hangs vertically. A weight of 1500 N is attached at the lower end of the chain. Find the work done in raising the weight through 20 m by winding the chain.*

Solution : Total weight of the chain = $40 \times 50 = 2000$ N

Weight attached = 1500 N

$\therefore$ Initial force required = Weight of chain + Attached weight

= 2000 + 1500 = 3500 N

When weight is lifted by winding the chain the length of the hanging portion of the chain decreases and hence force also decreases.

$\therefore$ Final force after winding 20 m length of the chain = Weight of (50–20) meter of chain + Attached weight

= Weight of 30 meter of chain + Attached weight

= (40×30) + 1500 N

= 1200 + 1500 N

= 2700 N

Weight is lifted through 20 m, hence displacement = 20 m.

Now, graph of force-displacement can be drawn. While drawing the graph, consider

Initial force = 3500 N

Final force = 2700 N

Displacement = 20 m

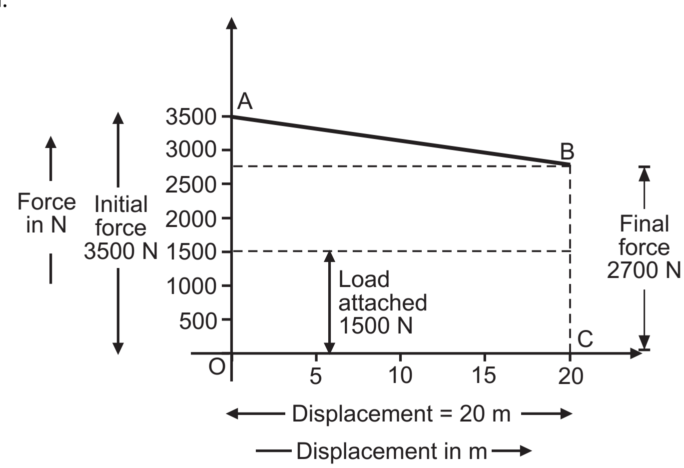


Fig. 4.16

Now, Work done for raising the weight through 20 m = Area under the graph

= Area of the trapezium OABC

Work done = $\frac{1}{2}$ (OA + CB) $\times$ OC

= $\frac{1}{2}$ (3500 + 2700) $\times$ 20

Work = 62000 Nm

Work = 62000 J

Example 8 : *A rocket motor exerts a thrust of 2.4 MN at a speed of 300 m/s. Find the power developed.*

Solution : Given : Force = Thrust = 2.4 MN

Velocity = Speed = 300 m/s

Power = ?

We have, $\text{Power} = \dfrac{\text{Work}}{\text{Time}}$

$$= \frac{(\text{Force} \times \text{Displacement})}{\text{Time}} = \text{Force} \times \frac{\text{Displacement}}{\text{Time}} = \text{Force} \times \text{Velocity}$$

$$= 2.4 \times 300$$

$$\boxed{\text{Power} = 720 \text{ MW}}$$

Example 9 : *A lift of mass 500 kg is being raised with a uniform velocity of 1.5 m/s. Find the power involved in it.*

Solution : Given : Mass = 500 kg

∴ Force = Weight = $m \times g = (500 \times 9.8)$

Velocity = 1.5 m/s

We have, Power = Force × Velocity = $(500 \times 9.8) \times (1.5)$

$$\boxed{\text{Power} = 7350 \text{ watt}}$$

Example 10 : *A water tank at ground level having volume capacity of 5 m^3 is completely filled. Water in it is to be lifted in another tank which is at a height of 24 m. This task is done in half an hour by a pump. If the efficiency of the pump is 60%, find the power required.*

Solution : Given : 5 m^3 of volume = 5000 litres = 5000 kg

∴ Mass m = 5000 kg, t = 30 min = (30×60) sec

$$\text{Efficiency} = 60\% = \frac{60}{100} = 0.6$$

Power of a pump = ?

We have, Work = mgh = $(5000 \times 9.8 \times 24) = 1176000$ J

$$\therefore \quad \text{Power} = \frac{1176000}{(30 \times 60)} \qquad \text{... (∵ time is half an hour)}$$

$$= 653.3 \text{ watt}$$

We have, $\text{Efficiency of a pump} = \dfrac{\text{Output power}}{\text{Input power of a pump}}$

$$\therefore \quad \text{Input power of a pump} = \frac{\text{Output power}}{\text{Efficiency of a pump}} = \frac{653.3}{(60/100)}$$

$$\boxed{\text{Input power of a pump} = 1088.89 \text{ watt}}$$

Example 11 : *A water tank of capacity 1000 litres is to be filled in 5 minutes by a pump. Water is required to be lifted through a height of 20 m. If the efficiency of the pump is 70%, find the power of pump.*

Solution : Given : 1000 litres = 1000 kg of water

t = 5 min = (5×60) sec

h = 20 m

$$\text{Efficiency} = 70\% = \frac{70}{100} = 0.7$$

Power of pump = ?

We have, Work = mgh = $1000 \times 9.81 \times 20 = 196200$ J

$$\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{196200}{(5 \times 60)} = 654 \text{ watt}$$

Now, $\text{Efficiency} = \dfrac{\text{Power}}{\text{Power of pump}}$

$$\therefore \quad \text{Power of pump} = \frac{\text{Power}}{\text{Efficiency}} = \frac{654}{(70/100)}$$

$$\boxed{\text{Power of pump} = 934.29 \text{ watt}}$$

Example 12 : *How many litres of water can be raised in 15 min to a height of 24 m by using a pump of 12 kW ?*

Solution : Given :

$$t = 15 \text{ min} = (15 \times 60) \text{ sec}$$
$$h = 24 \text{ m}$$
$$\text{Power} = 12 \text{ kW} = 12000 \text{ watt}$$
$$\text{Amount of water in litres} = ?$$

We have,

$$\text{Power} = \frac{\text{Work}}{\text{Time}}$$
$$\text{Power} = \frac{mgh}{t}$$
$$\therefore \quad \frac{\text{Power} \times t}{gh} = m$$
$$\therefore \quad m = \frac{\text{Power} \times t}{gh} = \frac{(12000)\,(15 \times 60)}{(9.81)\,(24)}$$
$$m = 45871.56 \text{ kg} \qquad (\because \text{ for water, 1 kg = 1 litre})$$
$$\therefore \quad \boxed{m = 45871.56 \text{ litres}}$$

Example 13 : *A steam boat moves at a speed of 30 km/hr. It requires a force of 90 kN to overcome the water resistance. Find the power developed.*

Solution : Given :

$$v = 30 \text{ km/hr} = \frac{30 \times 1000}{(60 \times 60)} = 8.33 \text{ m/s}$$
$$F = 90 \text{ kN} = 90 \times 10^3 \text{ N}$$
$$P = ?$$
$$\text{Power} = \text{Force} \times \text{Velocity} = (90 \times 10^3) \times (8.33)$$
$$\boxed{\text{Power} = 749.7 \times 10^3 \text{ watt}}$$

Example 14 : *A box of 15 kg falls down from a height of 4 m. Calculate the loss of potential energy.*

Solution : Given :

$$m = 15 \text{ kg}$$
$$h = 4 \text{ m}$$
$$\text{P.E.} = ?$$
$$\text{Loss of P.E.} = mgh = (15) \times (9.81)\,(4)$$
$$\boxed{\text{P.E.} = 588.6 \text{ J}}$$

Example 15 : *A vehicle of mass 82 kg is moving with a speed of 40 km/hr. Calculate its kinetic energy.*

Solution : Given :

$$m = 82 \text{ kg}$$
$$v = 40 \text{ km/hr} = \frac{40 \times 1000}{(60 \times 60)} = 11.11 \text{ m/s}$$
$$\text{K.E.} = ?$$
$$\text{K.E.} = \frac{1}{2} mv^2 = \frac{1}{2} (82)\,(11.11)^2$$
$$\boxed{\text{K.E.} = 5060.72 \text{ J}}$$

Example 16 : *A cubical water tank has a side of 2 m each. It is placed with its base 9 m above the ground level. Find the potential energy of the water tank when the tank is full.*

Solution : Given :

$$\text{Side} = 2 \text{ m}$$
$$\therefore \quad \text{Volume} = 2 \times 2 \times 2 = 8 \text{ m}^3$$
$$\therefore \quad \text{Mass} = 8 \times 10^3 \text{ kg} \qquad (\because 1 \text{ m}^3 = 1000 \text{ litres} = 1000 \text{ kg})$$

Base of the tank is of 9 m.

$\therefore$ Centre of the tank is (9 + 1) m away from the ground.

$$\therefore \quad h = 10 \text{ m}$$
$$\text{P.E.} = mgh = (8 \times 10^3)\,(9.81)\,(10)$$
$$\boxed{\text{P.E.} = 784.8 \times 10^3 \text{ J}}$$

Example 17 : *A train weighing 2000 kN is running at a speed of 36 km/hr. The train is brought to rest in 60 m. What is the resistance per kilo newton weight of the train ?*

Solution : Given :

$$\text{Weight} = 2000 \text{ kN} = 2000 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mass} = \frac{2000 \times 10^3}{9.81} = 203.87 \times 10^3 \text{ kg}$$

$$v = 36 \text{ km/hr} = \frac{36 \times 1000}{(60 \times 60)} = 10 \text{ m/s}$$

$$s = 60 \text{ m}$$

$$\therefore \quad \text{K.E. of the train} = \frac{1}{2} mv^2 = \frac{1}{2} (203.87 \times 10^3)\,(10)^2 = 10193.5 \times 10^3 \text{ J}$$

Now,

$$\text{K.E.} = \text{Work done}$$

$$(10193.5 \times 10^3) = F \times s$$

$$(10193.5 \times 10^3) = F \times 60$$

$$\therefore \quad F = \frac{(10193.5 \times 10^3)}{60} = 169.89 \times 10^3 \text{ N}$$

This is the total resistive force for 2000 kN train.

$$\therefore \quad \boxed{\text{Force per kN} = \frac{(169.89 \times 10^3)}{(2000)} = 84.95 \text{ N}}$$

Example 18 : *A nail is driven in a wall by a hammer of 1.5 kg. The nail penetrates 2 cm in the wall. The velocity of the hammer is 10 m/s. Find the resistance offered by the wall to penetration.*

Solution : Given :

$$m = 1.5 \text{ kg}$$

$$v = 10 \text{ m/s}$$

$$s = \text{Penetration} = 2 \text{ cm} = 0.02 \text{ m}$$

$$\text{Resistance of the wall} = ?$$

$$\text{K.E. of hammer} = \frac{1}{2} mv^2 = \frac{1}{2} (1.5)\,(10)^2 = 75 \text{ J}$$

But,

$$\text{Work done} = \text{Force} \times \text{Displacement}$$

Hence,

$$\text{Work done during penetration} = \text{Resistance of wall} \times \text{Penetration}$$

$$= F \times s$$

Now,

$$\text{K.E. of hammer} = \text{Work done during penetration}$$

$$75 = F \times (0.02)$$

$$\therefore \quad \boxed{F = 3750 \text{ N}} \quad \text{... (Resistive force of wall)}$$

Example 19 : *A force of 18 N is used to lift an object over a height of 2 m. Calculate the potential energy gained by the object.*

Solution : Given :

$$F = 18 \text{ N}$$

$$s = 2 \text{ m} \quad \text{(height = distance covered)}$$

$$W = \text{P.E.} = ?$$

$$\text{P.E.} = \text{Work done} = \text{Force} \times \text{Displacement}$$

$$= 18 \times 2$$

$$\boxed{\text{P.E.} = 36 \text{ J}}$$

Example 20 : *Porter lifts a suitcase weighing 20 kg from the platform and puts it on his head 2 m above the platform. Calculate the work done by the porter on the suitcase.*

Solution : Given :

$$m = 20 \text{ kg}$$
$$h = 2 \text{ m}$$
$$w = ?$$

$\therefore$ Work done = mgh

$$\boxed{\text{Work done} = 392 \text{ J}}$$

Questions

1. Define work. Give its S.I. unit.
2. Define power. Give its S.I. unit.
3. Define energy. State the two common forms of energies.
4. State the work-energy principle.
5. Explain the graphical representation of work done.

BTE Questions

1. Define :
   (i) Power
   (ii) Kinetic energy.
2. Define K.E. and P.E. State the equations.
3. Write any two points to distinguish between work and energy.
4. Define power. State its S.I. unit.
5. Define work and state its S.I. unit.
6. Which is meant by torque ? Give an equation for work done by a torque.

Unsolved Problems

1. A horse exerts a pull of 300 N on a cart and moves it through a distance of 40 m. Find the work done. (**Ans.** W = 12000 J)
2. 500 litres of water is pumped to a height of 50 m. Calculate the work done by the pump. (**Ans.** 245250 J)
3. A man pulls hand roller on a cricket pitch with a force of 125 N inclined at an angle of 40° to the horizontal. Find the work done in pulling the roller over a pitch of 21 m long. (**Ans.** 2010.87 J)
4. A constant force of 250 N moves a body through a distance of 90 m. Find the work done. (**Ans.** Work = Area under the graph = 22500 J)
5. The springs of chest expander are 30 cm when unstretched. It is stretched to 120 cm by applying a force which varies at the rate of 7 N per cm stretch. Find the work done in stretching the spring. (**Ans.** Work = Area under the graph = 283.5 J)
6. A 40 m long cable having weight of 14 N per meter hangs vertically from a drum. How much work is done in winding the cable completely ? (**Ans.** Work = Area under the graph = 11200 J)
7. A chain weighing 30 N/m has a length of 25 m. It hangs vertically. A weight of 1200 N is attached at the lower end of the chain. Find the work done in raising the weight through 15 m. (**Ans.** Work = Area under the graph = 25875 J)
8. A lift of mass 1500 kg is being raised with a uniform velocity of 1.3 m/s. Find the power involved in it. (**Ans.** 19129.5 watt)
9. A water tank at ground level having volume capacity of 10 m^3 is completely filled. Water in it is to be lifted in an other tank which is at a height of 20 m. This task is done in 1 hour by a pump. If the efficiency of the pump is 75%, find the power required. (**Ans.** 725.93 watt)
10. 300 cubic meters of water is to be raised to a reservoir at a height of 10 meters in 10 min. Calculate the power of the pump, if the efficiency of the pump is 80%. (**Ans.** 61312 watt)
11. A steam boat moves at a speed of 20 km/hr. It requires a force of 100 kN to overcome the water resistance. Find the power developed. (**Ans.** 555.56×10^3 watt)
12. A load of 21 kg falls down from a height of 11 m. Calculate the loss of potential energy. (**Ans.** 2266.1 J)
13. A vehicle of 120 kg is moving with a speed of 50 km/hr. Calculate its kinetic energy. (**Ans.** 11575.9 J)
14. A cubical water tank has side of 4 m each. It is placed with its base 11 m above the ground level. Find the potential energy of the water tank when the tank is full. (**Ans.** 8161.9×10^3 J)
15. A train weighing 1500 kN is running at a speed of 40 km/hr. The train is brought to rest in 50 m. What is the resistance per kilo newton of the train ? (**Ans.** Resistive force = 188734.1 N and resistance per kilo newton = 125.82 N)

16. A nail is driven in a wall by a hammer of 2.5 kg. The nail penetrates 3 cm in the wall. The velocity of the hammer is 10 m/s. Find the resistance offered by the wall to penetration. (**Ans.** 4166.7 N)

MCQs on Work, Power and Energy

1. *Work is given by relation ……*
   (a) $work = \frac{force}{displacement}$ (b) $force = work \times displacement$
   (c) $work = force + displacement$ (d) $work = force \times displacement$
2. *In the case of motion of hand roller, the work done is given by …*
   (a) $work = force \times displacement$ (b) $work = force/displacement$
   (c) $work = \text{component of force in the direction of motion} \times displacement$
   (d) $work = \frac{displacement}{force}$
3. *Power is defined as ……*
   (a) *time per work done* (b) *rate of work done w.r.t. time*
   (c) *amount of work done* (d) *work done per unit mass*
4. *Work is a …… quantity, power is a …… quantity.*
   (a) *scalar, scalar* (b) *scalar, vector* (c) *vector, vector* (d) *vector, scalar*
5. *S.I. unit of work done is ……*
   (a) *newton* (b) *dyne* (c) *watt* (d) *joule*
6. *S.I. unit of power is ……*
   (a) *newton* (b) *dyne* (c) *watt* (d) *joule*
7. *1 watt is given by ……*
   (a) $1\ W = 1\ J/1\ s$ (b) $1\ W = 1\ J \times 1s$ (c) $1\ W = 1\ s/1\ J$ (d) *none of these*
8. *The unit of work and energy are ……*
   (a) *joule, joule* (b) *joule, watt* (c) *watt, joule* (d) *joule, newton*
9. *Potential energy is a stored form of energy and given by ……*
   (a) $P.E. = mg/h$ (b) $P.E. = mgh$ (c) $P.E. = h/mg$ (d) $P.E. = m/gh$
10. *Kinetic energy is given by ……*
    (a) $K.E. = 2\ mv^2$ (b) $K.E. = \frac{1}{2}\ mv$ (c) $K.E. = mv^2$ (d) $K.E. = \frac{1}{2} mv^2$
11. *Work-energy principle states that work done by a system of forces acting on a body between any two points is equal to ……*
    (a) *change in P.E.* (b) *additions of K.E.* (c) *change in K.E.* (d) *additions of P.E.*
12. *Power is given by relation ……*
    (a) $power = force \times velocity$ (b) $power = \frac{force}{velocity}$ (c) $power = \frac{velocity}{force}$ (d) *none of these*
13. *Efficiency of a pump is given by ……*
    (a) $efficiency = \frac{input\ power}{output\ power}$ (b) $efficiency = \frac{output\ power}{input\ power}$
    (c) $efficiency = output\ power \times input\ power$ (d) $efficiency = output\ power + input\ power$
14. *Tank of volume 1 m^3 occupies ……*
    (a) *760 kg of water* (b) *1250 kg of water* (c) *1000 kg of water* (d) *1 gm of water*
15. *For water, 1 litre is equal to ……*
    (a) *1 kg* (b) *0.850 kg* (c) *1.25 kg* (d) *none of these*
16. *Force of 10 N applied on a body produces displacement of 10 m, the work done will be ……*
    (a) *1 J* (b) *100 J* (c) *20 J* (d) *200 J*
17. *A man pulls a hand roller on a cricket pitch with a force of 200 N inclined at an angle of 60° to the horizontal. The work done in pulling a roller over a pitch of 20 m long will be ……*
    (a) *100 J* (b) *200 J* (c) *500 J* (d) *2000 J*
18. *1000 litres of water is pumped to a height of 50 m. The work done by a pump is ……*
    (a) $9.8 \times 10^5\ J$ (b) $2 \times 10^5\ J$ (c) $4.9 \times 10^5\ J$ (d) $20 \times 10^5\ J$
19. *A rocket motor exerts a thrust of 2 MN at a speed of 250 m/s. Power developed in this case will be ……*
    (a) *100 MW* (b) *500 MW* (c) *1000 MW* (d) *1500 MW*
20. *A lift of weight 500 N is being raised with uniform velocity of 2 m/s. Power involved in it will be ……*
    (a) *1 kN* (b) *10 kN* (c) *100 kN* (d) *200 kN*

21. *Work of 1.5×10^6 J is done in half hour. If the efficiency of the pump is 70%, the power of the pump required will be ……*
    (a) 1190 watt (b) 510 watt (c) 1510 watt (d) 2090 watt
22. *A water tank of 1000 litres is to be lifted in 10 min to 'a' height of 40 m. If the efficiency of the pump is 80%, the required power of the pump will be ……*
    (a) 407 W (b) 512 W (c) 817 W (d) 911 W
23. *A vehicle of mass 100 kg is moving with a speed of 36 km/hr. Its kinetic energy will be ……*
    (a) 2000 J (b) 5000 J (c) 7000 J (d) 8000 J
24. *A cubical water tank has side 1 m each. It is placed with its base 9.5 m above the ground level. Potential energy of the tank when water tank is full will be ……*
    (a) 2.4×10^4 J (b) 4.9×10^4 J (c) 19.6×10^4 J (d) 9.8×10^4 J
25. *A train of mass 2×10^5 kg is running at a speed of 54 km/hr. Train is brought to rest in 100 m. Resistive force of the train will be ……*
    (a) 1×10^5 N (b) 2.25×10^5 N (c) 5×10^5 N (d) 10×10^5 N
26. *A nail is driven in a well by a hammer of 2 kg. The nail penetrates 3 cm in the wall. The velocity of the hammer is 10 m/s. Resistance offered by the wall to penetration is ……*
    (a) 3.33×10^3 N (b) 6.66×10^3 N (c) 9.99×10^3 N (d) 1.2×10^3 N
27. *A force of 24 N is used to lift an object over a height of 3 m. Potential energy gained by the object will be ……*
    (a) 8 J (b) 12 J (c) 72 J (d) 92 J
28. *Porter lifts a suitcase weighing 25 kg from the platform and puts on his head 2 m above the platform. Work done by the porter on the suitcase will be ……*
    (a) 110 J (b) 220 J (c) 390 J (d) 490 J
29. *If a graph of force on Y-axis and displacement on X-axis is plotted, then the area under the graph represents (gives) ……*
    (a) pressure (b) work done (c) force applied (d) total displacement
30. *Graph of force verses displacement for a work done by gradually increasing force is ……*
    (a) curved (b) horizontal line
    (c) straight line inclined with positive slope (d) straight line inclined with negative slope
31. *The force as a function of displacement of moving object is presented by a graph. How much work is done when the object moves from 0 m to 6 m ?*

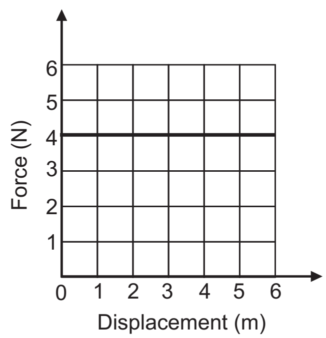


Fig. 4.17

    (a) 1.5 J (b) 10 J (c) 24 J (d) 42 J
32. *The force as a function of displacement of moving object is presented by the graph.*

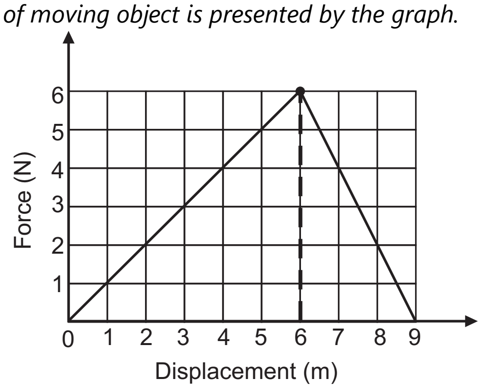


Fig. 4.18

How much work is done when the object moves from 0 to 6 m ?
    (a) 36 J (b) 12 J (c) 24 J (d) 18 J

33. *(Refer Fig. 4.18). How much work is done when the object moves from 6 m to 9 m ?*
    (a) 9 J *(b) 12 J* *(c) 24 J* *(d) 54 J*
34. *(Refer Fig. 4.18). How much work is done when the object moves from 0 to 9 m ?*
    (a) 0 J *(b) 12 J* *(c) 27 J* *(d) 54 J*
35. *The force as a function of displacement of a moving body is presented by the graph. How much work is done when the object moves from 0 m to 5 m ?*

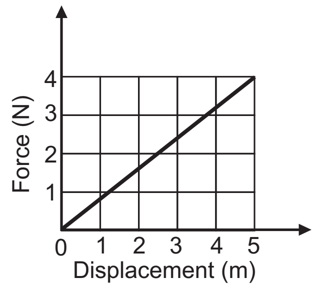


Fig. 4.19

(a) 2 m *(b) 5 m* *(c) 8 m* *(d) 10 m*

36. *Graph of force versus displacement for a work done by a gradually decreasing force is ……*
    (a) curved *(b) horizontal line*
    (c) straight line inclined with the slope *(d) straight line inclined with negative slope*

Answers and Hints on Unit-4 – Work, Energy and Power

1. (d)	2. (c)	3. (b)	4. (a)	5. (d)	6. (c)	7. (a)	8. (a)	9. (b)	10. (d)
11. (c)	12. (a)	13. (b)	14. (c)	15. (a)	16. (b)	17. (d)	18. (c)	19. (b)	20. (b)
21. (a)	22. (c)	23. (b)	24. (d)	25. (b)	26. (a)	27. (c)	28. (d)	29. (b)	30. (c)
31. (c)	32. (d)	33. (a)	34. (c)	35. (d)	36. (d)				

16. **Hint :** $W = f \times s = 10 \times 10 = \mathbf{100\ J}$
17. **Hint :** $W = F \cos\theta \times s = 200 \cos 60 \times 20 = \mathbf{2000\ J}$
18. **Hint :** $W = mgh = 1000 \times 9.8 \times 50 = 4.9 \times 10^5$ J
19. **Hint :** Power = Force × Velocity = $2 \times 250 =$ **500 MW**
20. **Hint :** Power = Force × Velocity = $5000 \times 2 = 10{,}000$ N
21. **Hint :** Output power $= \dfrac{\text{Work}}{\text{Time}} = \dfrac{1.5 \times 10^6}{(30 \times 60)} = \mathbf{833\ W}$

    Efficiency $= \dfrac{\text{Output power}}{\text{Input power}}$ $\therefore$ Input power $= \dfrac{\text{Output power}}{\text{Efficiency}} = \dfrac{833}{70/100} = 1190$ W
22. **Hint :** $W = mgh = 1000 \times 9.8 \times 40 = 392000$ J, Output power $= \dfrac{W}{t} = \dfrac{392000}{(10 \times 60)} = 653.33$ watt

    Input power $= \dfrac{\text{Output power}}{\text{Efficiency}} = \dfrac{653.33}{(80/100)} = \mathbf{816.7\ W}$
23. **Hint :** v = 36 km/hr = 10 m/s, K.E. $= \dfrac{1}{2} mv^2 = \dfrac{1}{2} (100)\ (10)^2 = \mathbf{5000\ J}$
24. **Hint :** $v = 1\ m^3$ $\therefore$ M = 1000 kg, height of centre of tank from ground = 9.5 + 0.5 = 10 m
    P.E. $= mgh = 1000 \times 9.8 \times 10 = \mathbf{9.8 \times 10^4\ J}$
25. **Hint :** K.E. = Work done, $\dfrac{1}{2} mv^2 = F \times s$, v = 54 km/hr = 15 m/s $\dfrac{1}{2} \times 2 \times 10^5 \times (15)^2 = F \times 100$ $\therefore F = \mathbf{2.25 \times 10^5\ N}$
26. **Hint :** $\dfrac{1}{2} mv^2 = Fs$, $\dfrac{1}{2} \times 2 \times 10^2 = F \times 0.03 = 3.33 \times 10^3$ N
27. **Hint :** P.E = Work = $F \times s = 24 \times 3 = 72$ J
28. **Hint :** Work = $mgh = 25 \times 9.8 \times 2 =$ **490 J**

❐❐❐

Chapter 5

GENERAL PROPERTIES OF MATTER

5.1 ELASTICITY

Introduction :

We know that in case of solids, the distance between two molecules is small and fixed. Solids have good amount of force of attraction between their molecules. The **molecules of solid are at fixed distance from each other** and have strong force of attraction between them and the **molecules behave as if they are connected with the help of springs**. Thus if **forces** from all sides are applied on a body then the **body gets compressed** and if **these forces are removed** then the **body regains its size** and **shape** and **becomes same as earlier**.

5.1.1 Elasticity - Molecular Theory of Elasticity (Explanation of Elasticity)

1. Consider an elastic body as shown in Fig. 5.1 (a). Molecules are at equal distances from each other. There is a force of attraction between the molecules. Hence, molecules are at equal distances.
2. Refer Fig. 5.1 (b). Consider an external force applied on a body. Because of external force, every **molecule starts moving undesirely** in the direction of force. They stop moving at a particular position where this applied force is balanced due to equal and opposite force.
3. Refer Fig. 5.1 (c). Due to shifting of molecules, a body changes its size and shape and the body is said to be **deformed**.

Deforming force (Definition) : *The* ***force*** *applied on body which is* ***responsible to deform*** *(change size and shape of the body) is called as deforming force.*

$$\text{Deformation} \propto \text{Deforming force}$$

Under deformed condition, every shifted molecule tries to achieve its original position due to which an **internal restoring force** is developed inside a body.

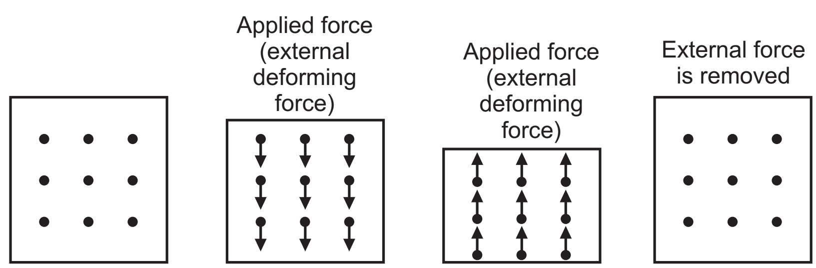
Internal restoring force : *Under deformed condition,* ***every shifted molecule tries to achieve its original position*** *(because of elastic property). When a body is deformed because of external force, internal molecular forces are set up within the body, which tend to oppose the changes in size and shape of the body.*

Internal restoring force (Definition) : *The* ***force*** *which is* ***responsible to restore original size and shape*** *of the body is called as* ***internal restoring force****.*

$$\text{Applied force} = \text{Internal restoring force}$$

4. Refer Fig. 5.1 (d). Now if external applied force is removed then because of internal restoring force, body regains its original size and shape i.e. *Elasticity*.

Body regains its original size and shape (exactly) on removal of external deforming force if and only if external force is within a certain limit called as *elastic limit*. If external force is too large then the intramolecular structure of body collapses and there will be permanent deformation (i.e. body does not regain its original size and shape on removal of external deforming force) and the body looses its elasticity.



(a) Original body **(b)** Particles start moving in the direction of applied force **(c)** Body is deformed (no more original form) **(d)** Body regains its original size and shape

Fig. 5.1

Elasticity, Plasticity and Rigidity :

(1) Elasticity : When a force is applied on a body, size and shape of a body changes and the body is said to be deformed. Under deformed condition, the internal force is developed which tries to restore the original (position) size and shape of a body called as internal restoring force. Because of this internal restoring force developed inside a body, the body regains its original size and shape on removal of external deforming force.

Elasticity (Definition) : *The property on account of which a body regains its original size and shape on removal of external deforming force is called as elasticity.*

The **body** which **regains** its original size and shape on removal of external deforming force is called as **elastic body**.

e.g. almost all metals – steel, brass, rubber, copper.

Some bodies which **exactly regain** their original dimensions on removal of external deforming forces are called **perfectly elastic bodies**.

e.g. quartz.

(2) Plasticity : Some bodies do not oppose the change in size and shape and thus deforms very easily. Such bodies do not regain their original size and shape even after the deforming force is removed. These are called plastic bodies.

e.g. clay, putty, chalk.

Plasticity (Definition) : *The property on account of which a body easily deforms and does not regain its original size and shape on removal of external deforming force is called as plasticity i.e. material undergo permanent deformation.*

(3) Rigidity : A body in which it is not possible to produce any relative displacement of the different particles inside it and hence size and shape of a body remains unchanged even after large force is applied on it, is called as rigid body. No body is perfectly rigid but **stone** is taken as a **rigid body**.

Rigidity (Definition) : *The property on account of which a body does not change its size and shape even when a large force is applied on it, is called as rigidity.*

5.1.2 Stress, Strain and Their Types

When external force is applied on a body, by keeping its other end or face fixed, then molecules start moving in the direction of force. They stop moving at particular position where this applied force is balanced due to equal and opposite forces. Due to shifting of molecules, body changes its size or shape and a body is said to be deformed. Under deformed condition, every shifted molecule tries to achieve its original position due to which an internal restoring force is developed inside a body. The force which is responsible to deform a body is called a deforming force. Thus

Applied force (i.e. deforming force) = Internal restoring force

5.1.2 (A) Stress and its Types

Stress (Definition) : *Stress is defined as* ***internal elastic restoring force per unit cross-sectional area of a body***.

i.e. $$\text{Stress} = \frac{\text{Internal elastic restoring force}}{\text{Cross-sectional area}}$$

But, Restoring force = Applied force

Thus stress can also be defined as *applied force per unit cross-sectional area of a body*.

$$\therefore \quad \text{Stress} = \frac{\text{Applied force}}{\text{Cross-sectional area}}$$

Thus, $$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Cross-sectional area}} = \frac{\text{Applied force}}{\text{Cross-sectional area}}$$

$$\text{Stress} = \frac{F}{A}$$

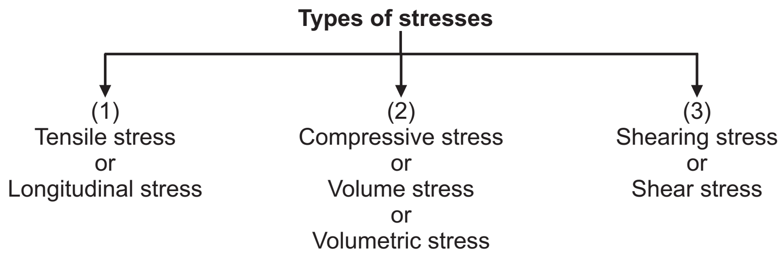
M.K.S. or **S.I.** unit of stress is **N/m²** or **(Pa) pascal**.

C.G.S. unit of stress is **dyne/cm²**.

The dimensions of stress are $[L^{-1} M^{1} T^{-2}]$.

[**Note :** Dimensions of stress and pressure are same since Pressure → Stress = $\frac{F}{A}$]

Types of Stresses :



1. Tensile stress or Longitudinal stress :

Definition : *When applied force produces change in length of a elongated body then the corresponding stress is called tensile stress or longitudinal stress.*

OR

Definition : *The stress that tends to change the length of a body is called longitudinal stress or tensile stress.*

$$\text{Tensile stress} = \frac{\text{Applied force}}{\text{Cross-sectional area}}$$

$$= \frac{F}{A}$$

$$= \frac{Mg}{\pi r^2} \quad \text{(where r is radius of the wire)}$$

M.K.S. unit of tensile stress is $\mathbf{N/m^2}$.

C.G.S. unit of tensile stress is $\mathbf{dyne/cm^2}$.

The dimensions of tensile stress are $[L^{-1} M^1 T^{-2}]$.

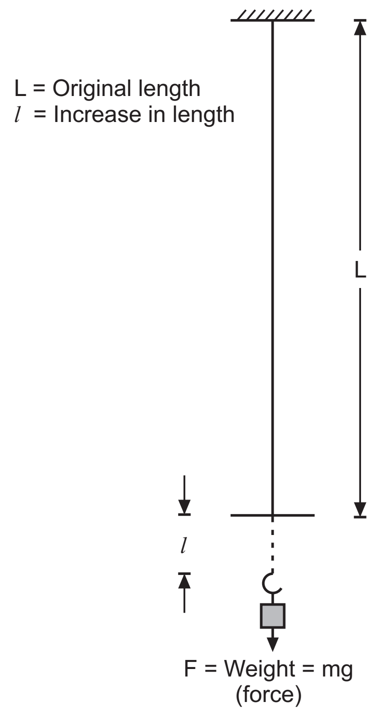


Fig. 5.2

Examples : Rope climbing, bicycle chain, cables of lift elevator.

2. Compressive stress or Volumetric stress :

Compressive stress (Definition) : *The stress which compresses the given body is called as compressive stress.*

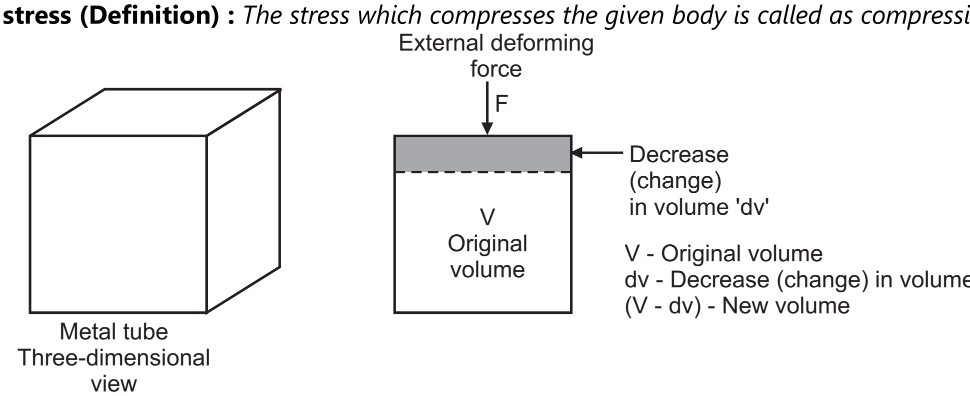


Fig. 5.3

Volume stress (Volumetric stress) (**Definition) :** *If a deforming force produces change in the volume of a body, then the corresponding stress is called as volume stress.*

OR

Definition : *The stress which changes volume of a body is called as volume stress.*

$$\text{Volume stress} = \frac{\text{Applied force}}{\text{Area}} = \text{Change in pressure} = dp$$

M.K.S. unit of volume stress is **N/m²**.

C.G.S. unit of volume stress is **dyne/cm²**.

Dimensions of volume stress are $[L^{-1} M^1 T^{-2}]$.

Examples : Piston, shock absorbers etc. Concrete is very strong in compression but it is weak while pulling.

3. Shear stress :

Consider a metal block ABCDEFGH of which the base DCGH is fixed. A tangential force 'F' is applied to the top surface as shown.

The block gets twisted (it changes its shape) and when equilibrium is reached, it takes the shape of parallelopiped DCGHA'B'F'E'.

Here volume of a body remains the same, shape of a body changes a little. Hence angle θ by which it is twisted is very small.

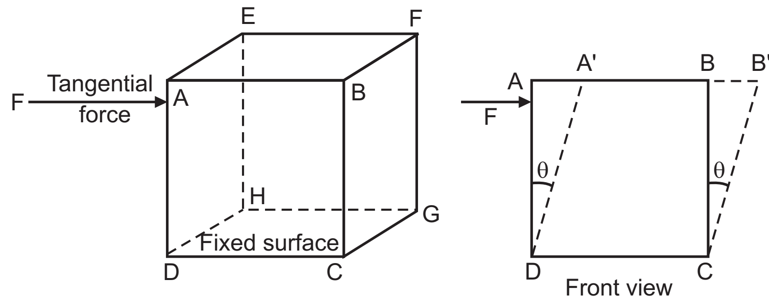


Fig. 5.4

Shear stress :

Definition : *The stress corresponding to change in shape of a body is called shear stress.*

OR

Definition : *If deforming force changes shape of a body, then the corresponding stress is called shear stress.*

OR

Definition : *The tangential force per unit area of the surface is called shearing stress.*

$$\text{Shear stress} = \frac{\text{Tangential force}}{\text{Area of surface}} = \frac{\text{Tangential force 'F'}}{\text{Area of surface AEDH}}$$

M.K.S. unit of shear stress is **N/m²**.

C.G.S. unit of shear stress is **dyne/cm²**.

Dimensions of shear stress are $[L^{-1} M^1 T^{-2}]$.

Examples : Torsional pendulum, metal sheet cutter, shaft connecting fly wheel, bone fracture because of bending.

5.1.2 (B) Strain and its Types

We have seen that when a force is applied on a body keeping its one of the end fixed then either size of the body changes or its shape changes i.e. its dimensions change.

Strain (Definition) : *The change in dimensions per unit original dimension is called strain.*

OR

Definition : *The strain is the amount of change in size (or shape) of material per unit original size (or shape) of it.*

OR

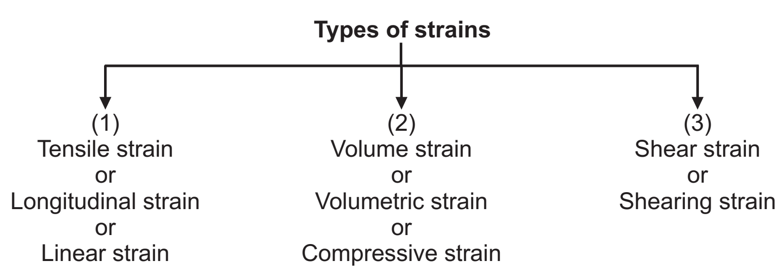
Definition : *Strain is the ratio of change in dimensions to the original dimension of the material.*

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

Since **strain** is the ratio of two similar quantities, **it has no unit**.

Strain is a pure number.

Types of Strains :



1. Tensile strain or Longitudinal strain or Linear strain :

If the applied (deforming) force produces change in the length of a body then the corresponding strain is the tensile strain.

Definition : *Tensile strain (longitudinal strain) is defined as change in length per unit original length.*

OR

Definition : *Tensile strain (longitudinal strain) is defined as the ratio of change in length to the original length.*

$$\text{Tensile strain} = \frac{\text{Change (increase) in length}}{\text{Original length}}$$

$$\text{Tensile strain} = \frac{\text{New length} - \text{Original length}}{\text{Original length}}$$

$$\text{Tensile strain} = \frac{l}{L}$$

It has **no unit**.

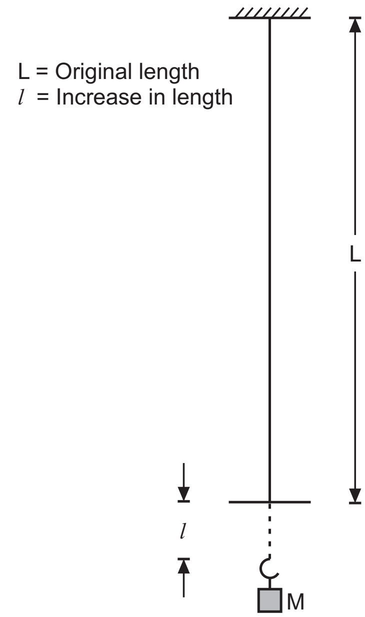


Fig. 5.5

2. Volume strain :

When applied force produces change in volume then the corresponding strain is the volume strain.

Volume strain (Definition) : *Volume strain is defined as the change in volume per unit original volume of a body.*

OR

Definition : *Volume strain is defined as the ratio of change in volume to the original volume of a body.*

$$\text{Volume strain} = \frac{\text{Change in volume}}{\text{Original volume}}$$

$$\text{Volume strain} = \frac{dv}{V}$$

It has **no unit**.

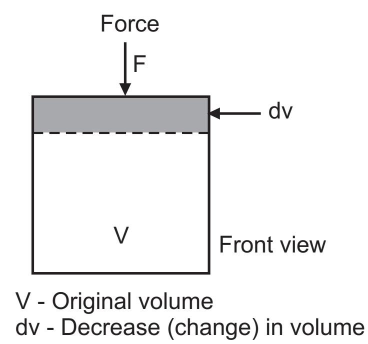


Fig. 5.6

3. Shearing strain :

If the applied force produces change in shape of a body (body gets twisted) then the corresponding strain produced is the shearing strain. As shown in Fig. 5.7 the cube is converted into parallelopiped.

Example : When spiral spring is stretched then the metal wire is twisted i.e. sheared (it does not elongate).

Shearing strain (Definition) : *Shearing strain is defined as the ratio of lateral displacement of any layer to its distance from the fixed layer.*

$$\text{Shearing strain} = \frac{\text{Lateral displacement of any layer}}{\text{Its distance from the fixed layer}}$$

$$\text{Shearing strain} = \frac{\text{Lateral displacement of upper layer}}{\text{Its distance from the fixed layer}}$$

$$\text{Shearing strain} = \frac{AA'}{AD}$$

$$\text{Shearing strain} = \tan\theta$$

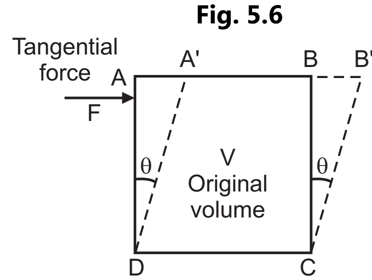


Fig. 1.7

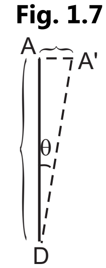


Fig. 1.8

But even a large tangential force is applied on a metal cube it gets twisted with a little amount i.e. θ is very small.

If θ is very small then mathematically $\tan\theta = \theta$.

$\therefore$ $\quad$ Shearing strain = Shear = θ (in radian)

5.1.3 Elastic Limit and Hooke's Law

Elastic limit (Definition) : "*The stress corresponding to the limiting value of the load, which when applied and subsequently released, does not produce permanent deformation is called as elastic limit*".

If this limit is crossed, the proportionality is lost and the stress is found to be less than what is expected and intramolecular structure of body collapses.

Hooke's law (Statement) : It states that, "*within elastic limit, strain is directly proportional to the stress.*"

$$\therefore \quad \text{Strain} \propto \text{Stress}$$

$$\therefore \quad \text{Strain} = \text{Constant} \times \text{Stress}$$

$$\text{OR} \quad \frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

The constant of proportionality is called as modulus of elasticity.

$$\therefore \quad \text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

S.I. unit of **modulus of elasticity** is **N/m^2**.

CGS unit is **dyne/cm^2**

Dimensions of modulus of elasticity are $[L^{-1} M^1 T^{-2}]$.

5.1.4 Types of Moduli of Elasticity

As we have seen when a force is applied on a body, then its length changes or volume changes or shape of a body changes. Accordingly, there are three types of modulus of elasticity.

1. Young's Modulus of Elasticity (Y) :

Definition : *Within elastic limit, the ratio of tensile stress to tensile strain is called Young's modulus of elasticity 'Y'.*

Young's modulus of elasticity measures the opposition (resistance) offered by material for its change in length.

We have,

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant} = \text{Modulus of elasticity}$$

$$\frac{\text{Tensile stress}}{\text{Tensile strain}} = \text{Young's modulus of elasticity} = Y$$

$$\therefore \quad Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F/A}{l/L}$$

$$= \frac{Mg/\pi r^2}{l/L}$$

$$\boxed{Y = \frac{Mg\,L}{\pi r^2\, l}}$$

where, M = Load attached

r = Radius of wire

L = Original length of wire

l = Extension produced

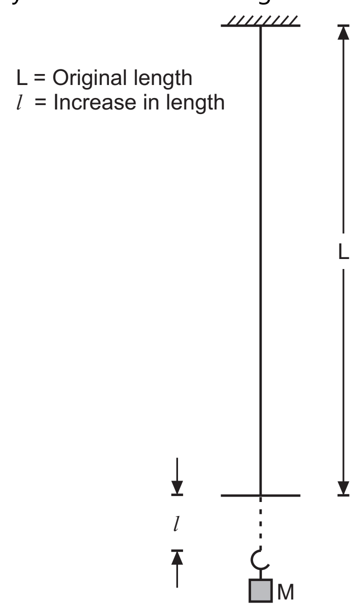


Fig. 5.9

Its **S.I.** or **MKS** unit is **N/m^2**. **CGS** unit is **dyne/cm^2**.

Dimensions are $[L^{-1}M^1T^{-2}]$. Dimensions of Y are same as stress.

[**Note :** Dimensions of stress and all moduli of elasticities (i.e. Y, K, η) are same.]

2. Bulk Modulus of Elasticity (K) :

Definition : *Within elastic limit, the ratio of volume stress to volume strain is called bulk modulus of elasticity 'K'.*

Bulk modulus measures the opposition (resistance) offered by a material during its change in volume.

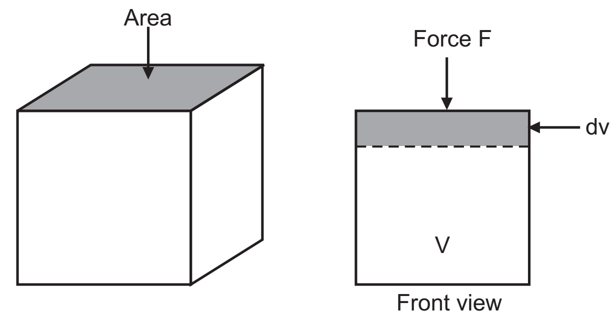


Fig. 5.10

Bulk modulus of elasticity,

$$K = \frac{\text{Volume stress}}{\text{Volume strain}} \text{ or } \frac{\text{Bulk stress}}{\text{Bulk strain}}$$

$$K = \frac{F/A}{dv/V}$$

$$K = \frac{dp}{dv/V}$$

$$\therefore \quad K = \frac{dp \times V}{dv}$$

where,

V = original volume
F = applied force
dv = decrease in volume
dp = increase in pressure
A = area of cross section on which force is applied

Its **S.I.** or **MKS** unit is **N/m²**. **CGS** unit is **dyne/cm²**.

Its dimensions are $[L^{-1} M^1 T^{-2}]$. Dimensions of K are same as stress, Y etc.

Compressibility : *The property on account of which a body can be compressed by the application of external force is called as compressibility i.e. it is the ability to compress.*

Definition : *The reciprocal of bulk modulus of elasticity is called compressibility.*

$$\text{Compressibility} = \frac{1}{\text{Bulk modulus}} = \frac{1}{K} = \frac{1}{\frac{dp}{dv} \times V} = \frac{dv}{dp \cdot V}$$

Bulk modulus measures the opposition made by the material to change its volume and compressibility measures the permission given by the material for its compression.

Compressibility represents strain per unit stress.

$$\text{Compressibility} = \frac{\text{Volume strain}}{\text{Volume stress}}$$

The **S.I.** unit of **compressibility** is **m²/N**.

The **C.G.S.** unit of **compressibility** is **cm²/dyne**.

Dimensions of compressibility are $[L^1 M^{-1} T^2]$. Dimensions of compressibility are exactly inverse of 'K'.

3. Modulus of Rigidity (η) :

Definition : *Within elastic limit, the ratio of shearing stress to shearing strain is called modulus of rigidity 'η'.*

Modulus of elasticity measures the opposition (resistance) offered by a material for its change in shape.

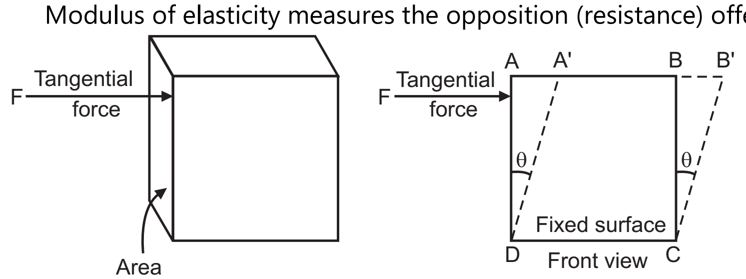


Fig. 5.11

where,

F = tangential force
A = area on which force is applied

Modulus of rigidity,

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\therefore \quad \eta = \frac{F/A}{\left[\frac{\text{Lateral displacement of layer}}{\text{Its distance from fixed layer}}\right]}$$

$$\therefore \quad \eta = \frac{F/A}{AA'/AD}$$

$$= \frac{F/A}{\tan\theta} \qquad \because \frac{AA'}{AD} = \tan\theta$$

$$\therefore \quad \boxed{\eta = \frac{F}{A\tan\theta}}$$

If 'θ' is small put tan θ = θ

$$\therefore \quad \eta = \frac{F}{A\theta}$$

Its **S.I.** or **MKS** unit is **N/m²**. **CGS** unit is **dyne/cm²**.

Dimensions are $[L^{-1} M^1 T^{-2}]$ i.e. Dimensions of η are same as stress, Y, K etc.

Relation between Y, K and η :

For a given material, there is certain relation between Y, K and η which is given by,

$$Y = \frac{9\eta K}{3K + \eta} \quad \text{or} \quad \frac{1}{Y} = \frac{1}{3\eta} + \frac{1}{9K}$$

where, Y = Young's modulus of elasticity of material

K = Bulk modulus

η = Modulus of rigidity

5.1.5 Some Important Definitions

Breaking stress : *The maximum stress upto which wire can be loaded or wire can bear is called breaking stress.* The corresponding point in the graph is the breaking point B.

Breaking stress (Definition) : *It is the maximum stress at which the wire brakes.*

Ultimate stress (Definition) : *It is the maximum stress the system is capable of withstanding.*

Ultimate stress (Definition) : *Ultimate stress is defined as the ratio of maximum load that the specimen (system) is capable of withstanding to its original cross-sectional area.*

$$\text{Ultimate stress} = \frac{\text{Maximum load the system can withstand}}{\text{Original cross-sectional area}}$$

Working stress (Definition) : *It is the actual practical stress on the system.*

Working stress (Definition) : *Working stress is defined as the ratio of actual load to the original cross-sectional area.*

$$\text{Working stress} = \frac{\text{Actual load on the specimen (system)}}{\text{Original cross-sectional area}}$$

Factor of safety : Working stress is the maximum allowed (permitted) stress on the system. For the safety of structure, this working stress should be less than elastic limit of the material. Working stress is determined by dividing the ultimate stress by a number called factor of safety.

Factor of safety (Definition) : *Factor of safety is defined as the ratio of ultimate stress to working stress.*

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}} \; \frac{\text{(load)}}{\text{(load)}}$$

OR

Factor of safety is also defined as the ratio of maximum load that the structure can bear to the actual load on the structure.

For example, the weighing balance used in grossary shop. Even though the weigh balance can bear a load of 10 kg, the message written on it may be "only for 5 kg".

Example : Message on lift may be only for 5 persons but actual capacity may be of 20 persons.

Important Points

- Elasticity is defined as the property by virtue of which body regains its original size and shape on removal of external deforming force.
- Stress is defined as internal resistive (restoring) force developed per unit cross-sectional area.
- Strain is defined as ratio of change in dimensions to original dimensions.
- **Hooke's law** states that, within elastic limit, stress is directly proportional to strain.
- Young's modulus, $Y = \frac{\text{Tensile (longitudinal) stress}}{\text{Tensile (longitudinal) strain}}$
- Compressibility $= \frac{1}{K}$
- Bulk modulus, $K = \frac{\text{Volume stress}}{\text{Volume strain}}$
- Modulus of rigidity, $\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$

- **Factor of safety** is defined as the ratio of maximum load that the structure can bear to the actual load on the structure. It should always be greater than one.
- **Breaking stress :** It is the maximum stress at which wire brakes.
- **Ultimate stress :** It is the maximum stress the system is capable of withstanding.
- **Working stress :** It is the actual practical stress on the system.
- **Poisson's ratio :** It is defined as the ratio of lateral strain to the longitudinal strain.

Formulae

1. Stress = $\frac{F}{A}$
2. Longitudinal strain = $\frac{l}{L}$

   Volume strain = $\frac{dv}{V}$

   Shearing strain = $\tan\theta$

   = θ (if θ is small)
3. $Y = \frac{F\,L}{Al} = \frac{MgL}{\pi r^2 l}$
4. $K = \frac{FV}{A\,(dv)} = \frac{PV}{dv}$
5. Compressibility = $\frac{1}{K}$
6. $\eta = \frac{F}{A\tan\theta} = \frac{F}{A\theta}$
7. $Y = \frac{9\,K\eta}{3\,K + \eta}$

   Or $\frac{1}{Y} = \frac{1}{3\eta} + \frac{1}{9K}$

where,

F – Force in newton 'N'

A – Area

l – Increase in length

L – Original length

dv – Decrease in volume

V – Original volume

M – Load in kg

r – Radius of wire

P – Pressure applied

Y – Young's modulus of elasticity

K – Bulk modulus of elasticity

η – Modulus of rigidity

SOLVED EXAMPLES

Example 1 : *Four wires of same metal and same diameter are stretched by same load. Length of each wire is given below. Which of them will elongate most ?*

(a) L = 0.5 m, (b) L = 1 m, (c) L = 1.5 m, (d) L = 2 m.

Solution : Four wires of same metal ∴ Y is same. Same diameter ∴ A is same Same load ∴ F is same.

Now only original length 'L' and elongation 'l' are variable (changing).

We have, $Y = \frac{F}{A} \times \frac{L}{l}$

∴ $\boxed{l \propto L}$ L = 2 m is maximum ∴ elongation in case of L = 2 m is most.

Ans. (d) L = 2 m

Example 2 : *Four wires of same metal are stretched by same load. The dimensions are given below. Which of them will elongate most ?*

(a) r = 0.5 mm, L = 50 cm; (b) r = 1 mm, L = 100 cm; (c) r = 1.5 mm, L = 150 cm; (d) r = 2 mm, L = 200 cm.

Solution : Y is same in all cases (since same metal). M is same $\therefore$ F is same.

Here r, L and l are variables.

We have, $$Y = \frac{F}{A} \times \frac{L}{l}$$

$$Y = \frac{F}{\pi r^2} \times \frac{L}{l} \qquad \boxed{\because \; l \propto \frac{L}{r^2}}$$

$\therefore$ (a) $$l \propto \frac{50}{(0.05)^2} \rightarrow 20{,}000$$ (here r = 0.5 mm = 0.05 cm)

(b) $$l \propto \frac{100}{(0.1)^2} \rightarrow 10{,}000$$

(c) $$l \propto \frac{150}{(0.15)^2} \rightarrow 6{,}666.67$$

(d) $$l \propto \frac{200}{(0.2)^2} \rightarrow 5{,}000$$

Thus elongation in the first case will be the most. $\therefore$ **Ans.** (a)

Example 3 : *Four wires of different metals are stretched by different load. The dimensions are given below. Which of the following has highest 'Y' elasticity ?*

(a) M = 1 kg, r = 1 mm, l = 1 mm, L = 1 m *(b) M = 2 kg, r = 2 mm, l = 2 mm, L = 2 m*

(c) M = 3 kg, r = 3 mm, l = 3 mm, L = 3 m *(d) M = 4 kg, r = 4 mm, l = 4 mm, L = 4 m*

Solution : We have relation, $$Y = \frac{Mg}{\pi r^2} \times \frac{L}{l} \qquad \therefore \; Y \propto \frac{ML}{r^2 l}$$

$\therefore$ In the case of (a) $$Y \propto \frac{1 \times 1}{1^2 \times 1} \rightarrow 1$$ (b) $$Y \propto \frac{2 \times 2}{2^2 \times 2} \rightarrow 0.5$$

(c) $$Y \propto \frac{3 \times 3}{3^2 \times 3} \rightarrow 0.33$$ (d) $$Y \propto \frac{4 \times 4}{4^2 \times 4} \rightarrow 0.25$$

$\therefore$ Highest Y will be in the first case. **Ans.** (a)

Example 4 : *The extension produced in a wire due to load is 2 mm. The extension in a wire of same material and length but half the radius by the same load will be (a) 4 mm, (b) 6 mm, (c) 8 mm, (d) 1 mm.*

Solution : We have, $$Y = \frac{mg}{\pi r^2} \times \frac{L}{l}$$

Relation between l and r is $$l \propto \frac{1}{r^2}$$

Now radius is half the earlier.

$\therefore$ $$l \propto \frac{1}{(1/2)^2}$$

$l \propto 4$ i.e. l is 4 times earlier = 8 mm **Ans.** (c) 8 mm

---

Example 5 : *If 'l' is the extension produced in the wire of length L, radius r with a force F, find the extension produced in the wire of same metal, of length 3 L, radius 3r and a force 3F.*

Solution : $$Y = \frac{F}{\pi r^2} \times \frac{L}{l} \qquad \therefore \; l \propto \frac{FL}{r^2} \qquad \ldots (1)$$

Now, $$l_2 \propto \frac{3F \times 3L}{(3r)^2}$$

$$\frac{l_1}{l} = \frac{3 \times 3}{9} = 1$$ **Ans.** Extension produced will be l same as earlier.

Example 6 : *A wire of length 3 m extends by 3 mm when a force is applied to it. Calculate stress produced in it, if $Y = 2 \times 10^{11}$ N/m^2.*

Solution : Given : L = 3 m, l = 3 mm = 3×10^{-3} m, F = 2 N

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \quad \text{Stress} = Y \times \text{Strain} = Y \times (l/L) = 2 \times 10^{11} \times \left(\frac{3 \times 10^{-3}}{3}\right)$$

$$\therefore \quad \boxed{\text{Stress} = 2 \times 10^{8}\ \text{N/m}^2}$$

Example 7 : *A wire of diameter 3 mm and length 4 m extends by 2.5 mm when a force of 10 N is applied. Find the Young's modulus of material of wire.*

Solution : Given : dia = 3 mm

$\therefore$ r = 1.5 mm = 1.5×10^{-3} m

L = 4 m

l = 2.5 mm = 2.5×10^{-3} m

F = 10 N

Y = ?

We have,

$$Y = \frac{F/A}{l/L} = \frac{FL}{(A)\ l} = \frac{FL}{(\pi r^2)\ l} = \frac{(10)\ (4)}{(3.142)\ (1.5 \times 10^{-3})^2\ (2.5 \times 10^{-3})}$$

$$\boxed{Y = 2.26 \times 10^{9}\ \text{N/m}^2}$$

Example 8 : *Calculate Young's modulus of elasticity for a wire having length 100 cm and cross-sectional area 1.96×10^{-5} m^2. The wire elongates by 2 mm when subjected to a load of 10 N.*

Solution : Given : Y = ?, L = 100 cm = 1 m, A = 1.96×10^{-5} m^2, l = 2 mm = 2×10^{-3} m, F = 10 N.

We have

$$Y = \frac{FL}{Al} = \frac{(10 \times 1)}{[1.96 \times 10^{-5} \times (2 \times 10^{-3})]}$$

$$\boxed{Y = 2.54 \times 10^{8}\ \text{N/m}^2}$$

Example 9 : *A metal bar has a maximum stress of 6×10^8 N/m^2. If the area of cross-section of the bar is 0.04 m^2, find the maximum force that the bar can withstand.*

Solution : Given : Maximum stress = 6×10^8 N/m^2, Area = 0.04 m^2, Maximum force = ?

We have,

$$\text{Maximum stress} = \frac{\text{Maximum force}}{\text{Area of cross-section}}$$

$$\therefore \quad \text{Maximum stress} \times \text{Area} = \text{Maximum force}$$

$$\therefore \quad \text{Maximum force} = \text{Maximum stress} \times \text{Area} = 6 \times 10^{8} \times 0.04$$

$$\boxed{\text{Maximum force} = 24 \times 10^{6}\ \text{N}}$$

Example 10 : *A copper wire is stretched by 5% of its length. Determine the stress produced in the wire. Given Y for copper = 1.2×10^{11} N/m^2.*

Solution : Given : Since wire is stretched 5%, take l = 5 and L = 100, Stress = ?

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = Y \times \text{Strain} = Y \times \left(\frac{l}{L}\right) = 1.2 \times 10^{11} \times \left(\frac{5}{100}\right)$$

$$\boxed{\text{Stress} = 6 \times 10^{9}\ \text{N/m}^2}$$

Example 11 : *A longitudinal stress of 8×10^7 N/m^2 produces an extension of 1 mm in a wire of length 2 metres. Find Young's modulus of the material of the wire.*

Solution : Given : Stress = 8×10^7 N/m^2, l = 1 mm = 1×10^{-3} m, L = 2 m, Y = ?

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Stress}}{l/L} = \frac{\text{Stress} \times L}{l} = \frac{8 \times 10^7 \times 2}{1 \times 10^{-3}}$$

$$\boxed{Y = 1.6 \times 10^{11} \text{ N/m}^2}$$

Example 12 : *Equal weights are attached to two wires of same metal. The length and radius of one wire is 1.5 times that of the other. Calculate ratio of their extensions.*

Solution : Given : 1st wire

$L_1 = 1.5 L_2$

$r_1 = 1.5 r_2$

$\frac{l_1}{l_2} = ?$

$$Y = \frac{MgL_1}{\pi r_1^2 l_1} \quad \dots (1)$$

2nd wire

Let length = L_2

Let radius = r_2

$$Y = \frac{MgL_2}{\pi r_2^2 l_2} \quad \dots (2)$$

Equating (1) and (2),

$$\frac{L_1}{r_1^2 l_1} = \frac{L_2}{r_2^2 l_2}$$

$$\therefore \quad \frac{L_1}{L_2} \times \frac{r_2^2}{r_1^2} = \frac{l_1}{l_2}$$

$$\frac{(1.5 L_2)}{L_2} \times \frac{r_2^2}{(1.5 r_2)^2} = \frac{l_1}{l_2}$$

$$\frac{1}{1.5} = \frac{l_1}{l_2}$$

i.e. $\boxed{\text{Ratio of elongations of two wires is 1 : 1.5}}$

Example 13 : *When a metal cube is subjected to a stress of 9×10^9 N/m^2, each side of the cube gets shortened by 1%. Find volume strain and bulk modulus of metal.*

Solution : Let original (length) side of the cube l = 100 units.

$\therefore$ Original volume = $100 \times 100 \times 100$

$$\boxed{V = 1000000 \text{ unit}^3}$$

But now due to applied stress, its side gets shortened by 1% i.e. becomes 99 units each.

$\therefore$ New volume = $99 \times 99 \times 99 = 970299$ unit^3

$\therefore$ Change in volume = 1000000 – 970299

$$\boxed{dv = 29701 \text{ unit}^3}$$

$$\therefore \quad \text{Volume strain} = \frac{dv}{V} = \frac{29701}{1000000} = 0.0297$$

$$\boxed{\text{Volume strain} = 0.03}$$

$$\text{Bulk modulus} = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{9 \times 10^9}{0.03}$$

$$\boxed{\text{Bulk modulus } K = 3 \times 10^{11} \text{ N/m}^2}$$

Example 14 : *A lead block of volume 1 m^3 is subjected to a pressure of 10^6 N/m^2. Calculate the change in volume of lead block. Also calculate compressibility of lead. (Given : Bulk modulus of lead K = 5×10^{10} N/m^2)*

Solution : Given : V = 1 m^3, dp = 10^6 N/m^2, dv = ?, Compressibility = ?

We have, Bulk modulus, $K = \frac{dp \times V}{dv}$

$$\therefore \quad dv = \frac{dp \times V}{K} = \frac{10^6 \times 1}{(5 \times 10^{10})}$$

$$\boxed{dv = 2 \times 10^{-5} \text{ m}^3}$$

$$\text{Compressibility} = \frac{1}{K} = \frac{1}{(5 \times 10^{10})}$$

$$\boxed{\text{Compressibility} = 0.2 \times 10^{-10} \text{ m}^2/\text{N}}$$

Example 15 : *6 cm thick aluminium square plate has area 2.5 m^2. The lower surface of plate is fixed and a tangential force is applied to the top surface and the top surface displaces by 0.08 mm. Calculate shearing stress, shearing strain.*

(Given : Modulus of rigidity of aluminium = $\eta = 3 \times 10^{10}$ N/m^2)

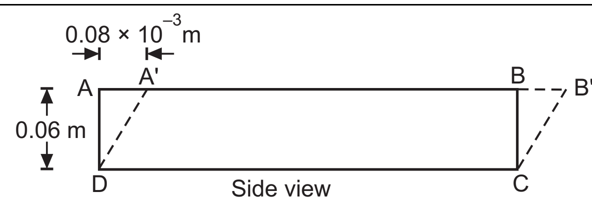


Fig. 5.12

Solution : Given : AD = 6 cm = 0.06 m, AA' = 0.08 mm = 0.08×10^{-3} m

$$\text{Shearing strain} = \frac{\text{Lateral displacement of any layer}}{\text{Its distance from fixed layer}} = \frac{AA'}{AD} = \frac{0.08 \times 10^{-3}}{0.06}$$

$$\boxed{\text{Shearing strain} = 1.33 \times 10^{-3}}$$

$$\text{Modulus of rigidity, } \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

$$\therefore \quad \text{Shearing stress} = \eta \times \text{Shearing strain} = (3 \times 10^{10}) \times (1.33 \times 10^{-3})$$

$$\therefore \quad \boxed{\text{Shearing stress} = 3.99 \times 10^{7} \text{ N/m}^2}$$

Practice Questions

1. Define the terms : (i) Elasticity, (ii) Stress, (iii) Strain, (iv) Elastic limit.
2. State and explain Hooke's law.
3. Define elastic body and plastic body and give one example of each.
4. Explain : (i) Young's modulus, (ii) Bulk modulus, (iii) Modulus of rigidity.
5. Define compressibility and state its S.I. unit.
6. Explain : (i) Longitudinal (Tensile) strain, (ii) Volume strain, (iii) Shearing strain.
7. Define factor of safety.
8. State whether rubber is more elastic or steel, why ?
9. Define ultimate and breaking stress.
10. Define factor of safety and elastic limit.
11. Define Young's modulus, Bulk modulus and Rigidity modulus of elasticity. State relation between them.
12. Distinguish between elastic bodies and plastic bodies.
13. Define : (i) Stress, (ii) Strain, (iii) Restoring force, (iv) Deforming force.
14. State and explain Hooke's law. Define modulus of rigidity. Write down its S.I. unit.
15. State Hooke's law of elasticity and define Young's modulus of elasticity.
16. Define deforming force and restoring force. State S.I. unit of stress.

17. State Hooke's law of elasticity and define elastic limit.
18. Define : (i) Young's modulus 'Y', (ii) Bulk modulus 'K', (iii) Modulus of rigidity 'η'.
19. Correct the equation $Y = \frac{9 K\eta}{3 (K + \eta)}$

B.T.E. Questions

1. Out of steel and rubber, which is more elastic and why ?
2. Define stress and factor of safety.
3. Define stress and strain.
4. Define rigidity and stress.
5. State and explain Hooke's law of elasticity. Hence define elastic limit.
6. Define modulus of rigidity. Give its units.
7. Define : (1) Elasticity, (2) Plasticity.
8. Define the term breaking stress.

Practice Problems

1. Calculate the strain produced in a wire under tension when its stress is 12×10^5 kg/m^2, Y for material is 2×10^6 kg/m^2. **Ans.** Strain = 0.6.
2. A wire of diameter 4 mm and of length 2 m extends by 1.99 mm applying a force of 10 N. Find Young's modulus of material of the wire. **Ans.** $Y = 7.996 \times 10^8$ N/m^2.
3. Calculate the strain produced in a material if the stress is 2000 kg/cm^2 and $Y = 2 \times 10^6$ kg/cm^2. **Ans.** Strain = 10^{-3}.
4. A wire of length 1.5 m extends by 1.5 mm when a force is applied to it. Calculate the stress produced in it. Given $Y = 2 \times 10^{11}$ N/m^2. **Ans.** Stress = 2×10^8 N/m^2.
5. A wire of length 1 m extends by 2 mm when stress acting on it is found to be 4000 kg/cm^2. Find Young's modulus of material of the wire. **Ans.** $Y = 1.96 \times 10^{11}$ N/m^2.
6. A wire gets elongated by 0.002 mm when a stress of 2000 N/m^2 is applied. Find Young's modulus of elasticity of material of wire if the original length of the wire is 4 m. (**Ans.** $Y = 4 \times 10^9$ N/m^2)
7. A steel wire of length 314 cm and diameter 1 mm has its upper end tied to a beam. If a load of 2 kg is attached to the lower end, find the extension produced in the wire. $Y = 1.96 \times 10^{11}$ N/m^2 **Ans.** l = 0.4 mm.
8. A weight exerts force of 120 N on steel wire of cross-sectional area 0.02 cm^2. Find extension produced if the length of wire is 5 m ($Y = 2 \times 10^{13}$ N/m^2). **Ans.** $l = 1.5 \times 10^{-5}$ m
9. Equal weights are attached to two wires of the same metal. The length and radius of one wire is twice the length and radius of the other wire. Calculate the ratio of their extensions. **Ans.** $l_1 : l_2 = 1 : 2$.
10. Find the weight attached to the lower end of the wire of length 1.5 m, radius 0.3 mm extends it by 0.6 mm, if $Y = 2 \times 10^{11}$ N/m^2. **Ans.** Weight = F = 22.62 N.
11. When a metal cube is subjected to a stress of 2×10^{10} N/m^2, each side of the cube is shortened by 2%. Find volume strain and bulk modulus of metal. **Ans.** Volume strain = 0.059, Bulk modulus = 3.39×10^{11} N/m^2.
12. A metal cube of volume 0.7 m^3 is subjected to a pressure of 1.2×10^6 N/m^2. Calculate the change in volume of metal block. Also calculate compressibility of metal. (Given : Bulk modulus of metal $K = 9 \times 10^{10}$ N/m^2)
    Ans. dv = 9.33×10^{-6} m^3, Compressibility = 0.11×10^{-10} m^2/N
13. 8 cm thick square metal plate has area 2 m^2. The lower surface of the plate is fixed and a tangential force is applied to the top surface and the top surface displaces by 0.06 mm. Calculate shearing stress, shearing strain.
    (Given : Modulus of rigidity of metal $\eta = 3.5 \times 10^{10}$ N/m^2)
    Ans. Shearing strain = 0.75×10^{-3}, Shearing stress = 2.62×10^7 N/m^2.
14. A metal wire of length 1.5 m and diameter 0.9 mm is stretched by a certain load. If extension produced in the wire is 1 mm and diameter of wire decreases by 0.12×10^{-3} mm, calculate Poisson's ratio of material of wire.
    Ans. Poisson's ratio, $\sigma = 0.2$.
15. An aluminium wire 3 mm in diameter and 4 m long is used to support a mass of 50 kg. What is the elongation of the wire if Young's modulus of aluminium is 7×10^{10} N/m^2 ?

5.2 SURFACE TENSION

5.2.1 Introduction

Matter is divided into solid, liquid and gas. Solids, liquids and gases can be compared as follows :

Solids : The intermolecular distance is small and fixed. Therefore intermolecular force is stronger in case of solids, hence it is not easy to change the size and shape of a body. Large force is required to change size and shape of a solid body.

Liquids : In case of liquids, intermolecular distance is more and not fixed. Therefore, liquid has no fixed shape. It takes the shape of the container. The intermolecular forces are comparatively weaker than solids. Hence shape of liquid can be easily changed. But size (volume) of the liquid is not so easy to change. Liquids need appreciable force (pressure) to change its volume.

Gases : The intermolecular distance is large and not fixed. It has no fixed size and shape. The intermolecular forces are very weak. Hence it is very easy to change shape and volume of gases.

Liquids and gases together are termed as fluids since they are able to flow.

5.2.2 Surface Tension - Introduction

Liquids do not have definite shape but they have definite volume. Therefore, liquids take the shape of a container and have free surfaces. The well-defined free surfaces of liquids have an interesting property which we are going to study in this chapter.

Free surface of liquids behaves like stretched elastic membrane. It has tendency to contract and occupy minimum surface area like stretched elastic membrane. This property is called *surface tension*.

The property of surface tension can be demonstrated by following examples :

1. A needle, if gently placed on the surface of water, floats even though it is heavier than water. It is due to surface tension of water. But the same needle sinks down if soap solution or salt is added to water. It is due to decrease of surface tension of net solution.
2. Insects like *'water spider'* can walk on water surface, as if they are walking on stretched elastic membrane.
3. Small drops of water and mercury are spherical in shape because sphere is the only shape which has minimum surface area.
4. Hail stones and planets are spherical because in the process of their formation they take spherical shape since they were in the liquid form before freezing.
5. A painting brush when dipped into water and taken out, their bristles are pulled together as water surface formed on the bristles contracts.
6. The following experiment illustrates the property of surface tension. A thread is tied to wire frame and immersed in soap solution and taken out so that film of soap solution is formed with the loose thread in the film. If the film is broken (using pin) on one side of the thread then the *remaining film contracts* and pulls the thread (film of soap solution behaves like surface of liquid).

In Fig. 5.13 (b), thread gets pulled by surface of liquid which states that surface of liquid has tendency to contract and occupy the minimum surface area.

Before going to *molecular theory* of surface tension, we will go through some definitions.

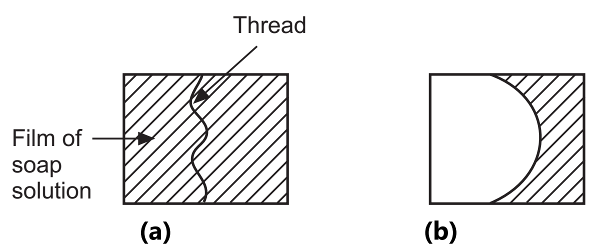


Fig. 5.13

Molecular force : *Every molecule attracts another molecule. This force of attraction is called molecular force.*

Adhesive force (Definition) : *It is the intramolecular force of attraction between two different types of molecules.*

e.g. Force of attraction between glass molecule and water molecule.

Cohesive force (Definition) : *It is the intramolecular force of attraction between two similar type of molecules.*

e.g. The molecules of liquid exert force of attraction on one another called cohesive force. e.g. force of attraction between two water molecules.

Molecular range (Definition) : *The maximum distance upto which cohesive force can act is called as molecular range.*

Sphere of influence (Definition) : *The imaginary sphere, surrounding a molecule, in which force of attraction is present is called the sphere of influence of that molecule.* **OR**

Defintion : *The imaginary sphere drawn with molecule as a centre and molecular range as a radius is called sphere of influence.*

5.2.3 Laplace's Molecular Theory of Surface Tension

Consider three molecules M_1, M_2, M_3 of liquid as shown in Fig. 5.14.

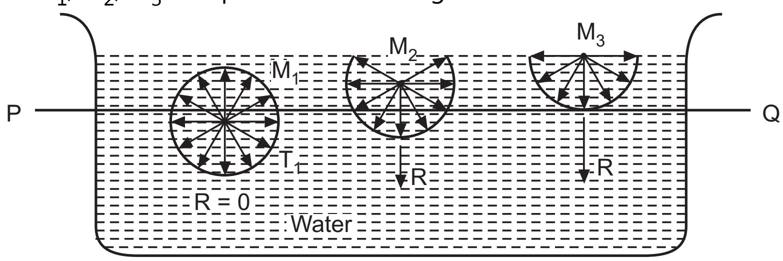


Fig. 5.14

Molecule M_1 : Its sphere of influence is totally inside liquid i.e. it is equally surrounded by neighbouring molecules. Therefore, molecule M_1 will be attracted by other molecules in its sphere of influence equally in all directions and the resultant force of attraction is zero.

Molecule M_2 : Its part of sphere of influence lies outside the liquid. Means it is not equally surrounded by neighbouring molecules. The molecules in the lower part of sphere are more. Therefore, it will get attracted downside and resultant force acting on molecule M_2 is in downward direction.

Molecule M_3 : Its half part of sphere of influence lies outside the liquid. Therefore there are no liquid molecules on the upper side, hence no upward force. Therefore, molecule M_3 experiences more downward resultant force than M_2.

Thus, in short, molecule M_1 experiences zero resultant force, M_2 experiences downward resultant force, M_3 experiences *more* downward resultant force. In short molecules below imaginary line PQ experience zero resultant force and molecules above line PQ experience some or more downward resultant force. Thus, molecules which lie on the surface of liquid (surface film) experience downward resultant force and are being pulled inside the liquid. To balance this downward force, molecules come closer to each other which reduce the surface area of liquid. This shows that there is a downward force of attraction on the liquid surface and it is under constant tension.

Potential Energy of Molecules : Molecules which are below line PQ have no potential energy. But the molecules above line PQ possess potential energy due to their position. As the natural tendency is to attain the position of minimum potential energy, these molecules are attracted downwards inside the liquid. As a result of this, the number of molecules on the surface of liquid come closer to each other and hence its surface area is reduced or surface of liquid contracts producing tension.

5.2.4 Definition and Unit of Surface Tension

Surface tension (Definition) : *The surface tension is defined as the property of liquids by virtue of which the surface of a liquid is under constant tension due to the tendency to contract and occupy minimum surface area.*

This is the reason for - Why hail stones and planets are spherical in shape, also small drops of mercury or water are spherical in shape. Because sphere is the only shape which has minimum surface area. This property of surface tension is used to prepare ball bearings or bullets which are spherical in shape.

Unit of Surface Tension :

Surface tension can also be defined as the force of contraction per unit length in the free surface of liquid.

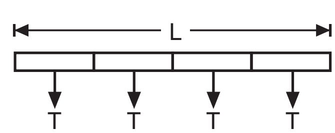


Fig. 5.15

S.I. unit of **surface tension** is **N/m**.

CGS unit is **dyne/cm**.

Dimensions of S.T. : Force length $\rightarrow \frac{\text{kg m/s}^2}{\text{m}} \rightarrow \text{kg/s}^2 \rightarrow [L^0M^1T^{-2}]$.

T is the force of contraction on unit length, therefore force F acting on total length 'L' is

$$\boxed{F = T \times L}$$

5.2.5 Angle of Contact and its Significance

Consider water in one glass container and mercury in another container.

The water level creeps up at the wall of container but mercury level depresses down at the wall of a container. Draw tangent to the curved part of the liquid surface at the point of contact.

Angle of contact θ (Definition) : *The angle made by the tangent to the curved part of liquid surface at the point of contact with wall of a container measured through the liquid is called angle of contact θ.*

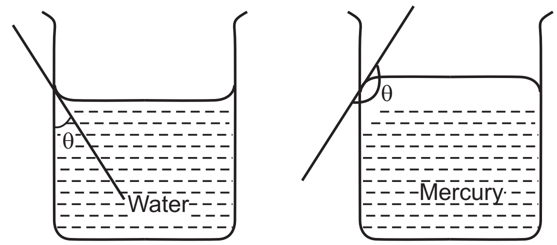


Fig. 5.16

S.I. unit of angle of contact **θ** is **radian**.

Factors affecting angle of contact : The angle of contact depends on **nature of liquid** and **nature of solid**. Also **θ** depends upon **purity of liquid** and **cleanliness of solid**. Thus for a given solid-liquid pair, the angle of contact is constant. Angle of contact θ may have values between 0° to 180°.

e.g. θ for pure water and clean glass = 0°

θ for pure mercury and clean glass = 140°

θ for most of liquids and clean glass < 90°

Significance of angle of contact :

- In case of **water** in **glass** container, **θ is acute** because force of **adhesion** is **stronger** than cohesion i.e. water can **stick** to the glass and therefore liquid creeps up at wall of the container.
- The case of **mercury** in **glass** container, **θ is obtuse** because force of **cohesion is stronger**. i.e. **mercury does not stick to glass** container and mercury depresses down at the wall of container.
- If liquid completely wets the solid → θ = 0 e.g. water in contact with hydrophilic solid like sugar, salt.
- If liquid does not wet the solid → θ is obtuse → e.g. mercury, (water on hydrophobic solid), water drop on a leaf surface, grass.
- If liquid wets the solid → θ is acute → e.g. water in glass.

5.2.6 Capillarity or Capillary Action

Consider a capillary tube (i.e. a glass tube with narrow bore) dipped in a liquid. It is observed that the liquid rises in the capillary above the general level of liquid in the beaker if the angle of contact is acute e.g. water rises up inside the capillary; and it is depressed down the general level of liquid if the angle of contact is obtuse e.g. mercury depresses down inside the capillary.

Capillarity or Capillary action (Definition) : *The rise or fall of liquid inside the capillary is called as capillarity. [Refer Fig. 5.17] e.g. Rise of oil upto wick end of oil lamp, i.e. the rise of oil in oil lamp upto the tip of wick is due to capillary action.*

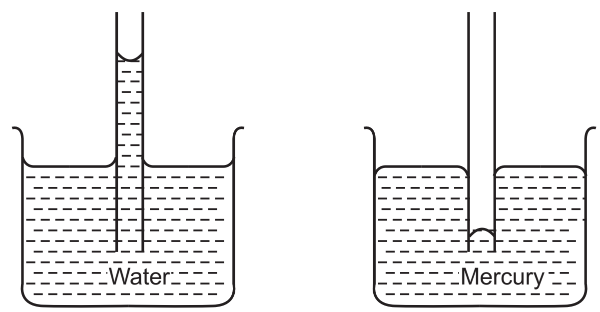


Fig. 5.17

Examples of capillary action (capillarity) :

(1) Rise of oil through the cotton wick of oil lamp → Oil rises through the cotton wick of a oil lamp or lantern or wick-stoves because of capillary action through the tiny capillaries formed by the threads of wick.

(2) Rise of ink through the pen nib → The tip of the nib of a pen is splitted which acts as a capillary. So ink is drawn upto the point continuously.

(3) A blotting paper absorbs ink by capillary action.

(4) Damping (absorbing moisture) of bricks because of rise of moisture due to capillary action through the small pores.

(5) Water rises from stem of tree towards the branches by capillary action.

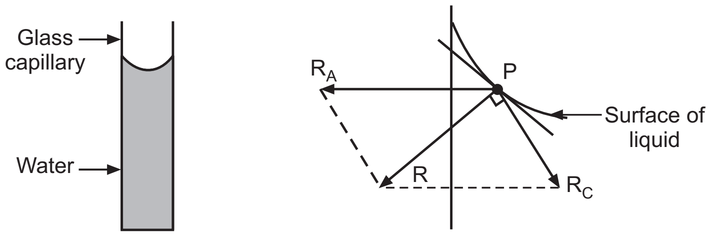
(6) Due to capillary action, water may rise over the surface of soil and evaporate easily. To prevent this evaporation the tiny capillaries inside the soil are broken by plouging the fields.

(7) Water stored in earthen vessel or canvas bag oozes out of the surface because of capillary action through pores.

5.2.7 Shape of Miniscus for Water and Mercury

It is observed that shape of liquid surface in a capillary tube may be plane or concave or convex depending upon the nature of liquid and container.

Shape of water surface in a glass capillary tube is concave as shown in Fig. 5.18 (a).



R_A = Resultant adhesive force

R_C = Resultant cohesive force

R = Resultant of R_A and R_C

(a) Shape of water surface in glass capillary is concave

(b) Diagram explaining why water surface is concave in glass capillary

Fig. 5.18

Explanation : 'P' is a water molecule near the glass wall. Molecule 'P' experiences two forces at a time.

(1) Force of adhesion i.e. force of attraction between water molecule 'P' and glass molecule.

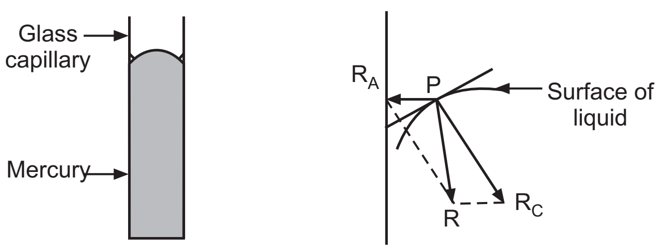
(2) Force of cohesion i.e. force of attraction between water molecule 'P' and other water molecules near to it.

In case of water in glass container, force $R_A > R_C$ (water sticks to glass).

R is the resultant of R_A and R_C as shown in Fig. 5.18 (b).

In order to balance this resultant force, the surface of liquid should be perpendicular to 'R'. Hence, the surface of water gets concave shape as shown.

Shape of mercury in a glass capillary tube is convex as shown in Fig. 5.19 (a).



R_A = Resultant adhesive force

R_C = Resultant cohesive force

R = Resultant of R_A and R_C

(a) Shape of mercury surface in glass capillary is convex

(b) Diagram explaining why mercury surface is convex in glass capillary

Fig. 5.19

Explanation : 'P' is a mercury molecule near the glass wall. Molecule 'P' experiences two forces at a time.

(1) Force of adhesion i.e. force of attraction between mercury molecule 'P' and glass molecules.

(2) Force of cohesion i.e. force of attraction between mercury molecule 'P' and other mercury molecules near to it.

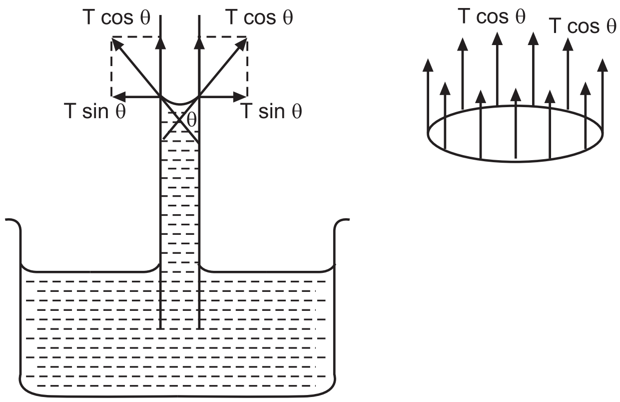
In case of mercury in glass container, force $R_C > R_A$ (mercury does not stick to glass).

R is the resultant of R_A and R_C as shown in Fig. 5.19 (b).

In order to balance this resultant force, the surface of liquid should be perpendicular to 'R'.

Hence, the surface of mercury gets convex shape as shown in Fig. 5.19 (b).

5.2.8 Relation between Surface Tension (T), Capillary Rise (h) and Radius of Capillary (r) (No Derivation)



where T = Force of surface tension

T cos θ = Vertical component of T

T sin θ = Horizontal component of T

Since total length is $2\pi r$

$\therefore$ Total upward force = $2\pi r \times T \cos \theta$

Fig. 5.20

$$T = \frac{rhdg}{2 \cos \theta}$$

where
- r – radius of capillary tube
- h – level difference or rise of liquid
- d – density of liquid
- g – gravitational acceleration
- θ – angle of contact

For a given liquid T, d and θ are constant then only r and h are variables then the relation becomes

$$rh = \text{constant}$$

i.e. $r_1 h_1 = r_2 h_2 = \ldots\ldots$ constant

5.2.9 Effect of Impurity and Temperature on Surface Tension

(a) Effect of Impurity (Contamination) :

Practically it is observed that, as soluble organic impurity like detergent powder is added in water then surface tension of net solution decreases i.e. soap solution has less surface tension than pure water. Thus as detergent powder is added in water, surface tension reduces i.e. contracting tendency decreases, i.e. spreading capacity increases.

$$\text{Surface tension of liquid} \propto \frac{1}{\text{Organic impurity in a liquid}}$$

(b) Effect of Temperature on Surface Tension :

As the temperature of liquid increases, the distance between the liquid molecules increases. This decreases the cohesive force of attraction between the molecules which ultimately decreases the surface tension of the liquid.

Thus the surface tension of the liquid decreases with increase of temperature.

For small temperature difference, surface tension decreases linearly.

$$\text{Surface tension of liquid} \propto \frac{1}{\text{Temperature of liquid}}$$

Thus as temperature of the liquid increases, its surface tension goes on decreasing and becomes zero at a particular temperature.

The surface tension of liquid becomes zero at a particular temperature, called critical temperature of that liquid.

For small temperature difference, the variation of surface tension of liquid with the temperature is almost linear.

5.2.10 Applications of Surface Tension

Note : *As per syllabus only list of applications may be expected but for understanding purpose, here these applications are explained.*

The surface tension property can be used as under :

1. **It is used to prepare ball bearings or bullets :**

   In order to prepare ball bearings, the hot molten metal is poured from a suitable height into the cold water. The liquid metal during its vertical fall breaks into smaller parts which automatically gain spherical shape due to *surface tension*, because surface tension is the property which tends to reduce surface area of liquid for given volume, *sphere is the only shape which has minimum surface area*.

2. **Use of detergent powder :**

   The detergent powder or surface active agents when dissolved in water reduce the surface tension of the resulting solution i.e. its contracting tendency decreases, in other words, its spreading and wetting capacity increase. Hence, soap solution easily spreads over the whole cloth and hence dirt gets easily removed from the clothes.

3. **Rise of oil upto wick end of oil lamp :**

   Oil rises through the cotton wick of oil lamp, lantern, wick-stoves due to capillary action through tiny capillaries formed by the threads of wick.

4. **Use of capillary action in liquid penetrant testing (NDT) :**

   A liquid of low surface tension is used to detect cracks in jobs; because it spreads easily in the cracks. In liquid penetrant inspection method, liquid is spread on the surface to be tested e.g. tile. Due to capillary action, liquid goes among the crack and crack gets visualized.

5. **Use of lubricants :**

   Oils of low surface tension are used as lubricants because they spread more in the parts of machineries. Formation of surface film of oil is important in reciprocating parts of hydraulic machine for its durability. If surface tension is less, oil film can produce easily.

6. **To check purity of water :**

   Purity of water can be checked with the help of value of surface tension.

7. **To stop breeding of mosquitoes and other insects :**

   Floating of mosquito eggs on the water surface helps them for breeding purpose. But if kerosene is sprayed on the surface of water, the surface tension of the net solution decreases and due to this eggs sink down the water and breeding of mosquito can be stopped.

8. **For cooling water :** Use of earthen vessel/canvas water bag.

   Water stored in earthen vessel or canvas bag oozes out of the surface because of capillary action through the tiny holes. This water gets easily evaporated by taking heat from the earthen vessel and cools water in the vessel.

9. **For drying :** Use of towel

   Use of towel to dry our body after bath. Towel soaks water due to capillary action.

10. **Use of blotting paper :**

    The tiny bores in the blotting paper form capillaries and when they are brought closer to ink, they absorb the ink.

11. Plugging of soil by a farmer – due to this capillaries in the soil are broken and this avoids water wastage which may come out by capillary action and may evaporate.

Important Points

- The imaginary sphere, surrounding a molecule, in which force of attraction is present is called sphere of influence.
- Surface tension is defined as the property of liquid by virtue of which the surface of liquid is under constant tension due to the tendency to contract and occupy minimum surface area.
- The force of contraction acting per unit imaginary length is the measure of surface tension. Hence its unit is N/m.
- The angle made by the tangent to the curved part of liquid surface at the point of contact with wall of container measured through the liquid is called angle of contact. It is shown by θ. Its unit is radian.
- The rise or fall of liquid inside the capillary is called capillarity.
- As impurity of organic substance in liquid increases, the surface tension of solution decreases.
- As temperature of liquid increases, its surface tension decreases.

Formulae

1. $T = \dfrac{rhdg}{2 \cos \theta}$

2. $r_1 h_1 = r_2 h_2$

where T–Surface tension

- r – Radius of capillary
- d – Density of liquid
- θ – Angle of contact of liquid
- h – Rise in capillary
- r_1 – Radius of 1^{st} capillary
- h_1 – Rise of liquid in 1^{st} capillary
- r_2 – Radius of 2^{nd} capillary
- h_2 – Rise of liquid in 2^{nd} capillary

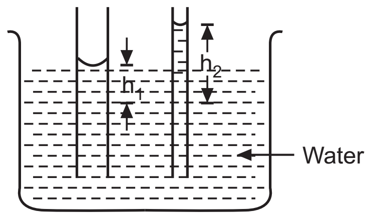


Fig. 5.21

SOLVED EXAMPLES

Example 1 : *A capillary tube of diameter 1 mm is dipped in water. How far will the water rise in tube if surface tension of water is 7×10^{-2} N/m ?*

Solution : Given : diameter = 1 mm

$\therefore$ radius r = 0.5 mm = 0.5×10^{-3} m

Surface tension T = 7×10^{-2} N/m

Find : h = ?

Assume density d = 1000 kg/m^3

$\theta = 0°$

We have,

$$T = \frac{rhdg}{2 \cos \theta}$$

$$h = \frac{2T \cos \theta}{rdg} = \frac{2\,(7 \times 10^{-2}) \cos 0}{(0.5 \times 10^{-3})\,(1000)\,(9.8)}$$

$$\boxed{h = 2.857 \times 10^{-2} \text{ m}}$$

$\therefore$ Liquid will rise by 2.857×10^{-2} m in a tube.

Example 2 : *A capillary tube of diameter 1 mm is dipped in water. How far will the water rise in the tube if surface tension of water is 7.2×10^{-2} N/m ? Density of water = 1×10^3 kg/m^3.*

Solution : Given : diameter = 1 mm

$\therefore$ r = 0.5 mm

= 0.5×10^{-3} m

h = ?

T = 7.2×10^{-2} N/m

d = 1×10^3 kg/m^3

$\theta = 0°$ ($\because$ water)

We have,

$$T = \frac{rhdg}{2 \cos \theta}$$

$\therefore$

$$h = \frac{2\,T \cos \theta}{rdg} = \frac{2 \times (7.2 \times 10^{-2}) \cos \theta}{(0.5 \times 10^{-3})\,(1 \times 10^3)\,(9.8)}$$

$$\boxed{h = 0.0293 \text{ m}}$$

Example 3 : *A capillary tube of diameter 0.2 mm is dipped into a liquid of density 0.85×10^3 kg/m³ and angle of contact 24°. If the liquid rises by 41 mm in the tube, find the surface tension of liquid.*

Solution : Given : diameter = 0.2 mm

$$\therefore \quad r = 0.1 \text{ mm} = 0.1 \times 10^{-3} \text{ m}$$

$$d = 0.85 \times 10^3 \text{ kg/m}^3$$

$$\theta = 24°$$

$$h = 41 \text{ mm} = 41 \times 10^{-3} \text{ m}$$

$$T = ?$$

$$T = \frac{rhdg}{2 \cos \theta} = \frac{(0.1 \times 10^{-3})\,(41 \times 10^{-3})\,(0.85 \times 10^3)\,(9.81)}{2 \cos 24°}$$

$$\boxed{T = 0.0187 \text{ N/m}}$$

Example 4 : *A liquid rises up by a height of 5.2 cm in a capillary tube of diameter 0.82 mm. How high will it rise in another tube of radius 0.025 cm ?*

Solution : Given :

$$h_1 = 5.2 \text{ cm} \qquad h_2 = ?$$

$$dia_1 = 0.82 \text{ mm} \qquad r_2 = 0.025 \text{ cm}$$

$$r_1 = 0.41 \text{ mm}$$

$$= 0.041 \text{ cm}$$

We have,

$$r_1 h_1 = r_2 h_2$$

$$h_2 = \frac{r_1 h_1}{r_2} = \frac{(0.041)\,(5.2)}{(0.025)}$$

$$\boxed{h_2 = 8.53 \text{ cm}}$$

Example 5 : *A liquid of density 1.1×10^3 kg/m³ and surface tension 31.5×10^{-3} N/m rises to a height of 0.15 cm in a tube of diameter 0.82 mm. Find the angle of contact of the liquid.*

Solution : Given :

$$d = 1.1 \times 10^3 \text{ kg/m}^3$$

$$T = 31.5 \times 10^{-3} \text{ N/m}$$

$$h = 0.15 \text{ cm} = (0.15 \times 10^{-2}) \text{ m}$$

$$\text{diameter} = 0.82 \text{ mm}$$

$$\therefore \quad r = 0.41 \text{ mm} = (0.41 \times 10^{-3}) \text{ m}$$

$$\theta = ?$$

We have,

$$T = \frac{rhdg}{2 \cos \theta}$$

$$\therefore \quad 2 \cos \theta = \frac{rhdg}{T}$$

$$\therefore \quad \cos \theta = \frac{rhdg}{2T} = \frac{(0.41 \times 10^{-3})\,(0.15 \times 10^{-2})\,(1.1 \times 10^3)\,(9.8)}{2 \times (31.5 \times 10^{-3})}$$

$$\cos \theta = 0.1052$$

$$\therefore \quad \theta = \cos^{-1}(0.1052)$$

$$\therefore \quad \boxed{\theta = 83.96°}$$

Example 6 : *Find the surface tension of mercury if mercury is depressed by 8.22 mm in a capillary tube, if diameter = 2.25 mm, angle of contact = 140°, density of mercury = 13.6×10^3 kg/m³.*

Solution : Given :

$$-h = -8.22 \text{ mm}$$

$$= -8.22 \times 10^{-3} \text{ m}$$

diameter = 2.25 mm

∴ $r = 1.125 \text{ mm} = 1.125 \times 10^{-3}$ m

$\theta = 140°$

$d = 13.6 \times 10^3 \text{ kg/m}^3$

$T = ?$

We have, $T = \frac{rhdg}{2\cos\theta} = \frac{(1.125 \times 10^{-3})(-8.22 \times 10^{-3})(13.6 \times 10^3)(9.8)}{2\cos 140°}$

$\boxed{T = 0.804 \text{ N/m}}$

Example 7 : *A liquid rises through a height of 4 cm in a capillary tube of radius 0.4 mm. How far will it rise in a capillary tube of radius 0.8 mm ?*

Solution : Given : radius, r_1 = 0.4 mm = 0.04 m $\quad r_2$ = 0.8 mm = 0.08 cm

rise of liquid, h_1 = 4 cm $\quad h_2$ = ?

$r_1h_1 = r_2h_2$

$h_2 = \frac{r_1h_1}{r_2} = \frac{(0.04) \times (4)}{(0.08)}$

$\boxed{h_2 = 2 \text{ cm}}$

∴ Liquid will rise by 2 cm in a capillary tube of radius 0.8 mm.

Example 8 : *A capillary tube of diameter 0.5 mm is dipped in a liquid of density 800 kg/m^3 and surface tension 0.028 N/m. Calculate the rise if angle of contact is 16.3°.*

Solution : Given : diameter = 0.5 mm

radius $r = 0.25 \text{ mm} = 0.25 \times 10^{-3}$ m

$d = 800 \text{ kg/m}^3$

$\theta = 16.3°$

$T = 0.028$ N/m

Find : $h = ?$

$T = \frac{rhdg}{2\cos\theta}$

∴ $h = \frac{2T\cos\theta}{rdg} = \frac{2 \times 0.028 \times \cos 16.3°}{(0.25 \times 10^{-3}) \times (800) \times (9.81)}$

∴ $\boxed{h = 0.028 \text{ m}}$

∴ Liquid will rise by 0.028 m in a capillary tube.

Example 9 : *Pure water rises to a height of 2.5 cm in a capillary tube of diameter 1 mm. Find the surface tension if density of water is 1000 kg/m^3.*

Solution : Given : $h = 2.5 \text{ cm} = 2.5 \times 10^{-2}$ m

diameter = 1 mm

∴ $r = 0.5 \text{ mm} = 0.5 \times 10^{-3}$ m

$T = ?$

$\theta = 0$ (for pure water)

$T = \frac{rhdg}{2\cos\theta} = \frac{(0.5 \times 10^{-3}) \times (2.5 \times 10^{-2}) \times (1000) \times (9.8)}{2\cos 0}$

$\boxed{T = 0.061 \text{ N/m}}$

Practice Questions

1. Define surface tension of a liquid.
2. Explain the phenomenon of surface tension with the help of Laplace's molecular theory.
3. Explain the molecular theory of surface tension with respect to the potential energy of surface molecules.
4. State unit of surface tension.
5. Define angle of contact. On what factors it depends ?
   OR Explain the significance of acute and obtuse angle.
6. State the formula for surface tension.
7. Define :
   (i) Cohesive force, (ii) Adhesive force, (iii) Sphere of influence, (iv) Angle of contact, (v) Capillary action.
8. State the effect of temperature and impurity on surface tension of the liquid.
9. Define surface tension. State the relation between surface tension, capillary rise and radius of capillary tube. State the meaning of symbols used in it.
10. Explain with neat sketch how liquid form concave and convex surfaces in a tube.
11. What is the effect of organic impurity and temperature on surface tension of the liquid ?
12. Define adhesive and cohesive molecular forces of attraction. Give one example of each force.
13. Define surface tension of a liquid and state its S.I. unit.

Practice Problems

1. A liquid rises to a height of 2 cm in a tube of diameter 2.5 mm. How far will it rise in a tube of diameter 1.5 mm ? (**Ans.** h_2 = 3.33 cm)
2. A glass capillary tube of diameter 0.02 cm is dipped into a liquid of density 0.85×10^3 kg/m^3. The liquid rises to a height of 14 cm. The angle of contact is 15°. Calculate the surface tension of the liquid. (**Ans.** T = 0.06 N/m)
3. The liquid rises to a height of 2.25 cm in a capillary tube of certain diameter, when the surface tension of the liquid is 6.45×10^{-2} N/m. Density of the liquid is 0.9×10^3 kg/m^3 and angle of contact is 2°. Calculate the diameter of tube. (**Ans.** r = 0.0649 cm and diameter = 0.1298 cm)
4. In a capillary tube of diameter 0.1 cm, the mercury level is depressed by 1.25 cm. Find surface tension of mercury. Density of mercury = 13.6×10^3 kg/m^3, Angle of contact = 128.67° (**Ans.** T = 0.66 N/m)
5. A capillary tube of radius 0.05 cm is dropped in water. How far will the water rise in the tube ? Given surface tension of water is 7×10^{-2} N/m, angle of contact is 0°, density of water is 10^3 kg/m^3, g = 9.8 m/s^2. (**Ans.** h = 2.857 cm)
6. A capillary tube of radius 0.06 cm is dipped in pure water. How far will the water rise in the tube if surface tension of water is 7.0×10^{-2} N/m. Density of water = 1×10^3 kg/m^3. (**Ans.** h = 2.38 cm)
7. A liquid of density 1200 kg/m^3 and surface tension 0.036 N/m rises to a height of 1.8 mm in a tube of radius 0.3 mm. Find the angle of contact of the liquid. (**Ans.** θ = 84.93°)

5.3 VISCOSITY

Introduction :

Liquids like honey, glycerine take more time to flow on the surface than that of water and kerosene, etc. i.e. Honey, glycerine are more *viscous* than that of water and kerosene. In other words, glycerine/honey are less mobile, hence more viscous and water/kerosene are more mobile, hence less viscous.

5.3.1 Viscosity

Consider flow of liquid on the horizontal plane surface. Imagine liquid is made up of number of horizontal layers. It is observed that different liquid layers move with different velocities.

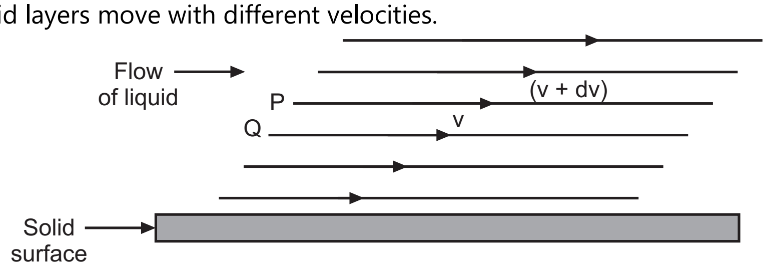


Fig. 5.22

The bottom-most liquid layer molecules have minimum speed on account of adhesion and speed of layer increases from bottom to top, then top-most layer has maximum speed.

Consider layers P and Q. Layer P which has more speed, tries to accelerate layer Q. At the same time, layer Q which has less speed, tries to decrease speed of layer P and *thus because of different tendencies of different layers, a frictional force (fluid friction) is created between two layers called as viscous force which tries to oppose the relative motion between different layers i.e. tries to reduce the difference between their speeds called viscosity.*

Viscosity (Definition) : *Viscosity is the property of liquid on account of which liquid tries to oppose the relative motion between its different layers.*

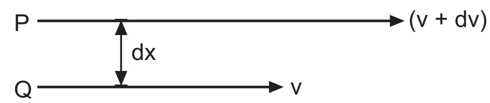


Fig. 5.23

Velocity gradient : Consider any two layers P and Q with velocities (v + dv) and v respectively. Here 'dv' is the change in velocity of layers and 'dx' is the change in vertical distance between the layers.

Then $\frac{dv}{dx}$ is called velocity gradient.

Velocity gradient (Definition) : *Velocity gradient is defined as the change in velocity per unit change in (vertical) distance of liquid layers.*

$$\text{Velocity gradient} = \frac{dv}{dx}$$

Unit of velocity gradient is per second i.e. 1/sec.

5.3.2 Newton's Law of Viscosity

The law states that the viscous force 'F' developed between two liquid layers is

(1) directly proportional to the surface area 'A' of liquid layer,

(2) directly proportional to the velocity gradient.

Thus, $F \propto A$

and $F \propto \frac{dv}{dx}$

$\therefore$ Combining, $F \propto A \times \frac{dv}{dx}$

$$F = \eta A \frac{dv}{dx}$$

where η is called coefficient of viscosity, 'η' is constant for a given liquid. It changes from liquid to liquid.

$$\therefore \quad \eta = \frac{F}{A \times \frac{dv}{dx}} \quad \text{i.e.} \quad \frac{\text{newton}}{m^2 \times \frac{m/s}{m}}$$

= $\mathbf{Ns/m^2}$ is the **SI unit of 'η'.**

or $\mathbf{dyne\text{-}s/cm^2}$ is the **CGS** unit of η.

or $$1 \text{ poise} = 1 \text{ dyne-s/cm}^2 = \frac{1}{10} \text{ N-s/m}^2$$

1 poise (Definition) : *The coefficient of viscosity 'η' is said to be 1 poise if 1 dyne viscous force is developed between two liquid layers of 1 cm² area for unit velocity gradient.*

To define 'η', we have $\eta = \frac{F}{A \times \frac{dv}{dx}}$ In order to define 'η', if A = 1, $\frac{dv}{dx} = 1$, then η = F.

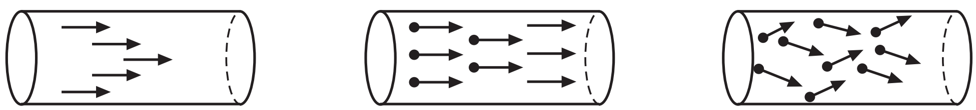
Coefficient of viscosity 'η' (Definition) : *Coefficient of viscosity 'η' of a liquid is defined as the viscous force developed between two liquid layers of unit surface area for unit velocity gradient.*

SBTE Questions

1. State Newton's law of viscosity.
3. State Newton's law of viscosity and obtain an expression for force of viscosity.
4. State Newton's law of viscosity. Define coefficient of viscosity. Give the S.I. unit of coefficient of viscosity.
5. Define viscosity and write S.I. unit of coefficient of viscosity.
6. State Newton's law of viscosity. Hence derive the unit of coefficient of viscosity.
7. Define coefficient of viscosity. Write down its S.I. unit.
8. Define velocity gradient and state its unit.
9. State unit of velocity gradient in viscosity.

5.3.3 Flow of Liquid through a Tube - Streamline Flow and Turbulent Flow

We have seen different layers of liquid flow with different velocities. In case of flow of liquid through a tube, the liquid layer which flows along the axis has maximum velocity and layers which are in contact with wall of tube has minimum velocity.



Flow of liquid through a tube **Streamline or Laminar flow** **Turbulent flow**

Fig. 5.24

Streamline or Laminar flow :

Streamline flow (Definition) : *Streamline flow is the flow of the liquid in which every particle of liquid moves in the same direction (i.e. parallel or inline) of flow of liquid.*

When liquid flows steadily, each particle follows its earlier particle. In streamline flow, the velocity at every point within the liquid remains constant. The flow remains streamline so long as the velocity is below certain velocity called critical velocity.

e.g.

(1) Flow of river water in summer season is slow and steady i.e. streamline.

(2) A gently flowing water stream.

Critical velocity :

Definition : *The value of velocity of flow of liquid upto which flow is streamline is called critical velocity 'v_c'.*

If velocity of flow of liquid increases and crosses critical velocity, then particle starts moving in the random direction i.e. flow becomes turbulent.

Thus, critical velocity is also defined as the velocity of flow of liquid at which streamline flow changes into turbulent flow.

Turbulent flow :

Definition : *The flow of liquid in which every particle is not moving in line and they move in random direction is called turbulent flow.*

This happens when speed of flow of water is more than certain value (i.e. more than critical velocity).

e.g.

(1) Flow of river water after very heavy rain i.e. during flood.

(2) Water fall.

(3) Non-steady flow as in tidal bore.

The velocity is given by the relation

$$v = \frac{\eta R}{\rho r}$$

where
- v – Velocity of flow of liquid (critical)
- η – Coefficient of viscosity of liquid
- R – Reynold's number
- ρ – Density of liquid
- r – Radius of the tube

Significance of Reynold's number :

On the basis of experiments, scientist Reynold observed that, Reynold's number determines the nature of flow of liquid through a tube. He observed that for a tube of radius 1 cm,

when R is less than 2000, then liquid flow is streamline.

when R is in between 2000 to 3000, the liquid flow is unstable.

when R is greater than 3000, the liquid flow is turbulent.

Comparison between Streamline flow and Turbulent flow :

Streamline flow	Turbulent flow
1. The flow of liquid in which every *particle* of the liquid *moves inline* of flow of liquid is called streamline flow.	1. The flow of liquid in which every *particle* of the liquid *moves* in any *random direction* is called turbulent flow.
2. This flow is *steady*.	2. This flow is speedy.
3. The *velocity* at every point within the liquid remains *constant*.	3. The *velocity* at every point within the liquid is *different*.
4. The velocity of flow of liquid is *less* than *critical velocity*. $v < v_C$	4. The velocity of flow of liquid is *more* than *critical velocity*. $v > v_C$
5. The value of Reynold's number '*R*' is *less* than 2000. $R < 2000$	5. The value of Reynold's number '*R*' is *more* than 3000. $R > 3000$
6. e.g. • Flow of river water during *summer*. • A *gently* flowing water stream.	6. e.g. • Flow of river water during *flood*. • *Water fall.*

SBTE Questions

1. Distinguish between streamline flow and turbulent flow.
2. What is Reynold's number ? State its significance.
3. Define streamline flow and turbulent flow and give one example of each.
4. Define types of flow of liquid using Reynold's number. Give its physical significance.
5. Define streamline flow and turbulent flow. State significance of Reynold's number.
6. Define critical velocity.
7. Give the significance of Reynold's number. How with the help of Reynold's number the nature of flow of liquid can be determined ?
8. Define streamline flow, turbulent flow and critical velocity. State the equation for critical velocity of liquid in tube with the meaning of the symbols used in the equation.
9. Give significance of Reynold's number in flow of liquid.

5.3.4 Free Fall of Spherical Body through Viscous Medium and Stoke's Law

When a body falls under gravitation, through a column of liquid, different liquid layers move with different speeds. Liquid layers which are in contact with body has maximum velocity and liquid layer in contact with the wall of container has zero velocity. Because of different speeds of different layers, a viscous force which opposes the relative motion is developed.

Practically, it is observed that after covering certain distance the body, which is freely falling, attains constant velocity called as terminal velocity 'v'.

Terminal velocity (Definition) : *The constant velocity with which a body falls through liquid column is called terminal velocity.*

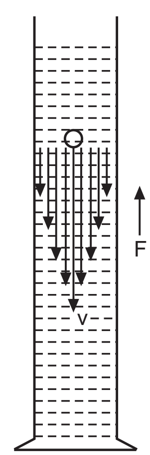


Fig. 5.25

Stoke's law (Definition) : *Stoke's law states that the force of viscosity experienced by a small metal sphere falling freely through a viscous medium, with terminal velocity is directly proportional to*

(1) radius of metal sphere 'r' (2) terminal velocity 'v' (3) coefficient of viscosity of liquid 'η'

i.e. $F \propto \eta r v$

$\boxed{F = 6\pi \eta rv}$ ... Stoke's law formula

5.3.5 Derivation of Coefficient of Viscosity 'η' by Stoke's Method

Consider a metal sphere placed on the surface of liquid taken in glass jar. It is observed that after covering certain distance, metal sphere attains a constant velocity called *terminal velocity.*

Metal sphere falling freely through a liquid experiences three forces :

(1) Weight of the metal sphere in the downward direction.

(2) Force of viscosity in the upward direction.

(3) Upthrust force (force of Buoyancy) in the upward direction.

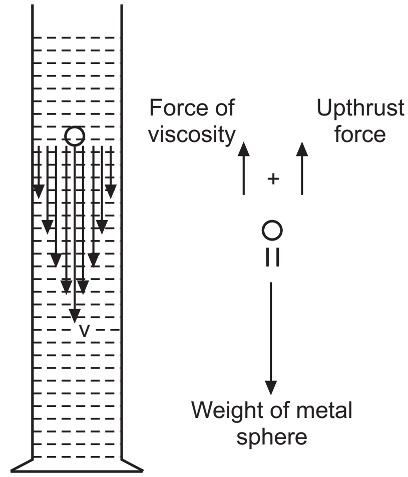


Fig. 5.26

If weight of a person is 65 kgf in air, it may be 45 kgf in liquid because liquid tries to lift the body and create upward force.

Buoyant force or upthrust force is the force with which liquid lifts the body dipped into it.

By Archimede's principle,

Upthrust force = Loss of weight of body in liquid

= Weight of displaced liquid

Since metal sphere falls with constant velocity, the total upward force is equal to the downward force.

∴ Total upward force = Downward force

∴ [Force of viscosity] + [Upthrust force] = [Weight of the metal sphere]

∴ [6πηrv] + [Weight of displaced liquid] = [Weight of the metal sphere]

∴ [6πηrv] + [(Mass of displaced liquid) × g] = [(Mass of metal sphere) × g]

$$\therefore \quad [6\pi\eta rv] + \left[(\text{Volume of displaced liquid})\begin{pmatrix}\text{Density}\\ \text{of liquid}\end{pmatrix} \times g\right] = \left[\begin{pmatrix}\text{Volume of}\\ \text{metal sphere}\end{pmatrix}\begin{pmatrix}\text{Density}\\ \text{of metal}\end{pmatrix} \times g\right]$$

$$[6\pi\eta rv] + \left[\left(\frac{4}{3}\pi r^3\right)(\rho)\, g\right] = \left[\left(\frac{4}{3}\pi r^3\right)(d)\, g\right]$$

$$(6\pi\eta rv) = \left(\frac{4}{3}\pi r^3\, dg\right) - \left(\frac{4}{3}\pi r^3 \rho g\right)$$

$$6\pi\eta rv = \frac{4}{3}\pi r^3\, g(d-\rho)$$

$$\eta = \frac{\frac{4}{3}\pi r^3\, g(d-\rho)}{6\pi rv}$$

$$\therefore \quad \boxed{\eta = \frac{2}{9}\frac{r^2\, g\,(d-\rho)}{v}}$$

$$\text{or} \quad \boxed{v = \frac{2}{9}\frac{r^2 g\,(d-\rho)}{\eta}}$$

where η – coefficient of viscosity of liquid
r – radius of metal sphere
d – density of metal (heavier)
ρ – density of liquid (lighter)
v – terminal velocity

SBTE Questions

1. Define terminal velocity.
2. State Stoke's law of viscosity.
3. Derive Stoke's formula in viscosity.
4. State Stoke's law of viscosity and obtain an equation for coefficient of viscosity of liquid.
5. State Stoke's law of viscosity and state the formula for coefficient of viscosity.

5.3.6 Effect of Temperature and Adulteration on Viscosity of Liquid

1. **Effect of temperature on viscosity :** Viscosity of a given liquid is affected by its temperature.

Viscosity of liquid decreases with increase in temperature.

$$\text{Viscosity of liquid} \propto \frac{1}{\text{Temperature}}$$

This is due to the reason that, the shear stress (opposition to change of shape i.e. viscosity) in liquid is due to intermolecular cohesion force. This cohesion force depends on the distance between two molecules. As the temperature of liquid increases, intermolecular distance increases which decreases the cohesion force and hence decreases viscosity.

In other words, *as temperature of liquid increases*, kinetic energy of molecules increases i.e. mobility of liquid increases i.e. viscosity of liquid decreases.

e.g. In summer season, speed of door closer increases.

2. **Effect of adulteration on viscosity of liquid :** Liquid adulteration means mixing of some other material.

Practically, it is observed that when *adulteration of soluble substance is added to liquid, its viscosity goes on increasing.*

$$\text{Viscosity} \propto \text{Adulteration}$$

Because of **adulteration**, mobility of liquid decreases i.e. **viscosity increases**.

e.g. If sugar is dissolved in pure water then viscosity of net solution increases.

SBTE Questions

1. State the effect of temperature and adulteration on viscosity of liquid.

Important Points

- The total force exerted by a liquid on a surface in contact is called thrust of a liquid.
- The force per unit area is called pressure.
- When liquid is in equilibrium, the pressure (force) of a liquid always act normal (perpendicular) to the surface.
- Pressure at a point inside the liquid depends on the depth. Pressure increases with the depth.
- Pressure at any point inside the liquid is same in all directions.
- There is equal pressure at all the points which are in the same level.
- The pressure exerted by the atmosphere is called atmospheric pressure.
- Pascal's law states that when the pressure in enclosed liquid at any point is changed by some amount, an equal amount of change in pressure is transmitted throughout the liquid. This change in pressure acts normal to the surface everywhere.
- Archimede's principle states that when a solid insoluble body is immersed completely or partly in a liquid, it losses its weight and loss of weight of the body is equal to the weight of displaced liquid.
- Viscosity is the property of liquid on account of which liquid tries to oppose the relative motion between its different layers.

- Newton's law states that the viscous force 'F' developed between two liquid layers is
  (1) directly proportional to the surface area 'A' of liquid layer,
  (2) directly proportional to the velocity gradient.
- The coefficient of viscosity 'η' is said to be 1 poise if 1 dyne viscous force is developed between two liquid layers of 1 cm^2 area for unit velocity gradient.
- Coefficient of viscosity 'η' of a liquid is defined as the viscous force developed between two liquid layers of unit surface area for unit velocity gradient.
- Streamline flow is the flow of the liquid in which every particle of liquid moves in the same direction (i.e. parallel or inline) of flow of liquid.
- The value of velocity of flow of liquid upto which flow is streamline is called critical velocity 'v_c'.
- When R is less than 2000, then liquid flow is streamline.
- When R is in between 2000 to 3000, the liquid flow is unstable.
- When R is greater than 3000, the liquid flow is turbulent.
- The constant velocity with which a body falls through liquid column is called terminal velocity.
- Stoke's law states that the force of viscosity experienced by a small metal sphere falling freely through a viscous medium, with terminal velocity is directly proportional to
  (1) radius of metal sphere 'r'
  (2) terminal velocity 'v'
  (3) coefficient of viscosity of liquid 'η'
- Buoyant force or upthrust force is the force with which liquid lifts the body dipped into it.
- Viscosity of liquid decreases with increase in temperature.
- Practically, it is observed that when adulteration of soluble substance is added to liquid, its viscosity goes on increasing.

Formulae

(1) $F = \eta A \frac{dv}{dx}$ … Newton's formula

where F – Force of viscosity
η – Coefficient of viscosity
A – Area of liquid layer
$\frac{dv}{dx}$ – Velocity gradient

(2) $F = 6\pi\eta rv$ … Stoke's formula

where F – Force of viscosity
η – Coefficient of viscosity
r – Radius of metal sphere
v – Terminal velocity

(3) $\eta = \frac{2}{9}\frac{r^2 g\,(d - \rho)}{v}$

where r – Radius of metal sphere
d – Density of metal (heavier)
ρ – Density of liquid (lighter)
v – Terminal velocity

(4) $v = \frac{R\eta}{\rho r}$

where v – Velocity of flow of liquid
R – Reynold's number
η – Coefficient of viscosity
ρ – Density of liquid
r – Radius of the tube (pipe)

SOLVED EXAMPLES

Example 1 : *A spherical ball of radius 0.15 cm takes 6 seconds to travel a distance of 80 cm through a viscous liquid. If the density of a ball is 8×10^3 kg/m^3 and that of liquid is 1.2×10^3 kg/m^3, find the viscosity of the liquid.*

Solution : Given :

$$r = 0.15 \text{ cm} = 0.15 \times 10^{-2} \text{ m}$$

$$v = \frac{\text{distance}}{\text{time}} = \frac{80 \text{ cm}}{6 \text{ sec}} = (13.33) \text{ cm/s} = (0.133) \text{ m/s}$$

$$d = 8 \times 10^3 \text{ kg/m}^3,\ \rho = 1.2 \times 10^3 \text{ kg/m}^3,\ \eta = ?$$

$$\eta = \frac{2}{9}\frac{r^2 g (d - \rho)}{v} = \frac{2}{9}\frac{(0.15 \times 10^{-2})^2 (9.8)(8 \times 10^3 - 1.2 \times 10^3)}{(0.133)}$$

$$\boxed{\eta = 0.25 \text{ Ns/m}^2}$$

Example 2 : *A spherical ball of radius 2.2 mm and density 8×10^3 kg/m^3 falls through a liquid of density 1.3×10^3 kg/m^3. Find terminal velocity.*

*(**Given :** η for liquid = 0.45 Ns/m^2).*

Solution : Given :

$$r = 2.2 \text{ mm} = 2.2 \times 10^{-3} \text{ m},\ d = 8 \times 10^3 \text{ kg/m}^3,\ \rho = 1.3 \times 10^3 \text{ kg/m}^3$$

$$\eta = 0.45 \text{ Ns/m}^2,\ v = ?$$

We have,

$$\eta = \frac{2}{9}\frac{r^2 g (d - \rho)}{v}$$

$$v = \frac{2}{9}\frac{r^2 g (d - \rho)}{\eta} = \frac{2}{9} \times \frac{(2.2 \times 10^{-3})^2 (9.8)(8 \times 10^3 - 1.3 \times 10^3)}{0.45}$$

$\therefore$

$$\boxed{v = 0.157 \text{ m/s}}$$

Example 3 : *A metal plate of area 0.25 m^2 rests on a layer of oil 0.003 m thick with coefficient of viscosity 1.56 Ns/m^2. If the plate is moved with a velocity of 0.05 m/s, calculate the horizontal force acting on the plate.*

Solution : Given : A = 0.25 m^2, dx = 0.003 m, η = 1.56 Ns/m^2, dv = 0.05 m/s, F = ?

We have,

$$F = \eta A \frac{dv}{dx} = (1.56)(0.25)\frac{(0.05)}{(0.003)}$$

$$\boxed{F = 6.5 \text{ N}}$$

Example 4 : *A plate of metal having 100 sq. cm. area rests on a layer of paraffin oil 2 mm thick. If the horizontal force required to move the plate with velocity 3 cm/s is 0.24 newton, find the coefficient of viscosity.*

Solution : A = 100 cm^2 = 100×10^{-4} m^2, dx = 2 mm = 2×10^{-3} m, dv = 3 cm/s = 3×10^{-2} m/s, F = 0.24 N, η = ?

By Newton's law of viscosity,

$$F = \eta A \times \frac{dv}{dx}$$

$\therefore$

$$\eta = \frac{F}{A \times \frac{dv}{dx}} = \frac{(0.24)}{(100 \times 10^{-4})\frac{(3 \times 10^{-2})}{(2 \times 10^{-3})}}$$

$$\boxed{\eta = 1.6 \text{ Ns/m}^2}$$

Example 5 : *Find the velocity of flow of a liquid of viscosity 0.02 Ns/m^2 and density 1.2 gm/cm^3 through a pipe of radius 0.05 m, Reynold's number = 2000.*

Solution : Given : η = 0.02 Ns/m^2, ρ = 1.2 gm/cm^3 = 1.2×10^3 kg/m^3, r = 0.05 m, R = 2000, v = ?

$$v = \frac{\eta R}{\rho r} = \frac{(0.02)(2000)}{(1.2 \times 10^3)(0.05)}$$

$$\boxed{v = 0.667 \text{ m/s}}$$

Example 6 : *A liquid flows through a pipe of diameter 10 cm with a speed of 0.5 m/s. The density of a liquid is 0.8×10^3 kg/m^3 and the coefficient of viscosity is 0.4 Ns/m^2. Determine Reynold's number and state whether the flow is turbulent or streamline.*

Solution : Given : diameter = 10 cm

$$\text{radius } r = 5 \text{ cm} = 0.05 \text{ m}$$

$$v = 0.5 \text{ m/s}$$

$$\rho = 0.8 \times 10^3 \text{ kg/m}^3$$

$$\eta = 0.4 \text{ Ns/m}^2$$

$$R = ?$$

$$v = \frac{\eta R}{\rho r}$$

$$\therefore \quad R = \frac{v\rho r}{\eta} = \frac{(0.5) \times (0.8 \times 10^3) \times (0.05)}{0.4}$$

$$R = 50$$

Since R < 2000, the liquid flow is streamlined.

Example 7 : *A steel ball of density 8×10^3 kg/m^3 falls vertically in a tall jar containing an oil of density 1.5×10^3 kg/m^3 and acquires a terminal velocity 0.2 m/s. If radius of the ball is 2 mm, find coefficient of viscosity of oil. (g = 9.81 m/s^2)*

Solution : Given : $d = 8 \times 10^3$ kg/m^3, $\rho = 1.5 \times 10^3$ kg/m^3, v = 0.2 m/s, r = 2 mm = 0.002 m, η = ?

$$\eta = \frac{2}{9} \frac{r^2 g (d - \rho)}{v} = \frac{2}{9} \times \frac{(0.002)^2 (9.81) (8 \times 10^3 - 1.5 \times 10^3)}{0.2}$$

$$\boxed{\eta = 0.28 \text{ Ns/m}^2}$$

Example 8 : *Calculate the viscous force on a steel ball of radius 2 mm, falling freely in a tall jar containing oil with a terminal velocity 0.2 m/s, coefficient of viscosity of oil is 0.28 Ns/m^2.*

Solution : F = ?, r = 2 mm = 2×10^{-3} m, v = 0.2 m/s, η = 0.28 Ns/m^2

We have Stoke's formula for viscous force.

$$F = 6\pi\eta r v = 6 \times 3.142 \times (0.28)\,(2 \times 10^{-3})\,(0.2)$$

$$\boxed{F = 2.11 \times 10^{-3} \text{ N}}$$

Example 9 : *Rain drop of diameter 0.03 cm is falling with velocity 2.2 m/s. If coefficient of viscosity of air is 1.75×10^{-4} Ns/m^2, calculate viscous force on the rain drop.*

Solution : Given : diameter = 0.03 cm $\therefore$ r = 0.015 cm = 0.015×10^{-2} m, $\eta = 1.75 \times 10^{-4}$ Ns/m^2, v = 2.2 m/s, F = ?

$$F = 6\pi\eta r v = 6 \times 3.142 \times (1.75 \times 10^{-4}) \times (0.015 \times 10^{-2})\,(2.2)$$

$$\boxed{F = 1.09 \times 10^{-6} \text{ N}}$$

Example 10 : *A rain drop with radius 0.2 mm, is falling through air with terminal velocity v. Calculate v if coefficient of viscosity of air is 1.8×10^{-4} Ns/m^2 and viscous force is 0.14 dyne.*

Solution : Given : r = 0.2 mm = 0.2×10^{-3} m, v = ?, $\eta = 1.8 \times 10^{-4}$ Ns/m^2, F = 0.14 dyne = 0.14×10^{-5} N

We have, $F = 6\pi\eta r v$

$$\therefore \quad v = \frac{F}{6\pi\eta r} = \frac{(0.14 \times 10^{-5})}{[(6 \times 3.142) \times (1.8 \times 10^{-4}) \times (0.2 \times 10^{-3})]}$$

$$\boxed{v = 2.06 \text{ m/s}}$$

Example 11 : *Assuming the Reynold's number to be 1000, calculate the critical velocity for glycerine in the pipe of diameter 2 cm. Density and viscosity of glycerine are 1.36×10^3 kg/m^3 and 0.85 Ns/m^2 respectively.*

Solution : Given : R = 1000, diameter = 2 cm, r = 1 cm = 1×10^{-2} m, density $\rho = 1.36 \times 10^3$ kg/m^3,

$\eta = 0.85$ Ns/m^2, $v = v_c = ?$

$$v = v_c = \frac{R\eta}{\rho r} = \frac{(1000) \times (0.85)}{(1.36 \times 10^3) \times (1 \times 10^{-2})}$$

$$\therefore \quad \boxed{v_c = 62.5 \text{ m/s}}$$

Example 12 : *An air bubble of radius 1 cm rises steadily through solution of density 1.75 × 10³ kg/m³ at the steady velocity of 0.35 m/s. Calculate coefficient of viscosity of the solution neglecting density of air.*

Solution : Given : $r = 1 \text{ cm} = 1 \times 10^{-2}$ m

density of liquid, $d = 1.75 \times 10^3$ kg/m^3

$v = 0.35$ m/s

$$\eta = \frac{2}{9}\frac{r^2 g (d - \rho)}{v} = \frac{2}{9}\frac{(1 \times 10^{-2})^2 \times (9.8) \times (1.75 \times 10^3 - 0)}{0.35}$$

$$\therefore \quad \boxed{\eta = 1.08 \text{ Ns/m}^2}$$

Practice Questions

1. Explain the property of viscosity of liquid.
2. State Newton's law of viscosity.
3. Define coefficient of viscosity and state its unit.
4. Define velocity gradient.
5. State Stoke's law and state formula for terminal velocity.
6. Define terminal velocity.
7. What is streamline flow and turbulent flow ?
8. Define critical velocity.
9. Explain significance of Reynold's number.
10. State Stoke's law of viscosity and state the formula for coefficient of viscosity.
11. Define critical velocity and streamline flow.
12. State Newton's law of viscosity and hence define coefficient of viscosity. Give its S.I. unit.
13. Define velocity gradient and state its unit.
14. What is Reynold's number ? State its significance.
15. Define velocity gradient and terminal velocity.
16. State Newton's law of viscosity. Define velocity gradient. State the formula for viscous force, hence define 1 poise.

Practice Problems

1. A spherical liquid drop of diameter 0.02 cm is falling freely through air. Find the terminal velocity of the drop. The density of the liquid is 500 kg/m^3 and the viscosity of air is 2×10^{-5} Ns/m^2. Neglect density of air.

   (**Ans.** v = 0.54 m/s)

2. With what terminal velocity will an air bubble of radius 0.1 cm rise through water ? Coefficient of viscosity of water is 10^{-2} poise. Neglect density of bubble. (**Ans.** v = 218 cm/s)

3. A spherical steel ball of radius 0.3 mm falls vertically through water. Find the coefficient of viscosity of water.

   Given : terminal velocity acquired by the ball = 9.8×10^{-2} m/s, density of water = 1×10^3 kg/m^3 and density of the ball = 1.5×10^3 kg/m^3. (**Ans.** $\eta = 1 \times 10^{-3}$ Ns/m^2)

4. Calculate the viscous force on raindrop of diameter 4 mm falling with a constant velocity of 4 m/s through air. Coefficient of viscosity of air is 1.8×10^{-4} poise. (**Ans.** F = 0.27 dyne)
5. A spherical ball of diameter 3 mm and density 7.8×10^3 kg/m^3 falls vertically through the oil of tall jar and attains a terminal velocity of 0.231 m/s. If density of oil is 1.2×10^3 kg/m^3, find coefficient of viscosity of oil.
(**Ans.** η = 0.14 Ns/m^2)
6. A spherical steel ball of diameter 6 mm and density 7.2×10^3 kg/m^3 falls with terminal velocity of 0.24 m/s through glycerine. Find the coefficient of viscosity of glycerine. Given : density of glycerine = 1.26×10^3 kg/m^3 and g = 9.8 m/s^2. (**Ans.** η = 0.485 Ns/m^2)
7. A force of 8 N is required to move a solid horizontal surface over a liquid of surface area 0.25 m^2 with a velocity of 0.05 m/s. If the thickness of the liquid layer is 2 mm, calculate the coefficient of viscosity. (**Ans.** η = 1.28 Ns/m^2)
8. A sphere of density 8×10^3 kg/m^3 falls vertically in a jar containing oil of density 1.5×10^3 kg/m^3 and acquires a terminal velocity of 0.25 m/s. If the radius of a sphere is 2 mm, find the coefficient of viscosity of oil.
(**Ans.** η = 0.2265 Ns/m^2)
9. Assuming the Reynold's number to be 1200, calculate the critical velocity for liquid in the pipe of radius 1.2 cm. Density and viscosity of liquid are 1300 kg/m^3 and 0.8 Ns/m^2 respectively. (**Ans.** v_c = 61.54 m/s)
10. A liquid flows through a pipe of radius 4 cm with a speed of 12 m/s. The density of liquid is 0.85×10^3 kg/m^3 and coefficient of viscosity is 0.6 Ns/m^2. Determine Reynold's number and state whether the flow is turbulent or streamline. (**Ans.** R = 680 which is less than 2000, hence flow is streamlined)

IMP : (Last hour quick note) Quantity with units to be remembered (generally asked in MCQs) :

Sr. No.	Quantity	MKS unit	CGS unit
1.	Stress (all types)	N/m^2	dyne/cm^2
2.	Strain (all types)	No unit	No unit
3.	Compressibility	m^2/N	cm^2/dyne
4.	Surface tension	N/m	dyne/cm
5.	Velocity gradient	1/sec	1/sec
6.	Coefficient of viscosity (η)	Ns/m^2	dyne s/cm^2 or poise

MCQs on General Properties of Matter

1. *When an external force is applied on an elastic body, then body changes its size and shape and the body is said to be ……*
   (a) regained *(b) restored* *(c) deformed* *(d) plastic*
2. *The force applied on a body which is responsible for change of size and shape of the body is called as ……*
   (a) restoring force *(b) deforming force* *(c) internal force* *(d) regaining force*
3. *In the case of elastic body, under deformed condition, every shifted molecule tries to achieve its original position due to which a force is developed inside a body is called as ……*
   (a) internal restoring force *(b) deforming force* *(c) external force* *(d) applied force*
4. *The force which is responsible to restore original size and shape of the body is called as ……*
   (a) applied force *(b) deforming force* *(c) external force* *(d) internal restoring force*
5. *Force applied on elastic body is within elastic limit and if now force is removed then the body ……*
   (a) regains its original size and shape *(b) changes its size and shape*
   (c) opposes change in size and shape *(d) does not regain its size and shape*
6. *In the case of elastic body, body regains its original size and shape on removal of external deforming force if and only if the external force is ……*
   (a) within elastic limit *(b) more than elastic limit*
   (c) too large *(d) equal to deforming force*

7. *In the case of elastic body, if external applied force is too large and it is more than elastic limit, then there will be ……*
   (a) permanent retention (b) more opposition (c) permanent deformation (d) less opposition
8. *The property on account of which body regains its original size and shape on removal of external deforming force is called as ……*
   (a) plasticity (b) elasticity (c) rigidity (d) ductility
9. *Almost all metals are ……*
   (a) elastic (b) plastic (c) rigid (d) none of these
10. *Clay, putty, chalk are the examples of ……*
   (a) elastic body (b) plastic body (c) rigid body (d) none of these
11. *The body which easily deforms when external force is applied and does not regain its original size and shape on removal of external deforming force is called as ……*
   (a) elastic body (b) rigid body (c) plastic body (d) none of these
12. *The property on account of which body does not change its size and shape even when large force is applied on it, is called ……*
   (a) elasticity (b) plasticity (c) ductility (d) rigidity
13. *Stone is ……*
   (a) elastic body (b) plastic body (c) rigid body (d) none of these
14. *Stress is defined as ……*
   (a) internal elastic restoring force per unit area (b) area per unit internal elastic restoring force
   (c) product of internal elastic restoring force and area (d) none of these
15. *The unit of stress is ……*
   (a) m^2/N (b) N/m^2 (c) Nm^2 (d) J/m^2
16. *Dimensions of stress are ……*
   (a) $[L^1\ M^{-1}\ T^2]$ (b) $[L^1\ M^1\ T^{-2}]$ (c) $[L^{-1}\ M^{-1}\ T^2]$ (d) $[L^{-1}\ M^1\ T^{-2}]$
17. *Stress is equal to ……*
   (a) A/F (b) $F \times A$ (c) F/A (d) $F + A$
18. *Tensile stress is also called as ……*
   (a) lateral stress (b) longitudinal stress (c) volume stress (d) shear stress
19. *The stress that tends to change the length of the body is called as ……*
   (a) tensile stress (b) lateral stress (c) volume stress (d) shearing stress
20. *Cable of lift elevator is the example of ……*
   (a) longitudinal stress (b) volume stress (c) lateral stress (d) shearing stress
21. *If a deforming force produces change in volume of a body, then the corresponding stress is called as ……*
   (a) longitudinal stress (b) tensile stress (c) volume stress (d) shear stress
22. *Shock absorber in a vehicle is an example of ……*
   (a) longitudinal stress (b) tensile stress (c) volume stress (d) shear stress
23. *Volume stress is equal to ……*
   (a) change in pressure (b) product of force and area
   (c) area per unit force (d) addition of force and area
24. *The stress corresponding to change in shape of a body is called as ……*
   (a) longitudinal stress (b) tensile stress (c) volume stress (d) shear stress
25. *Torsional pendulum is the example of ……*
   (a) tensile stress (b) volume stress (c) shear stress (d) longitudinal stress

26. *One of the following is the example of shear stress.*
(a) *bycycle chain* (b) *shock absorber in vehicle* (c) *cable of lift elevator* (d) *metal sheet cutter*

27. *Shear stress is equal to ......*
(a) *area/tangential force* (b) *tangential force/area* (c) *tangential force* $\times$ *area* (d) *tangential force + area*

28. *The change in dimensions per unit original dimension is called as ......*
(a) *stress* (b) *strain* (c) *modulus of electricity* (d) *shear stress*

29. *The unit of strain is ......*
(a) N/m^2 (b) Nm^2 (c) J/m^2 (d) *no unit*

30. *Tensile strain is defined as ......*
(a) *change in length per unit original length* (b) *original length per unit change in length*
(c) *product of original length and change in length* (d) *addition of original length and change in length*

31. *Longitudinal strain is equal to ......*
(a) *F/A* (b) *A/F* (c) *l/L* (d) *L/l*

32. *Volume strain is defined as ......*
(a) *change in length per unit original length* (b) *change in volume per unit original volume*
(c) *original volume per unit change in volume* (d) *original length per unit change in length*

33. *Shear strain is defined as ......*
(a) *force per unit area* (b) *area per unit force*
(c) *product of lateral displacement to distance from fixed layer*
(d) *ratio of lateral displacement of layer to its distance from fixed layer*

34. *The ratio of lateral displacement of any layer to its distance from fixed layer is called as ......*
(a) *lateral strain* (b) *shear strain* (c) *lateral stress* (d) *shear stress*

35. *The stress corresponding to limiting value of the load, which when applied and subsequently released, does not produce permanent deformation is called as ......*
(a) *elastic limit* (b) *plastic limit* (c) *breaking stress* (d) *ultimate stress*

36. *Within elastic limit, strain is directly proportional to stress is the ......*
(a) *Boyle's law* (b) *Newton's law* (c) *Pascal's law* (d) *Hooke's law*

37. *Modulus of elasticity is equal to ......*
(a) *strain/stress* (b) *stress/strain* (c) *stress* $\times$ *strain* (d) *none of these*

38. *Within elastic limit, the ratio of tensile stress to tensile strain is called as ......*
(a) *Young's modulus of elasticity* (b) *Bulk modulus of elasticity*
(c) *modulus of rigidity* (d) *Poisson's ratio*

39. *Formula for Young's modulus of elasticity is ......*
(a) $Y = \frac{F}{A} \times \frac{l}{L}$ (b) $Y = \frac{A}{F} \times \frac{l}{L}$ (c) $Y = \frac{F}{A} \times \frac{L}{l}$ (d) $Y = \frac{A}{F} \times \frac{L}{l}$

40. *Within elastic limit, the ratio of volume stress to volume strain is called as ......*
(a) *Young's modulus of elasticity* (b) *Bulk modulus of elasticity*
(c) *Modulus of rigidity* (d) *none of these*

41. *Bulk modulus of elasticity is given by*
(a) $K = \frac{dv}{V} \times dp$ (b) $K = \frac{dv}{V \times dp}$ (c) $K = dp \times dv \times V$ (d) $K = \frac{dp \times V}{dv}$

42. *The compressibility is defined as ......*
(a) *reciprocal of bulk modulus of elasticity* (b) *reciprocal of Young's modulus of elasticity*
(c) *reciprocal of modulus of rigidity* (d) *none of these*

43. *Modulus of rigidity is the ……*
    (a) *product of shear stress to shear strain*
    (b) *ratio of shear strain to shear stress*
    (c) *ratio of shear stress to shear strain*
    (d) *ratio of tensile strain to tensile stress*
44. *The relation between Young's modulus (Y), bulk modulus (K) and modulus of rigidity (η) is given by ……*
    (a) $\frac{1}{Y} = \frac{1}{3\eta} + \frac{1}{9K}$
    (b) $\frac{1}{3Y} = \frac{1}{\eta} + \frac{1}{9K}$
    (c) $\frac{1}{K} = \frac{1}{Y} + \frac{1}{\eta}$
    (d) $\frac{1}{Y} = \frac{1}{9\eta} + \frac{1}{3K}$
45. *The maximum stress the system is capable of withstanding is called as ……*
    (a) *breaking stress* (b) *ultimate stress* (c) *working stress* (d) *tensile stress*
46. *Actual practical stress on the system is called as ……*
    (a) *breaking stress* (b) *ultimate stress* (c) *working stress* (d) *tensile stress*
47. *Factor of safety is defined as ……*
    (a) *ratio of ultimate stress to working stress*
    (b) *ratio of working stress to ultimate stress*
    (c) *ratio of breaking stress to ultimate stress*
    (d) *ratio of ultimate stress to breaking stress*
48. *Lateral strain is the ……*
    (a) *ratio of change in length to original length*
    (b) *product of decrease in diameter to original diameter*
    (c) *ratio of original diameter to decrease in diameter*
    (d) *ratio of decrease in diameter to original diameter*
49. *Elasticity of steel is ……*
    (a) *more than rubber*
    (b) *less than rubber*
    (c) *same as that of rubber*
    (d) *more or less than rubber-depends upon dimensions*
50. *If we take 1 m long steel wire and 2 m long steel wire then ……*
    (a) *elasticity of 1 m will be more than 2 m*
    (b) *elasticity of 2 m will be more than 1 m*
    (c) *elasticity of 1 m and 2 m wire will be the same*
    (d) *depends on diameter of wire too*
51. *If two different wires of steel and aluminium of same dimensions are taken then ……*
    (a) *elasticity of both the wires will be same*
    (b) *elasticity of both the wires will be different*
    (c) *elasticity depends upon what dimensions it has*
    (d) *none of these*
52. *Four wires of same metal and same diameter are stretched by same load. Length of each wire is given below. Which of them will elongate least ?*
    (a) *L = 0.5 m* (b) *L = 1 m* (c) *L = 1.5 m* (d) *L = 2 m*
53. *Four wires of same metal are stretched by same load. The dimensions are given below. Which of them will elongate least ?*
    (a) *r = 0.5 mm, L = 50 cm* (b) *r = 1 mm, L = 100 cm* (c) *r = 1.5 mm, L = 150 cm* (d) *r = 2 mm, L = 200 cm*
54. *Four wires of different metals are stretched by different load. The dimensions are given below, which of the following has lowest elasticity ……*
    (a) *M = 1 kg, r = 1 mm, l = 1 mm, L = 1 m*
    (b) *M = 2 kg, r = 2 mm, l = 2 mm, L = 2 m*
    (c) *M = 3 kg, r = 3 mm, l = 3 mm, L = 3 m*
    (d) *M = 4 kg, r = 4 mm, l = 4 mm, L = 4 m*
55. *The extension produced in a wire due to a load is 3 mm. The extension in a wire of same material and length but half the radius will be ……*
    (a) *10 mm* (b) *12 mm* (c) *14 mm* (d) *16 mm*
56. *If 'l' is the extension produced in the wire of length L, radius r, with a force F, find the extension produced in the wire of same metal of length 2L, radius 2r and force 2F will be ……*
    (a) *l/2* (b) *l* (c) *2l* (d) *3l*
57. *A wire of length 2 m extends by 2 mm when a force is applied to it. Calculate stress produced in it if* $Y = 2 \times 10^{11}$ N/m^2.
    (a) 1×10^8 N/m^2 (b) 2×10^8 N/m^2 (c) 3×10^8 N/m^2 (d) 4×10^8 N/m^2

58. *A wire of diameter 2 mm and length 3 m extends by 2 mm when a force of 5 N is applied. Find Young's modulus of material of the wire.*
    (a) 1.2×10^9 N/m^2 (b) 2.39×10^{10} N/m^2 (c) 2.39×10^9 N/m^2 (d) 2.39×10^8 N/m^2

59. *Calculate Young's modulus of elasticity for a wire having length 150 cm and cross-sectional area 2×10^{-5} m^2. The wire elongates by 3 mm when subjected to a load of 10 N.*
    (a) 2.5×10^8 N/m^2 (b) 2.5×10^9 N/m^2 (c) 1×10^8 N/m^2 (d) 1×10^9 N/m^2

60. *A metal bar has a maximum stress of 9×10^8 N/m^2. If area of cross-section of the bar is 0.02 m^2, find maximum force that the bar can withstand.*
    (a) 0.18×10^9 N/m^2 (b) 0.18×10^6 N/m^2 (c) 0.18×10^7 N/m^2 (d) 0.18×10^8 N/m^2

61. *A metal wire is stretched by 3% of its length. Determine the stress produced in the wire. Given Y for metal = 2.4×10^{11} N/m^2.*
    (a) 8×10^8 N/m^2 (b) 8×10^{10} N/m^2 (c) 7.2×10^9 N/m^2 (d) 7.2×10^8 N/m^2

62. *A longitudinal stress of 8×10^8 N/m^2 produces an extension of 2 mm in a wire of length 3 m. Find Young's modulus of material of the wire.*
    (a) 12×10^{11} N/m^2 (b) 12×10^{12} N/m^2 (c) 12×10^{10} N/m^2 (d) 1.2×10^{11} N/m^2

63. *Equal weights are attached to two wires of the same metal. The length and radius of one wire is twice the other. Calculate ratio of their extensions.*
    (a) $\frac{l_1}{l_2} = \frac{1}{4}$ (b) $\frac{l_1}{l_2} = \frac{1}{3}$ (c) $\frac{l_1}{l_2} = \frac{1}{2}$ (d) $\frac{l_1}{l_2} = \frac{1}{1}$

64. *Calculate the compressibility of metal, if bulk modulus of elasticity K = 2×10^{10} N/m^2.*
    (a) 0.2×10^{-10} m^2/N (b) 0.5×10^{-10} m^2/N (c) 0.2×10^{10} m^2/N (d) 0.5×10^{10} m^2/N

65. *Calculate shearing strain if 5 cm thick metal plate is sheared and top surface displaces by 0.06 mm.*
    (a) 2.4×10^3 (b) 2.4×10^{-3} (c) 1.2×10^3 (d) 1.2×10^{-3}

66. *Calculate Poisson's ratio if metal wire of length 3 m and diameter 0.3 mm is stretched by 2 mm and lateral contraction is 1.5×10^{-4} mm.*
    (a) 0.25 (b) 0.5 (c) 0.75 (d) 1

MCQs on Surface Tension

67. *The angle of contact for liquid on a solid surface is the angle between ……*
    (a) the tangent to the liquid surface at the point of contact and the solid surface
    (b) the tangent to the solid surface at the point of contact and the liquid surface
    (c) the liquid surface and the solid surface at the point of contact
    (d) none of the above

68. *When impurity is added to a liquid, its surface tension ……*
    (a) decreases (b) first decreases and then increases
    (c) increases (d) remains same

69. *By which phenomenon does the water rise from roots to leaves of plants ?*
    (a) capillary action (b) surface tension (c) Bernoulli's theorem (d) viscosity

70. *When a capillary tube of radius r is dipped in a liquid of density ρ and surface tension S, the liquid rises or falls through a distance ……*
    (a) $h = \frac{2S \cos \theta}{r\rho g}$ (b) $h = \frac{S \cos \theta}{r^2\rho g}$ (c) $h = \frac{2S \cos \theta}{r\rho g}$ (d) $h = \frac{2S^2 \cos \theta}{r\rho g}$

71. *S.I. unit of surface tension is ……*
    (a) N.m^2 (b) N.m (c) N/m (d) N/m^2

72. *Water rises to a height of 10 mm in a capillary. If the radius of the capillary is made 1/3rd of its previous value, to what height will the water now rise in the tube ?*
    (a) 30 mm (b) 40 mm (c) 50 mm (d) 60 mm

73. *One end of a towel dips into a bucket full of water and other end hangs over the bucket. It is found that after some time the towel becomes fully wet. It happens ……*
    (a) because viscosity of water is high (b) because of capillary action of cotton threads
    (c) because of gravitational force (d) because of evaporation of water
74. *For pure water and clean glass, the angle of contact is ……*
    (a) $0°$ (b) $90°$ (c) $140°$ (d) $8°$
75. *The angle of contact between a glass capillary tube of length 10 cm and a liquid is 90°. If the capillary tube is dipped vertically in the liquid, then the liquid ……*
    (a) will rise in the tube (b) will get depressed in the tube
    (c) will rise upto 10 cm in the tube and will overflow (d) with neither rise nor fall in the tube
76. *When there are no external forces, the shape of a liquid drop is determined by ……*
    (a) surface tension of the liquid (b) density of the liquid
    (c) viscosity of the liquid (d) temperature of air only
77. *Choose the wrong statement from the following ……*
    (a) small droplets of a liquid are spherical due to surface tension
    (b) oil rises through the wick due to capillarity
    (c) in drinking the cold drinks through a straw, we use the phenomenon of capillarity
    (d) gum is used to stick two surfaces, in this process we use the property of adhesion
78. *When the angle of contact between a solid and a liquid is 90°, then ……*
    (a) cohesive force > adhesive force (b) cohesive force < adhesive force
    (c) cohesive force = adhesive force (d) cohesive force >> adhesive force
79. *Rain drops are spherical in shape because of ……*
    (a) surface tension (b) capillarity
    (c) downward motion (d) acceleration due to gravity
80. *Which of the following is not based on the principle of capillarity ?*
    (a) floating of wood on water surface (b) ploughing of soil
    (c) rise of oil in a wick of lantern (d) soaking of ink by bloating paper
81. *The rise of liquid in a capillary tube does not depend upon ……*
    (a) angle of contact (b) density of the liquid
    (c) radius of the capillary tube (d) atmospheric pressure
82. *The height of water in a capillary tube of radius 2 cm is 4 cm. What should be the radius of capillary, if the water rises to 8 cm in the tube ?*
    (a) 1 cm (b) 0.1 cm (c) 2 cm (d) 4 cm
83. *For a liquid, which is rising in a capillary tube, the angle of contact is ……*
    (a) $90°$ (b) $180°$ (c) acute (d) obtuse
84. *A capillary tube is placed vertically in a liquid. If the cohesive force is less than the adhesive force, then ……*
    (a) the miniscus will be convex upwards (b) the liquid will wet the solid
    (c) the angle of contact will be obtuse (d) the liquid will drip in the capillary tube
85. *The surface tension for pure water in a capillary tube experiment is ……*
    (a) $\frac{3g}{2hr}$ (b) $\frac{3}{hr\rho g}$ (c) $\frac{r\rho g}{2h}$ (d) $\frac{hr\rho g}{2}$
86. *A liquid is kept in a glass vessel. If the liquid solid adhesive force between the liquid and the vessel is very weak as compared to the cohesive force in the liquid, then the shape of the liquid surface near the solid should be ……*
    (a) concave (b) convex (c) horizontal (d) almost vertical

87. *In a capillary tube, fall of liquid is possible when angle of contact is ……*
   (a) acute angle (b) right angle (c) obtuse angle (d) none of these
88. *Water rises upto a height of 5 cm in a capillary tube of radius 2 mm. What is the radius of the capillary tube if the water rises upto a height of 10 cm in another capillary ?*
   (a) 4 mm (b) 1 mm (c) 3 mm (d) 1 cm
89. *Surface tension of liquid is independent of the ……*
   (a) temperature of the liquid (b) area of the liquid surface
   (c) nature of the liquid (d) impurities present in the liquid
90. *For a liquid which is rising in a capillary, the angle of contact is ……*
   (a) obtuse (b) acute (c) 180° (d) 90°
91. *Two capillary tubes of the same material but of different radii are dipped in a liquid. The heights to which the liquid rises in the two tubes are 2.2 cm and 6.6 cm. The ratio of radii of the tubes will be ……*
   (a) 1 : 9 (b) 1 : 3 (c) 9 : 1 (d) 3 : 1
92. *The dimensions of surface tension are ……*
   (a) $[M^1L^1T^{-1}]$ (b) $[M^1L^2T^{-2}]$ (c) $[M^1L^0T^{-2}]$ (d) $[M^1L^{-1}T^{-2}]$
93. *Kerosene in the wick of lantern rises up because ……*
   (a) of negligible viscosity (b) the diffusion of the oil through the wick
   (c) of surface tension of the oil (d) wick attracts the kerosene
94. *At the boiling point of water, its surface tension ……*
   (a) is infinite (b) is zero
   (c) is the same as that at room temperature (d) is maximum
95. *The surface tension of liquid is 10^7 dyne/cm. It is equivalent to ……*
   (a) 10^4 N/m (b) 10^5 N/m (c) 10^9 N/m (d) 10^{11} N/m
96. *The surface of water in contact with glass wall is ……*
   (a) plane (b) concave (c) convex (d) both (b) and (c)
97. *More liquid rises in a thin tube because of ……*
   (a) larger value of radius (b) larger value of surface tension
   (c) smaller value of surface tension (d) smaller value of radius
98. *When two capillary tubes of different diameters are dipped vertically the rise of the liquid is ……*
   (a) same in both the tubes (b) more in tube of larger diameter
   (c) less in tube of smaller diameter (d) more in the tube of smaller diameter

MCQs on Viscosity

99. *Viscosity is the property of liquid on account of which liquid tries to ……*
   (a) help the relative motion between its different layers
   (b) accelerates the relative motion between its different layers
   (c) stops the relative motion between its different layers
   (d) opposes the relative motion between its different layers
100. *Velocity gradient is defined as the ……*
   (a) change in velocity/distance (b) distance/change in velocity
   (c) change in velocity × distance (d) change in velocity + distance
101. *Newton's law of viscosity states that, the viscous force 'F' developed between two liquid layers is ……*
   (a) directly proportional to area of layer and inversely proportional to velocity gradient
   (b) inversely proportional to area of liquid layer and directly proportional to velocity gradient
   (c) directly proportional to area of layer and directly proportional to velocity gradient
   (d) none of these

102. *The unit of coefficient of viscosity of liquid is ……*
   (a) Ns m^2 (b) m^2/s N (c) Ns/m^2 (d) m^2s/N
103. *The constant velocity with which a body falls through liquid column is called as ……*
   (a) critical velocity (b) terminal velocity (c) radial velocity (d) angular velocity
104. *Stoke's law states that the force of viscosity 'F' experienced by a small metal sphere falling freely through a viscous medium, with terminal velocity is directly proportional to ……*
   (a) radius of metal sphere 'r' (b) terminal velocity 'v'
   (c) coefficient of viscosity (d) all of the above
105. *Stoke's formula is given by ……*
   (a) $F = 6\pi/\eta rv$ (b) $F = 3\pi/\eta rv$ (c) $F = 3\pi\eta rv$ (d) $F = 6\pi\eta rv$
106. *The formula for coefficient of viscosity of liquid is given by ……*
   (a) $\eta = \frac{2}{9}\frac{r^2 g (d-\rho)}{v}$ (b) $\eta = \frac{9}{2}\frac{r^2 g (d-\rho)}{v}$ (c) $\eta = \frac{2}{9}\frac{g (d-\rho)}{r^2 v}$ (d) $\eta = \frac{9}{2}\frac{g (d-\rho)}{r^2 v}$
107. *As temperature of liquid increases, viscosity of liquid ……*
   (a) increases (b) remains same (c) decreases (d) none of these
108. *As adulteration of soluble substance in liquid increases then viscosity of net solution ……*
   (a) increases (b) remains same (c) decreases (d) none of these
109. *If sugar is dissolved in pure water then viscosity of net solution is ……*
   (a) less than pure water (b) same as pure water (c) more than pure water (d) none of these
110. *A spherical steel bar of diameter 2 mm is falling through a liquid with terminal velocity of 0.1 m/s. The terminal velocity of steel ball of diameter 4 mm will be ……*
   (a) 0.1 m/s (b) 0.2 m/s (c) 0.3 m/s (d) 0.4 m/s
111. *A spherical steel ball of 'r' m is falling through a liquid with terminal velocity of 'v' m/s. Terminal velocity of metal sphere of half the radius will be ……*
   (a) v m/s (b) $\frac{v}{2}$ m/s (c) $\frac{v}{4}$ m/s (d) 2v m/s
112. *If the force of 10 N is required to move the plate of area 100 m^2 over a liquid, the force required to move a plate with same velocity over a same liquid of area 200 m^2 will be ……*
   (a) 10 N (b) 20 N (c) 30 N (d) 40 N
113. *Force of 'F' N is required to move plate of area 'A' m^2 over a liquid. The force required to move plate of half the earlier area will be ……*
   (a) 'F/2' N (b) 'F' N (c) '2F' N (d) '3F' N
114. *Viscous force 'F' is experienced by rain drop of radius 'r'. Viscous force experienced by a double radius rain drop moving with same speed will be ……*
   (a) F/2 (b) '2F' (c) 'F' (d) '3F'

MCQ Answers and Hints on Chapter 5

1. (c)	2. (b)	3. (a)	4. (d)	5. (a)	6. (a)	7. (c)	8. (b)	9. (a)	10. (b)	11. (c)	12. (d)
13. (c)	14. (a)	15. (b)	16. (d)	17. (c)	18. (b)	19. (a)	20. (a)	21. (c)	22. (c)	23. (a)	24. (d)
25. (c)	26. (d)	27. (b)	28. (b)	29. (d)	30. (a)	31. (c)	32. (b)	33. (d)	34. (b)	35. (a)	36. (d)
37. (b)	38. (a)	39. (c)	40. (b)	41. (d)	42. (a)	43. (c)	44. (a)	45. (b)	46. (c)	47. (a)	48. (d)
49. (a)	50. (c)	51. (b)	52. (a)	53. (d)	54. (d)	55. (b)	56. (b)	57. (b)	58. (c)	59. (a)	60. (d)
61. (c)	62. (a)	63. (c)	64. (b)	65. (d)	66. (c)	67. (a)	68. (c)	69. (b)	70. (a)	71. (c)	72. (a)
73. (b)	74. (a)	75. (d)	76. (a)	77. (c)	78. (c)	79. (a)	80. (a)	81. (d)	82. (a)	83. (c)	84. (b)
85. (d)	86. (b)	87. (c)	88. (b)	89. (b)	90. (b)	91. (d)	92. (c)	93. (c)	94. (b)	95. (a)	96. (b)
97. (d)	98. (d)	99. (d)	100. (a)	101. (c)	102. (c)	103. (b)	104. (d)	105. (d)	106. (a)	107. (c)	108. (a)
109. (c)	110. (d)	111. (c)	112. (b)	113. (a)	114. (b)						

52. **Hint :** $l \propto L$, 0.5 m is minimum $\therefore$ least extension.

53. **Hint :** $l \propto \frac{L}{r^2}$, $l \propto \frac{200}{(0.2)^2} \rightarrow$ 5000 is least

54. **Hint :** $l \propto \frac{Ml}{r^2 l}$, $l \propto \frac{4 \times 4}{4^2 \times 4} \rightarrow$ 0.25 lowest

55. **Hint :** $l \propto \frac{1}{r^2}$, $l \propto \frac{1}{(1/2)^2}$, $l \propto$ 4 times earlier

56. **Hint :** $\frac{l_1}{l} = \frac{2F \times 2L}{(2r)^2} = 1 \;\therefore\; l_1 = l$

57. **Hint :** Stress = Y × strain = $2 \times 10^{11} \times \frac{(2 \times 10^{-3})}{2}$

58. **Hint :** $Y = \frac{FL}{(\pi r^2)\, l} = \frac{(5 \times 3)}{[3.142 \times (1 \times 10^{-3})^2 \times 2 \times 10^{-3}]}$

59. **Hint :** $Y = \frac{FL}{Al} = \frac{(10 \times 1.5)}{(2 \times 10^{-5} \times 3 \times 10^{-3})}$

60. **Hint :** Maximum force = Maximum stress × Area = $9 \times 10^8 \times 0.02 = 0.18 \times 10^8$ N/m^2

61. **Hint :** Stress = Y × Strain = $2.4 \times 10^{11} \times \left(\frac{3}{100}\right) = 7.2 \times 10^9$ N/m^2

62. **Hint :** $Y = \frac{\text{Stress} \times L}{l} = \frac{8 \times 10^8 \times 3}{(2 \times 10^{-3})} = 12 \times 10^{11}$ N/m^2

63. **Hint :** $l \propto \frac{L}{r^2} \;\therefore\; \frac{l_1}{l_2} = \frac{L_1}{r_1^2} \times \frac{r_2^2}{L_2} = \frac{(2L_2)}{(2r_2)^2} \times \frac{r_2^2}{L_2} = \frac{1}{2}$

64. **Hint :** Compressibility = $\frac{1}{K} = \frac{1}{(2 \times 10^{10})}$

65. **Hint :** Shearing strain = $\frac{\text{Lateral displacement of top layer}}{\text{Its distance from fixed layer}} = \frac{0.06 \times 10^{-3}}{5 \times 10^{-2}} = 1.2 \times 10^{-3}$

66. **Hint :** $\sigma = \frac{d/D}{l/L} = \frac{1.5 \times 10^{-4}/0.3}{2/3000} = \frac{1.5 \times 10^{-4} \times 3000}{0.3 \times 2} = 0.75$

110. **Hint :** $V \propto r^2 \;\therefore\; V_2 = \frac{V_1 \times dia_2^2}{dia_1^2} = 0.1 \times \frac{4^2}{2^2} = 0.4$ m/s

111. **Hint :** $V \propto r^2 \;\therefore\; V \propto \left(\frac{1}{2} r\right)^2 \rightarrow \left(\frac{1}{4}\right)$ times earlier

112. **Hint :** $F \propto A \;\therefore\; \frac{F_2}{F_1} = \frac{A_2}{A_1} \;\therefore\; F_2 = F_1 \times \frac{A_2}{A_1} = 10 \times \frac{200}{100} = 20$ N

113. **Hint :** $F \propto A$ $\therefore$ if A is half, force will be half $\rightarrow$ F/2

114. **Hint :** $F \propto r$ $\therefore$ if radius doubled, viscous force also gets doubled to maintain same speed $\rightarrow$ 2F.

❒❒❒

Chapter 6

HEAT

6.1 TRANSMISSION OF HEAT AND EXPANSIONS OF SOLIDS

Heat – Introduction :

Heat : Heat is a energy. Sun is the natural source of heat energy.

Heat (Definition) : *Heat is the form of energy which produces sensation of hotness or warmness.*

Units of heat :

joule … J … SI unit
kilocalorie … kcal … MKS unit
Calorie … cal … CGS unit

All molecules of a body possess some amount of kinetic energy.

Definition : *Heat energy or thermal energy of a system is defined as the sum of energies of all molecules of the body or system.*

Heat energy possessed by a body is mainly due to kinetic energy of molecules of the body. Hotter the object, faster is the motion of molecules. Heat is a form of energy and as per the law of conservation of energy, energy is conserved and can be converted from one form to another.

For example,

- Heat engine converts heat (thermal) energy into mechanical energy.
- Electric heater converts electrical energy into heat energy.
- In winter we rub our palms to each other, here mechanical frictional energy is converted into heat energy.

Heating a body produces physical and chemical changes in a body i.e. heating produces **expansion** of a body, **change of state** like solid to liquid and liquid to gases, change in electrical properties, change in colour of a body, change in chemical properties etc.

We can define heat as a energy which is transferred from higher temperature to lower temperature.

Heat is a non-mechanical energy concerned with temperature difference between the system and surroundings (environment). If temperature of the system is more than the surroundings, then heat flows from system to surroundings and vice a versa i.e. heat flows from higher temperature to lower temperature.

Heat is an **extensive property** i.e. heat produced by one cup of boiling water is different than that of boiling water in a huge boiler.

OR

Heat required to boil 1 cup milk and heat required to boil 5 litres of milk is different. Thus heat is an **extensive property**.

6.2 TRANSMISSION OF HEAT

Heat is a form of energy. Energy always flows from higher energy level to lower energy level. For example, water flows from higher level to lower level. Heat energy flows from body at higher temperature to body at lower temperature.

Sun is the source of heat. We receive heat on earth from sun. We feel much hotter in houses with tin roofs (shade). Use of woolen sweater in winter, use of thermocole pot as a ice box, use of copper in boiler, are few examples of day-to-day life. Hence it is interesting to study conduction, convection, radiation, conductor, insulator etc.

Types of transmission of heat

(1) Conduction (2) Convection (3) Radiation

6.2.1 Conduction

Conduction (Definition) : *It is the process of transfer of heat in which heat is transferred from a part of body at higher temperature to a part of body at lower temperature without bodily movement of particles (molecules) from one place to the other.*

When one end of a metal rod is heated, other end also gets heated after some time.

Let us assume that the rod is divided into different compartments P, Q, R, S.

Before heating, molecules in all compartments are vibrating about their mean position with same amplitude. If now end A is heated, then molecules in compartment 'P' receive heat and start vibrating with more amplitude. These molecules collide on molecules in the next compartment 'Q'. Now molecules in compartment 'Q' receive vibrational energy and they also start vibrating with more amplitude and this process continues. Thus, heat travels through the rod. In this process, the individual molecules do not travel from one end to other end. They only vibrate about their mean positions and vibrations are passed on from one end to the other.

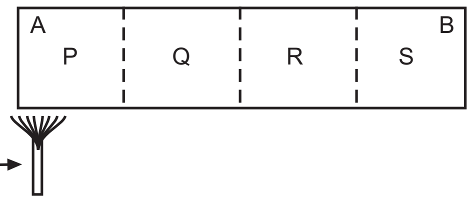


Fig. 6.1

If holes are drilled in each compartment and if thermometers are introduced in each compartment i.e. P, Q, R, S say T_1, T_2, T_3, T_4. It is observed that as time increases, temperature shown by each thermometer goes on increasing which is called as variable state.

Variable state (Definition) : *The state in which temperature of rod goes on increasing is called as variable state.*

In variable state, Heat absorbed by material > Heat given out by material.

Steady state (Definition) : *The state in which temperature of the rod remains constant and will not increase further is called steady state.*

In steady state, Heat absorbed by material = Heat given out by material.

6.2.1.1 Thermal Conductivity and Coefficient of Thermal Conductivity (Law of Thermal Conductivity)

Factors Affecting the Conduction of Heat :

Suppose AB is a bar of metal, of cross-sectional area A as shown in Fig. 6.2. Consider the bar to be in *steady state*.

No amount of heat is lost to the surroundings by means of radiation.

Consider two planes C and D in the bar.

Let, Q = the amount of heat flowing from C to D

d = distance between C and D or distance between two thermometers

θ_1 = temperature of plane C

θ_2 = temperature of plane D

$\theta_1 > \theta_2$

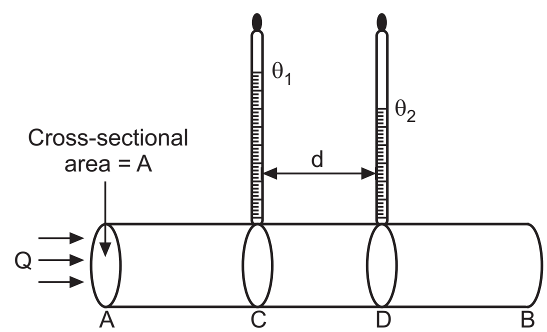


Fig. 6.2

Then, amount of heat flowing (Q) from C to D at steady state is *directly proportional* to

1. Cross-sectional area 'A' of rod,
2. Temperature difference between two planes i.e. $(\theta_1 - \theta_2)$,
3. Time 't' for which heat flows,

and *inversely proportional* to

1. Distance 'd' between two planes or distance between two thermometers.

Thus,
$$Q \propto A$$
$$Q \propto (\theta_1 - \theta_2)$$
$$Q \propto t$$
$$Q \propto \frac{1}{d}$$

$\therefore$ Combining,
$$Q \propto \frac{A \times (\theta_1 - \theta_2)\, t}{d}$$
$$Q = \text{Constant} \times \frac{A \times (\theta_1 - \theta_2)\, t}{d}$$
$$\boxed{Q = K \times \frac{A \times (\theta_1 - \theta_2) \times t}{d}} \qquad \ldots (1)$$

where K is the constant of proportionality which is called *coefficient of thermal conductivity*. 'K' depends on material of a bar.

Coefficient of Thermal Conductivity (K) :

We have,
$$Q = K \frac{A\,(\theta_1 - \theta_2)\, t}{d} \qquad \ldots \text{from equation (1)}$$

If
$$A = 1$$
$$(\theta_1 - \theta_2) = 1$$
$$t = 1$$
$$d = 1$$

then
$$Q = \frac{K \times 1 \times 1 \times 1}{1}$$
$$Q = K$$

Coefficient of thermal conductivity (K) (Definition) : *The coefficient of thermal conductivity is defined as the amount of heat conducted in one second, in steady state of temperature through unit cross-sectional area of an element of material of unit thickness with unit temperature difference between its opposite faces.*

Unit of 'K' :
$$K = \frac{Q \times d}{A\,(\theta_1 - \theta_2)\, t} = \frac{\text{cal} \times \text{m}}{\text{m}^2\,(^\circ\text{C})\,\text{sec}} = \frac{\text{cal}}{\text{m}\,^\circ\text{C sec}}$$
$$= \frac{\text{cal}}{\text{cm}\,^\circ\text{C sec}} \qquad \ldots \text{CGS unit}$$
$$= \frac{\text{kcal}}{\text{m}\,^\circ\text{C sec}} \qquad \ldots \text{MKS unit}$$
$$= \frac{\text{watt}}{\text{m}\,^\circ\text{K}} \qquad \ldots \text{S.I. unit}$$

Temperature Gradient :

The factor $\frac{(\theta_1 - \theta_2)}{d}$ in equation (1) is called temperature gradient.

Here
θ_1 – temperature at plane C
θ_2 – temperature at plane D
d – distance between C and D

Definition : *The temperature gradient is defined as the change in temperature per unit length of rod.*

The unit of temperature gradient is **°C/m** or **°K/m**.

Using $\text{T.G.} = \frac{(\theta_1 - \theta_2)}{d}$ equation (1) becomes
$$Q = K \times A \times \text{T.G.} \times t$$

If A = 1, T.G. = 1, and t = 1

then
$$K = Q$$

Coefficient of thermal conductivity (K) (Definition) : *The coefficient of thermal conductivity (K) of material can also be defined as the amount of heat flowing through a rod of unit area, in one second for unit temperature gradient at steady state.*

Important Questions

1. Define : (i) Steady state, (ii) Temperature gradient.
2. State law of thermal conductivity. Define coefficient of thermal conductivity. State its S.I. unit.
3. Write S.I. unit for coefficient of thermal conductivity.
4. What is steady state of temperature ? Define temperature gradient. Give its unit.
5. Define temperature gradient and mention its unit.
6. Define coefficient of thermal conductivity. State/Obtain its S.I. unit.
7. State the factors affecting conduction of heat and state the relation between them.
8. Write equation for heat conducted by a rod.
9. 'We do not receive heat from the sun by conduction', give reason.

Ans. *It is the process of transfer of heat in which heat is transferred from a part of body at higher temperature to a part of body at lower temperature without bodily movement of particles (molecules) from one place to the other.*

Thus reason – **conduction needs medium** for transfer of heat and between sun and earth the millions of kilometers, there is **no medium** present. Hence conduction is not possible.

6.2.1.2 Good Conductors of Heat and Insulator with Suitable Examples

Good conductor of heat (Definition) : *A material that easily transfers heat energy by conduction is called good conductor of heat (good thermal conductor).*

The value of coefficient of thermal conductivity 'K' of a good conductor is high.

Some examples of good conductors with their value of 'K' are as given below :

Good conducting material	Coefficient of thermal conductivity 'K' in (W/m °K) S.I.
Aluminium	235
Copper	401
Silver	428

6.2.1.3 Insulator or Bad Conductor of Heat with Suitable Examples

Insulator (Definition) : *A material which does not conduct heat energy through them is called insulator or bad conductor of heat (thermal insulator).*

For this we must know the concept of **thermal resistance R.**

$$R \propto \frac{1}{K}$$

where, R = Thermal resistance

K = Thermal conductivity

Thus poor thermal conductor means good thermal insulator. i.e. bad conductor of heat has very low value of 'K' or high value of 'R'.

Some examples of insulators with their values of 'K'.

Insulating material	Value of 'K' in (W/m °K) S.I.
Fiber glass	0.048
Rock wool	0.043
Dry air	0.026

6.2.1.4 Applications of Conduction

(A) Good Conductor :

1. **Good conducting material is used as a heat sink in electronic circuits** : In order to protect costly electronic components from overheating, a good conducting material like aluminium piece of large area is kept in contact with components. Such a piece absorbs heat and radiates to atmosphere quickly.

2. **Spiral tube covering the coil of electric heater is made of good conductor :** So that heat developed is conducted to liquid in contact quickly.

3. **Condenser coil in a refrigerator is ideally made up of copper (good conductor) :** For better conduction.

4. Cooling of electrical machines by blowing hydrogen gas through machines cools machine speedily : Reason is thermal conductivity of hydrogen is more than air.

5. Davy's safety lamp : In Davy's safety lamp, flame of lamp is covered by iron-gauze of good conductor. Uniform heating of iron-gauze due to its conductivity avoids explosion and maintains safetyness of lamp.

(B) Bad Conductor :

1. Ice box : A bad conducting material like thermocole is used in ice box because of which ice melts very slowly.

2. Handle of cooker is made up of bad conducting material like plastic : Because of bad conducting material, heat is not conducted and handle can be easily held.

3. Use of insulation in refrigerator : The pipeline connection between expansion valve outlet and evaporator inlet is thermally insulated to avoid thermal loss.

4. Use of double walled flask or thermos flask : Flasks having double layers and air gap in between avoid thermal loss, hence warm food or liquid inside the flask remains warm for longer time.

Important Questions

1. Explain conduction of heat. Define good and bad conductors of heat with one example each.
2. State two applications of thermal conductivity.
3. Explain good and bad conductors of heat with two examples of each.
4. Why two shirts keep the body warmer than single shirt of the same material and double thickness

Ans. Reason : We know that **dry air** acts as a **insulation** of heat. The air between two shirts hence does not allow body heat to go outside and keep body warmer.

6.2.2 Convection

The meaning of convection is *carrying*.

Definition : *Convection is a process of transfer of heat from a part of body at higher temperature to a part of body at lower temperature with bodily (actual) movement of particles.*

OR

Definition : *Convection is the process by which heat is transmitted through a substance from one point to another due to the bodily motion of heated particles of a substance.*

Process : When water is taken in a glass beaker and if heat is supplied at the bottom of beaker then the water molecules which are closer to heat source get heated first, become lighter and rise upward. While the colder water moves downward, receives heat and becomes lighter and goes up. This cycle continues. Thus currents are set up in the heated water. These are called as convection currents.

There are two types of convection : (1) Free convection and (2) Forced convection.

1. Free convection : The convection that takes place in a **fluid which is at rest** is called free convection.

For example,

(1) The still air in a closed room.

(2) The trade winds – wind blowing from north east to equator.

(3) Monsoons.

2. Forced convection : The convection that takes place in an externally controlled steady stream of fluid which passes through the present fluid. e.g. ventilated cooling of a room using exhaust fans at windows at higher position.

6.2.2.1 Applications of Convection

1. Room ventilation : Ventilators (exhaust fans) are provided at high position in the room. Warm air and carbon dioxide go out through ventilators. Fresh and cool air come in through doors and windows.

2. Formation of trade winds : The earth's surface near the equator get heated strongly and this hot air goes up and forms low pressure region at the surface of equator. On the other hand, cool air at higher side of pole goes down and forms high pressure region at the surface of poles. Because of high pressure region at pole surface and low pressure region at equator surface, wind blows from pole to equator, there it becomes hot, goes up and circulation of air takes place i.e. convection takes place. Also spinning of the earth is responsible for motion of wind.

3. Formation of sea breeze and land breeze : Near the seaside location, in the day time, the land surface gets heated more strongly than the sea water. Thus hot air at land surface goes up and forms low pressure region at land surface air. The cold air from sea surface rushes towards the land surface and forms **sea breeze** and air circulation takes place.

On the other hand, at night the land surface cools more speedily than the sea surface. Thus wind blows from land (high pressure region) to sea (low pressure region) which forms **land breeze** and air circulation takes place.

4. Gas filled coiled electric lamps : Gas filled coiled electric lamps reduce convection current and increase life of the lamp.

5. Cooling system in automobile engines : The hot water around the engine block goes into radiator and then this water is brought down through a honey-comb spiral pipelines where this water is cooled down using fans.

Important Questions

1. Define convection.
2. State applications of convection.

6.2.3 Radiation

Definition : *Radiation is the process of transmission of heat in the form of electromagnetic waves from a body at higher temperature to a body at lower temperature without heating the intervening medium.*

Heat energy emitted by a body because of its temperature is known as **thermal radiation**.

As electromagnetic waves travel also through vacuum, no material medium is necessary for radiation. The heat from the sun reaches the earth by means of radiation. We know that around the earth upto certain region there is presence of medium. But beyond this there is vacuum. Heat from the sun reaches the earth after travelling several kilometers through vacuum because electromagnetic waves can also travel through vacuum. Even if the material medium is present, it is not necessary that it should be first get heated because thermal radiation does not affect the medium through which they pass.

6.2.3.1 Applications of Radiation

1. White or light coloured clothes are preferred in summer : because they are good reflector and poor absorber of heat which keep our body cool. **Coloured clothes are preferred in winter** because they absorb most of the heat from sun and which keep our body warm.

2. Heat radiators in cars, machines are painted black : which radiate most of the heat and maintain cooling effect.

3. High absorbing power of water vapour is a natural gift : presence of water vapour in the atmosphere protects us by absorbing heat radiations from sun in the day time and preventing loss of heat radiations in the night.

4. Aeroplanes and ships are painted white : to minimize absorption of heat.

5. The polished surface of space craft reflect most of the heat radiated from sun.

6. Base of the cooking utensils is made black : which is a good absorber of heat radiation and which accelerate cooking.

7. Inactivation of HIV by application of heat radiations.

8. Teapots has bright shining surface : because brightly polished objects retain their heat for longer time.

Comparison between Conduction, Convection and Radiation

Parameter	Conduction	Convection	Radiation
1. Definition	1. It is the process of transfer of heat in which heat is transferred from a part of a body at higher temperature to a part of body at lower temperature **without bodily (actual) movement** of particles.	1. It is the process of transfer of heat from a part of a body at higher temperature to a part of a body at lower temperature **with bodily (actual) movement** of particles.	1. It is the process of transfer of heat in the **form of electromagnetic waves** from a body at higher temperature to a body at lower temperature **without heating intervening medium.**
2. Example	2. If **metal rod** is **heated** at one end, its other end gets heated.	2. **Heating of water** in a beaker.	2. **Heat from sun reaches** the **earth.**
3. Necessity of material medium	3. **Material medium** is **essential.**	3. Material **medium** is **essential**.	3. Material **medium** is **not essential**.

… Contd.

4. Material medium	4. Metal **rod itself** acts as a **medium**.	4. **Liquid (fluid)** itself acts as a **medium**.	4. Medium may be present like air or no medium i.e. vacuum.
5. Connectivity	5. There is a **connection** between high temperature and low temperature region.	5. There is a **connection** between high temperature and low temperature region.	5. Body at higher temperature is **isolated** from a body at lower temperature.
6. Typical applications	6. It has typical applications like • Heat sink in electronic circuits. • Condenser coil in refrigerator. • Safety lamp. • Ice box. • Double walled flask.	6. It has applications like • Room ventilation system. • Formation of trade winds. • Formation of sea breeze and land breeze. • Monsoons.	6. It has applications like • Use of white clothes • Heat radiators in car. • In activation of HIV. • Use of cooking utensils with black coloured base. • White painting of aeroplane.

Important Questions

1. State and explain modes of transmission of heat.
2. Distinguish between conduction, convection and radiation processes for heat exchanges.
3. Define any two modes of transfer of heat.
4. Explain modes of transmission of heat.

6.3 EXPANSION IN SOLIDS

Whenever a solid is heated, it expands. If solid is in thin rod form, then after heating, its length increases. If it is in thin sheet form, then its area increases and if it is in cube form, then its volume increases.

Accordingly, there are three types of coefficient of expansion.

6.3.1 Coefficient of Linear Expansion (α) [Alpha]

Consider a metal rod of length L_o.

Let, L_o be the original length at 0°C. It is heated upto t°C. Let, the new increased length at t°C be L_t. Then the the increase in length is $(L_t - L_o)$.

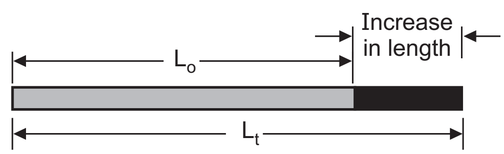


Fig. 6.3

Practically, it is observed that

$$(L_t - L_o) \propto L_o \text{ i.e. original length}$$

and also $(L_t - L_o) \propto (t - 0)$ i.e. change in temperature

$$\therefore \quad (L_t - L_o) \propto L_o(t - 0)$$

$$\therefore \quad (L_t - L_o) = \text{constant} \times L_o t$$

$$(L_t - L_o) = \alpha L_o t \qquad \text{... (6.1)}$$

where α is the constant of proportionality called *coefficient of linear expansion*.

It is constant for a given material. It changes if metal changes.

$$\therefore \quad \alpha = \frac{(L_t - L_o)}{L_o \times t} \quad \text{or} \quad \alpha = \frac{(L_2 - L_1)}{L_1 (t_2 - t_1)}$$

Thus, coefficient of linear expansion 'α' of material is defined as increase in length per unit original length at 0°C, per unit increase in temperature.

6.3.2 Coefficient of Aerial Expansion (Surface Expansion) or (Superficial Expansion) (β)

Consider a thin metal sheet PQRS.

Let A_o be the original area at 0°C. It is heated upto t°C. Let area of sheet at t °C be A_t. Then the increase in area of sheet is $(A_t - A_o)$.

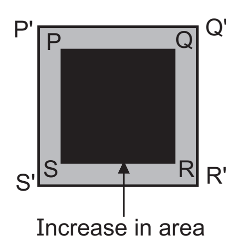


Fig. 6.4

Then $(A_t - A_o) \propto A_o \times (t - 0)$

$$\therefore \quad (A_t - A_o) = \text{Constant} \times A_o t$$

$\therefore \quad (A_t - A_o) = \beta \times A_o t$ ... (6.2)

where β is the constant of proportionality called *coefficient of aerial expansion*. It is constant for a given metal.

$$\therefore \quad \beta = \frac{(A_t - A_o)}{A_o t} \quad \text{or} \quad \beta = \frac{(A_2 - A_1)}{A_1 (t_2 - t_1)}$$

Thus, coefficient of aerial expansion 'β' of a material is defined as increase in area per unit original area at 0°C, per unit increase in temperature.

6.3.3 Coefficient of Cubical (Volume) Expansion (γ)

Consider a metal cube.

Let V_o be the original volume of cube at 0°C. It is heated upto t°C. Let V_t be the volume at t°C. Then the increase in volume is $(V_t - V_o)$.

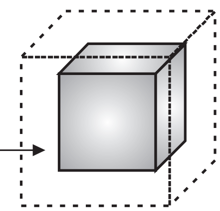


Fig. 6.5

Then $(V_t - V_o) \propto V_o \times (t - 0)$

$(V_t - V_o) = \text{Constant} \times V_o t$

$(V_t - V_o) = \gamma \times V_o t$ ... (6.3)

where γ is the constant of proportionality called *coefficient of cubical expansion*.

$$\therefore \quad \gamma = \frac{(V_t - V_o)}{V_o t} \quad \text{or} \quad \gamma = \frac{(V_2 - V_1)}{V_1 (t_2 - t_1)}$$

Thus, coefficient of cubical (volume) expansion 'γ' of a material is defined as increase in volume per unit original volume at 0°C, per unit increase in temperature.

6.3.4 Relation between α, β and γ (No Derivation)

We have

from equation (6.1), $(L_t - L_o) = \alpha L_o t \quad \therefore \quad L_t = L_o (1 + \alpha t)$ ... (6.4)

from equation (6.2), $(A_t - A_o) = \beta A_o t \quad \therefore \quad A_t = A_o (1 + \beta t)$ ... (6.5)

from equation (6.3), $(V_t - V_o) = \gamma V_o t \quad \therefore \quad V_t = V_o (1 + \gamma t)$ ... (6.6)

where, α is coefficient of linear expansion

β is coefficient of aerial expansion

γ is coefficient of cubical expansion

1. **Relation between α and β :** $\boxed{\beta = 2\alpha}$

   i.e. coefficient of aerial expansion 'β' is twice that of linear expansion (α) of same metal.

2. **Relation between α and γ :** $\boxed{\gamma = 3\alpha}$

   Coefficient of cubical expansion 'γ' is thrice that of linear expansion (α) of same metal.

3. **Relation between β and γ :** $\boxed{\gamma = \frac{3}{2}\beta}$

   Coefficient of cubical expansion (γ) is equal to 3/2 times coefficient of linear expansion (α) of same metal.

4. **Relation between α, β and γ :**

   For a given metal, ratio α : β : γ is 1 : 2 : 3

   i.e. $\frac{\alpha}{\beta} = \frac{1}{2}, \ \frac{\beta}{\gamma} = \frac{2}{3}, \ \frac{\alpha}{\gamma} = \frac{1}{3}$ $\boxed{\beta = 2\alpha}$

   i.e. coefficient of aerial expansion is *twice* that of coefficient of linear expansion.

6.3.5 Applications of Expansion

1. Expansion property is used in mercury thermometers and liquid thermometers.
2. Suitable gap between rails of railway track is kept to provide scope for expansion.
3. Long compound walls are also build by keeping certain gap.
4. Isothermal and adiabatic expansions and compressions of gas are used in studying heat cycles and it is used to study heat engines.
5. In manufacturing of heat exchangers, designer has to take into account the expansion of components used in it.
6. Depending upon values of linear expansion, aerial expansion or cubical expansion, thermal properties of material can be studied.

Important Points

- **Conduction** is a process of transfer of heat from a part of body at higher temperature to a part of body at lower temperature without bodily (actual) movement of particles.
- **Convection** is a process of transfer of heat from a part of body at higher temperature to a part of body at lower temperature with bodily (actual) movement of particles.
- **Radiation** is the process of transfer of heat in which heat is transferred from one place to other directly without the necessity of intervening medium. Heat is transferred in the form of electromagnetic waves.
- In variable state, heat absorbed by a material is more than heat given out.
- In steady state, heat absorbed by a material is equal to heat given out.
- Amount of heat flowing through a material is directly proportional to (1) its cross-sectional area, (2) temperature difference, (3) time and inversely proportional to distance between its ends.
- Coefficient of thermal conductivity 'K' of material is defined as heat conducted in one second in steady state through unit cross-sectional area for unit temperature gradient.
- Temperature gradient is defined as change in temperature per unit length of rod.
- Coefficient of linear expansion 'α' of material is defined as increase in length per unit original length at 0°C, per unit increase in temperature.
- Coefficient of aerial expansion 'β' of material is defined as increase in area per unit original area at 0°C per unit increase in temperature.
- Coefficient of cubical expansion 'γ' of material is defined as increase in volume per unit original volume at 0°C, per unit increase in temperature.
- For a given material, the ratio of $\alpha : \beta : \gamma$ is 1 : 2 : 3.

Important Formulae

1. $Q = \frac{KA(\theta_1 - \theta_2)\,t}{d}$ where, Q = Amount of heat conducted

   A = Cross-sectional area

   d = Thickness of material

3. Temperature gradient $= \frac{(\theta_1 - \theta_2)}{d}$ where,

   θ_1 and θ_2 = Temperatures of two faces or

   d = Distance between two (thermometers) faces

   If 'K' is given in cal/cm °C sec – then unit of calculated 'Q' will be cal.

   If 'K' is given in kcal/m °C sec – then unit of calculated 'Q' will be kcal.

   If 'K' is given in W/m °K – then unit of calculated 'Q' will be joule.

4. $\alpha = \frac{(L_2 - L_1)}{L_1(t_2 - t_1)}$

5. $\alpha : \beta : \gamma$ i.e. 1 : 2 : 3 i.e. $\frac{\alpha}{\beta} = \frac{1}{2}$, $\frac{\beta}{\gamma} = \frac{2}{3}$, $\frac{\alpha}{\gamma} = \frac{1}{3}$

SOLVED EXAMPLES

Examples on Conductivity :

Example 1 : *Find the quantity of heat conducted in 5 minutes across a silver sheet of size 40 cm × 30 cm of thickness 3 mm. If its two faces are at temperatures of 40°C and 25°C, K for silver = 0.1 kcal/m °Cs.*

Solution : Given : Q = ?, t = 5 min = (5 × 60) sec, A = 40 cm × 30 cm = 1200 cm^2 = 0.12 m^2, d = 3 mm = 3×10^{-3} m

$$(\theta_1 - \theta_2) = (40 - 25) = 15°C$$

We have,

$$Q = \frac{KA(\theta_1 - \theta_2)\,t}{d} = \frac{(0.1)(0.12)(15)(5 \times 60)}{(3 \times 10^{-3})}$$

$$\boxed{Q = 18000 \text{ kcal}}$$

[**Note :** Here unit of K is given in kcal/m °C sec. Therefore, answer of 'Q' will be in kcal.]

Example 2 : *A nickel plate of thickness 4 mm has temperature difference of 32°C between its faces. It transmits 200 kcal per hour through an area of 5 cm^2. Calculate the conductivity of nickel.*

Solution : Given : d = 4 mm = 4×10^{-3} m, $(\theta_1 - \theta_2)$ = 32°C, Q = 200 kcal, A = 5 cm^2 = 5×10^{-4} m^2 K = ?

t = 1 hr = (60 × 60) sec

We have,

$$Q = \frac{KA(\theta_1 - \theta_2)\,t}{d}$$

∴

$$K = \frac{Q \times d}{A(\theta_1 - \theta_2)\,t} = \frac{(200)(4 \times 10^{-3})}{(5 \times 10^{-4})(32)(60 \times 60)}$$

$$\boxed{K = 0.0139 \text{ kcal/m °C sec}}$$

Example 3 : *A metal rod of length 0.20 m has one of its ends at 20°C, while the other is at 50°C. Find the temperature gradient.*

Solution : Given : d = 0.20 m (i.e. distance between two thermometers)

θ_1 = 20°C, θ_2 = 50°C

Temperature gradient = ?

$$\text{Temperature gradient} = \frac{(\theta_2 - \theta_1)}{d} = \frac{50 - 20}{0.2}$$

$$\boxed{\text{Temperature gradient} = 150 \text{ °C/m}}$$

Example 4 : *If the thickness of the plate is 8 cm and temperature of two faces are 100°C and –20°C, find the temperature gradient.*

Solution : Temperature difference = $\theta_2 - \theta_1$ = [100 – (– 20)] = 120°C

Distance between two points at which temperature difference is known = thickness = 8 cm

$$\text{We have Temperature gradient} = \frac{\text{Temperature difference}}{\text{Distance}} = \frac{120°C}{8 \text{ cm}}$$

$$\boxed{\text{Temperature gradient} = 15°C/cm}$$

Example 5 : *A copper rod 19 cm long and area of cross-section 0.79 cm^2, which is thermally insulated, is heated at one end to a temperature of 100°C while the other end is kept at 30°C. Calculate the amount of heat which will flow in 5 min. along the rod if K for copper is 380 W/m°K.*

Solution : Given : d = 19 cm (length of rod or distance between two thermometers) = 0.19 m

A = 0.79 cm^2 = 0.79×10^{-4} m^2, θ_1 = 100°C, θ_2 = 30°C, Q = ?, t = 5 min = 5 × 60 = 300 sec., K = 380 W/m °K

We have,

$$Q = \frac{KA(\theta_1 - \theta_2)\,t}{d} = \frac{380 \times 0.79 \times 10^{-4} \times (100 - 30) \times 300}{0.19} = 3318 \text{ J}$$

OR

$$Q = \frac{3318}{4.2} \quad \boxed{Q = 790 \text{ cal.}}$$

Example 6 : *Find the amount of heat conducted in one hour by a window pane 60 cm × 30 cm and thickness 3 mm, if the difference between temperatures is 5°C and K = 0.0002 kcal/m °C sec.*

Solution : Given : Q = ?, t = 1 hr = 60 × 60 sec, A = 60 cm × 30 cm = 0.6 × 0.3 m^2 = 0.18 m^2

d = 3 mm = 3×10^{-3} m, $(\theta_1 - \theta_2)$ = 5°C

We have,
$$Q = \frac{KA(\theta_1 - \theta_2)\,t}{d} = \frac{0.0002 \times 0.18 \times (5) \times 60 \times 60}{3 \times 10^{-3}} \quad \therefore \boxed{Q = 216 \text{ kcal}}$$

Example 7 : *Heat is conducted through a slab of two layers of different metals of conductivities 0.1 and 0.15 CGS units respectively. The thickness of the layers is 4 cm and 6 cm respectively. If the temperatures of the outer faces are 100°C and 0°C respectively, find the temperature of the interface.*

Solution :

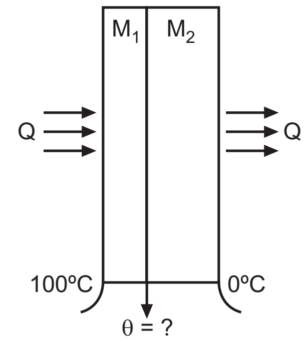


Fig. 6.6

Metal 1	Metal 2
K_1 = 0.1 CGS	K_2 = 0.15 CGS
d_1 = 4 cm	d_2 = 6 cm
Temperature difference between its faces = (100 – θ)°C	Temperature difference between its faces = (θ – 0)°C

Since they are in contact, cross-sectional area of them will be equal.

∴ Heat conducted through M_1 is $Q = \frac{K_1 A (100 - \theta)\, t}{d_1}$

and heat conducted through M_2 is $Q = \frac{K_2 A (\theta - 0)\, t}{d_2}$

But heat conducted through two metals is same because layers are in contact.

$$\therefore \quad \frac{K_1 A (100 - \theta)\, t}{d_1} = \frac{K_2 A (\theta - 0)\, t}{d_2}$$

$$\therefore \quad \frac{K_1 (100 - \theta)}{d_1} = \frac{K_2 (\theta - 0)}{d_2}$$

$$\frac{(0.1) \times (100 - \theta)}{4} = \frac{(0.15) \times (\theta - 0)}{6}$$

$$(6) \times (0.1) \times (100 - \theta) = (4) \times (0.15) \times (\theta - 0)$$

$$60 - 0.6\,\theta = 0.6\,\theta$$

$$60 = 1.2\,\theta$$

$$\therefore \quad \theta = \frac{60}{1.2} = 50°C \;\therefore \boxed{\text{Temperature of interface is } \theta = 50°C}$$

Example 8 : *The length of copper rod at 10°C is 80 cm. When heated to 100°C it increases by 0.12 cm. Find the coefficient of linear expansion of copper.*

Solution :

Initial	Final
t_1 = 10°C	t_2 = 100°C
∴ L_1 = 80 cm	Let length is L_2
	Here $(L_2 - L_1)$ = 0.12 cm

Coefficient of linear expansion
$$\alpha = \frac{(L_2 - L_1)}{L_1 (t_2 - t_1)} = \frac{0.12}{80\,(100 - 10)} = \frac{0.12}{80 \times 90} = 0.000016 \quad \boxed{\alpha = 1.66 \times 10^{-5}/°C}$$

Example 9 : *The coefficient of cubical expansion of a metal is 46 $\times 10^{-6}$/°C. Find the coefficient of aerial expansion.*

Solution : Given : $\gamma = 46 \times 10^{-6}$/°C, β = ?

We have $\beta : \gamma$ is 2 : 3

i.e. $\frac{\beta}{\gamma} = \frac{2}{3}$

$\therefore$ $\beta = \frac{2}{3} \times \gamma = \frac{2}{3} \times (46 \times 10^{-6})$ $\boxed{\beta = 30.66 \times 10^{-6}/°C}$

Example 10 : *The coefficient of aerial expansion of a certain metal is 25 $\times 10^{-6}$/°C. Find coefficient of linear expansion.*

Solution : Given : α = ?, $\beta = 25 \times 10^{-6}$/°C

We have : $\frac{\alpha}{\beta} = \frac{1}{2}$

$\therefore$ $\alpha = \frac{\beta}{2} = \frac{25 \times 10^{-6}}{2}$ $\boxed{\alpha = 12.5 \times 10^{-6}/°C}$

Example 11 : *The coefficient of linear expansion of metal is 40 $\times 10^{-6}$/°C. Find coefficient of cubical expansion.*

Solution : Given : $\alpha = 40 \times 10^{-6}$/°C, γ = ?

We have : $\frac{\alpha}{\gamma} = \frac{1}{3}$

$\therefore$ $\gamma = 3\alpha = 3 \times 40 \times 10^{-6}$ $\boxed{\gamma = 120 \times 10^{-6}/°C}$

Example 12 : *How much gap should be left in between two rails if each rail is 14 m long and the maximum rise in temperature is 25°C ? The coefficient of linear expansion of material is 20 $\times 10^{-6}$/°C.*

Solution : Given : L_o = 14 m, (t – 0) = 25°C, $\alpha = 20 \times 10^{-6}$/°C

Gap = $(L_t - L_o)$ = ?

$\alpha = \frac{(L_t - L_o)}{L_o (t - 0)}$

$\therefore$ $\alpha \times L_o \times t = (L_t - L_o)$

$\therefore$ $(L_t - L_o) = \alpha \times L_o \times t = 20 \times 10^{-6} \times 14 \times 25 = 7000 \times 10^{-6} = 0.007$ m $\boxed{\text{Gap should be 0.007 m}}$

Practice Questions

1. Explain conduction of heat on the basis of molecular theory.
2. Define temperature gradient and mention its unit.
3. Define coefficient of thermal conductivity of a material. State its unit and dimensions.
4. State the factors affecting conduction of heat and state relation between them.
5. Explain the modes of transfer of heat and define temperature gradient.
6. State the law of thermal conductivity of heat and define coefficient of thermal conductivity. State its S.I. units.
7. Explain conduction of heat along a bar. What is steady state of temperature ?
8. Define : (a) Coefficient of linear expansion, (b) Coefficient of aerial expansion, (c) Coefficient of cubical expansion.
9. State the relation between coefficient of linear expansion (α) and coefficient of aerial expansion (β).
10. State the relation between coefficient of linear expansion (α) and coefficient of cubical expansion (γ).
11. State relation between coefficient of aerial expansion (β) and coefficient of cubical expansion (γ).
12. State relation between α, β and γ coefficient of linear, aerial and cubical expansion.

MCQs on Transmission of Heat

1. *The energy which flows from body at higher temperature to body at lower temperature is ....*
   (a) sound *(b) light* *(c) heat* *(d) wind*
2. *Heat is ……*
   (a) an intensive property *(b) an extensive property*
   (c) an intensive as well as extensive property *(d) none of these*
3. *Which of the following is not a unit of heat ?*
   (a) joule *(b) Fahrenheit* *(c) calorie* *(d) kilocalorie*

4. *1 kcal is equal to ……*
   (a) 4.184 J (b) 1.484 J (c) 4184 J (d) 1484 J
5. *1 cal = ———— J.*
   (a) 4.186 (b) 6.63 (c) 4186 (d) 6630
6. *The process of transfer of heat in which heat is transferred from a part of body at higher temperature to a part of body at lower temperature without bodily movement of particles (molecules) from one place to other is known as ————.*
   (a) conduction (b) convection (c) radiation (d) refraction
7. *Conduction is the process of transfer of heat in which heat is transferred from a part of body at higher temperature to the part of body at lower temperature ————.*
   (a) with bodily movement of particles (b) without bodily movement of particles
   (c) with as well as without bodily movement of particles (d) none of these
8. *The process of transfer of heat in which heat is transferred from a part of body at higher temperature to a part of body at lower temperature with bodily movement of particles (molecules) from one place to other is known as ---————.*
   (a) conduction (b) convection (c) radiation (d) refraction
9. *In convection, there is ……*
   (a) no bodily movement of particles (b) vibrational movement of particles
   (c) bodily movement of particles (d) none of these
10. *Heat transfer when metal rod is heated at one end is by the way of ……*
    (a) conduction (b) convection (c) radiation (d) none of these
11. *Heat transfer when water in beaker is heated at bottom is by way of ……*
    (a) conduction (b) convection (c) radiation (d) none of these
12. *The process of transmission of heat in the form of electromagnetic waves from a body at higher temperature to a body at lower temperature without heating the intervening medium is known as ---------*
    (a) conduction (b) convection (c) radiation (d) refraction
13. *Transfer of heat from sun to the earth takes place by the way of ————.*
    (a) conduction (b) convection (c) radiation (d) reflection
14. *Transmission of heat energy through liquids or gases takes place by way of ————.*
    (a) conduction (b) convection (c) radiation (d) melting
15. *Transmission of heat energy through metals takes place by way of ————.*
    (a) conduction (b) convection (c) radiation (d) refraction
16. *Out of the following processes of heat transfer for which process material medium is not required is called ——.*
    (a) conduction (b) convection (c) radiation (d) refraction
17. *Only ———— takes place in vacuum as well as material medium.*
    (a) conduction (b) convection (c) radiation (d) refraction
18. *The way by which heat reaches to earth from sun is ————.*
    (a) conduction (b) convection (c) radiation (d) refraction
19. *Out of the following the fastest process of heat transfer is ————.*
    (a) conduction (b) convection (c) radiation (d) refraction
20. *Out of the following the slower process of heat transfer is*
    (a) conduction (b) convection (c) radiation (d) refraction
21. *Out of the following surface which radiates more heat at a given temperature?*
    (a) black and smooth (b) black and rough
    (c) white and smooth (d) white and rough

22. *Material medium is not necessary in ……*
    (a) conduction (b) convection (c) radiation (d) none of these
23. *In case of radiation, heat transfer is in the form of ……*
    (a) stationary waves (b) electromagnetic waves (c) transverse waves (d) longitudinal waves
24. *Which of the following is not a unit of coefficient of thermal conductivity K ?*
    (a) cal/cm °C sec (b) kcal/m °C sec (c) watt/sec °K (d) watt/m °K
25. *Temperature gradient is equal to ……*
    (a) $\frac{\text{change in temperature}}{\text{time}}$ (b) $\frac{\text{time}}{\text{change in temperature}}$
    (c) $\frac{\text{distance}}{\text{change in temperature}}$ (d) $\frac{\text{change in temperature}}{\text{distance}}$
26. *Temperature gradient is defined as ___________*
    (a) change in temperature per unit change in time
    (b) change in time per unit change in temperature
    (c) change in temperature per unit change in distance in the direction of heat flow
    (d) change in distance per unit change in temperature
27. *Unit of temperature gradient is ————.*
    (a) m/°C (b) °C/m (c) °C/sec (d) sec/°C
28. *The state in which temperature of substance goes on increasing with respect to time is called as ————.*
    (a) variable state (b) steady state (c) normal state (d) critical state
29. *Heat absorbed by the material > heat given out by the material is concerned with ————.*
    (a) normal state (b) critical state (c) variable state (d) steady state
30. *Heat absorbed by the material = Heat given out by the material is concerned with ————.*
    (a) normal state (b) critical state (c) variable state (d) steady state
31. *Amount of heat flowing through material of a rod of unit area, in one second for unit temperature gradient at steady state is known as ————.*
    (a) conductivity (b) heat constant
    (c) coefficient of thermal conductivity (d) thermal constant
32. *As per law of thermal conductivity, amount of heat flowing through a rod is ……*
    (a) directly proportional to cross-sectional area (b) directly proportional to temperature gradient
    (c) directly proportional to time (d) all of these
33. *The SI unit of coefficient of thermal conductivity is ————.*
    (a) watt-m-°K (b) watt/m-°K (c) m°K/watt (d) m/watt°K
34. *The coefficient of thermal conductivity of good conductors of heat is ————.*
    (a) low (b) medium (c) high (d) none of these
35. *Which of the following is not a good conductor of heat ?*
    (a) thermocole (b) mica
    (c) thermocole and mica both (d) copper
36. *Which of the following is not a bad conductor of heat ?*
    (a) plastic (b) wood (c) mica (d) plastic and mica both
37. *Thermal resistor is …… the thermal conductivity.*
    (a) reciprocal of (b) equal to (c) addition of (d) none of these

38. *Which type of material is used as a heat sink in electronic circuits ?*
    (a) bad conducting (b) conducting (c) semiconducting (d) all of these
39. *Condenser coil in a refrigerator is ideally made up of ……*
    (a) bad conductor (b) insulator (c) semiconductor (d) good conductor
40. *Davy's safety lamp is covered by ……*
    (a) insulating material (b) good conductor material
    (c) semiconducting material (d) none of these
41. *Which type of material is used in ice box ?*
    (a) bad conducting (b) good conducting (c) semiconducting (d) none of these
42. *Handle of cooker is made up of ……*
    (a) good conducting material (b) semiconducting material
    (c) aluminium (d) bad conducting material
43. *Room ventilation, formation of trade winds, formation of sea breeze are the applications of ……*
    (a) conduction (b) radiation (c) convection (d) all of the above
44. *Radiation can ……*
    (a) travel through vacuum (b) travel with speed of light
    (c) reflect, refract (d) all of these
45. *Heat radiations in car, use of white clothes in summer are the applications of ……*
    (a) conduction (b) convection (c) radiation (d) none of these
46. *Thickness of a plate is 10 cm. The temperatures of the two faces of element are 90°C and 60°C. Find temperature gradient.*
    (a) 30°C/cm (b) 3°C/cm (c) 1°C/cm (d) 7°C/cm
47. *A metal rod 10 cm long, of area 0.9 cm^2 has temperature difference 60°C. Calculate heat flowing in 1 minute. (Given : K = 0.14 cal/cm °C sec)*
    (a) 45.36 cal (b) 23.6 cal (c) 57.8 cal (d) none of these
48. *Calculate amount of heat conducted in 1 minute through a rod of cross-sectional area 0.2 m^2, temperature gradient 50°C/m. (Given : K = 0.08 kcal/m °C sec)*
    (a) 24 kcal (b) 48 kcal (c) 72 kcal (d) 59 kcal
49. *Thickness of plate is 8 cm. Temperatures of two faces are 100°C and – 20°C. Calculate temperature gradient.*
    (a) 10°C/cm (b) 20°C/cm (c) 25°C/cm (d) 15°C/cm
50. *When the temperature of a rod of copper is increased, its length ……*
    (a) stays the same (b) increases (c) decreases (d) increases and then decreases
51. *The amount by which unit length of a material increases when the temperature is raised by one degree is called the coefficient of ……*
    (a) cubical expansion (b) superficial expansion (c) linear expansion (d) aerial expansion
52. *The symbol used for volumetric expansion is ……*
    (a) γ (b) β (c) L (d) α
53. *The symbol used for linear expansion is ……*
    (a) γ (b) β (c) L (d) α
54. *The symbol used for aerial expansion is ……*
    (a) γ (b) β (c) L (d) α
55. *A material of length L_1 at temperature t_1 is subjected to a temperature rise of t. The coefficient of linear expansion of the material is α. The material expands by ……*
    (a) $L_2 (1 + \alpha t)$ (b) $L_1 \alpha(t - t_1)$ (c) $L_1 [1 + \alpha(t - t_1)]$ (d) $\alpha L_1 t$
56. *Some iron has a coefficient of linear expansion of 12×10^{-6} °C. A 100 mm length of iron piping is heated through 20°C. The pipe extends by ……*
    (a) 0.24 mm (b) 0.024 mm (c) 2.4 mm (d) 0.0024 mm

57. *If the coefficient of linear expansion is A, the coefficient of superficial expansion is B and the coefficient of cubic expansion is C, which of the following is false ?*
    (a) C = 3A (b) A = B/2 (c) B = 3C (d) A = C/3
58. *The length of a 100 mm bar of metal increases by 0.3 mm when subjected to a temperature rise of 100 K. The coefficient of linear expansion of the metal is ……*
    (a) $3 \times 10^{-3}/°C$ (b) $3 \times 10^{-4}/°C$ (c) $3 \times 10^{-5}/°C$ (d) $3 \times 10^{-6}/°C$
59. *A liquid has a volume V_1 at temperature t_1. The temperature is increased to t_2. If γ is the coefficient of cubic expansion, the increase in volume is given by ……*
    (a) $\gamma V_1 (t_2 - t_1)$ (b) $V_1 \gamma t_2$ (c) $V_1 + V_1 \gamma t_2$ (d) $V_1 [1 + \gamma(t_2 - t_1)]$
60. *Which of the following statements is true ?*
    (a) Gaps need to be left in lengths of rail to provide ventilation.
    (b) Gaps need to be left in lengths of rail to provide scope for expansion.
    (c) Gaps need to be left in lengths of rail to avoid electrical conductivity.
    (d) Gaps need to be left in lengths of rail to save metal material.
61. *On heating, amplitude of vibration of atoms or molecules of an object ……*
    (a) increases (b) decreases (c) remains constant (d) none of above
62. *Coefficient of volume expansion of solids is ……*
    (a) greater than liquids (b) equal to gases (c) less than liquids (d) equal to liquids
63. *Gaps are left in railway tracks to compensate thermal expansion during ……*
    (a) rainy season (b) winter (c) hot season (d) wind
64. *Molecules of a solid vibrate with larger amplitude at ……*
    (a) zero temperature (b) lower temperature (c) higher temperature (d) pressure
65. *An increase in breadth, length and thickness of a substance is due to ……*
    (a) fusion (b) thermal expansion (c) stress (d) boiling
66. *Thermal expansion of a material has units as ……*
    (a) J/kg-K (b) J/mol-K (c) $J.ohm/sec.K^2$ (d) 1/°C
67. *The coefficient of linear expansion of a steel is 0.000012 per °C. The coefficient of volume expansion in/°C – 1, is ……*
    (a) (0.000012) 3 (b) $(4\pi/3)$ (0.000012) 3 (c) 3×0.000012 (d) 0.000012

Answers

1. (c)	2. (b)	3. (b)	4. (c)	5. (a)	6. (a)	7. (b)	8. (b)	9. (c)	10. (a)
11. (b)	12. (c)	13. (c)	14. (b)	15. (a)	16. (c)	17. (c)	18. (c)	19. (c)	20. (a)
21. (b)	22. (c)	23. (b)	24. (c)	25. (d)	26. (c)	27. (b)	28. (a)	29. (c)	30. (d)
31. (c)	32. (d)	33. (b)	34. (c)	35. (a)	36. (c)	37. (a)	38. (b)	39. (d)	40. (b)
41. (a)	42. (d)	43. (c)	44. (d)	45. (c)	46. (b)	47. (a)	48. (b)	49. (d)	50. (b)
51. (c)	52. (a)	53. (d)	54. (b)	55. (d)	56. (b)	57. (c)	58. (c)	59. (a)	60. (b)
61. (a)	62. (c)	63. (c)	64. (c)	65. (b)	66. (d)	67. (c)			

46. **Hint :** T.G. = $\frac{(90 - 60)}{10}$ = 3°C/m

47. **Hint :** $Q = \frac{kA(\theta_1 - \theta_2)t}{d} = \frac{0.14 \times 0.9 \times (60) \times 60}{10} = 45.36$ cal

48. **Hint :** Q = kA (T.G.) t = $0.08 \times 0.2 \times 50 \times 60$ = 48 kcal

49. **Hint :** T.G. = $\frac{100 - (-20)}{8}$ = 15°C/m

❐❐❐

Chapter 7

ACOUSTICS

7.1 SOUND

7.1.1 Concept of a Wave Motion

Wave motion is a form of disturbance which travels through the medium due to repeated periodic motion of the medium particle about its mean position.

It is one of the methods of transfer of energy.

A wave motion can be explained on the basis of SHM.

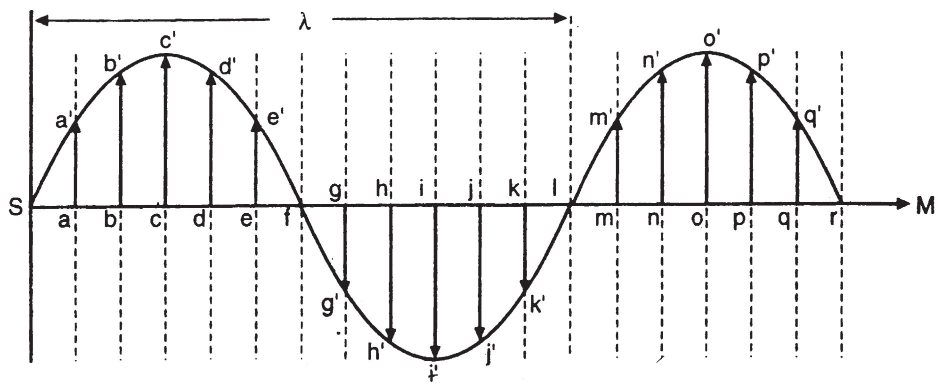


Fig. 7.1

Suppose S is a source of light, SM is one of the rays; a, b, c, d, ... are particles of material medium along SM which are equidistant.

When light energy starts from S, it sets 'a' into vibrations. 'a' in turn sets 'b' into vibrations, 'b' in turn sets 'c' into vibrations and so on. Thus energy travels from one point to other.

Each particle of medium performs SHM of same period and amplitude. As the particle starts vibrating after some lapse of time, they are not in same phase.

The above phenomenon is considered in another way. The oscillations of a, b, c, … are shown at a moment i.e. at one moment positions of particles of material medium will be as shown in Fig. 7.1.

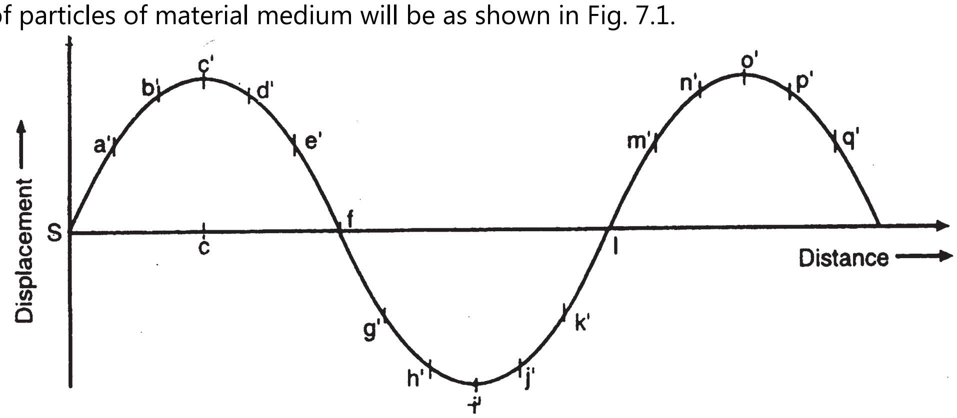


Fig. 7.2

The resulting curved line joining these positions at the same moment represents a wave.

The propagation of wave requires medium having *elasticity, inertia* and *little resistance.*

(i) Amplitude (a) : The maximum displacement of the oscillating particle in the wave is called amplitude. In Fig. 7.2, cc' is the amplitude.

(ii) Periodic time of period (T) : All oscillations of points a, b, c, ... take the same time to complete one oscillation.

The time taken by a particle to complete one oscillation in the wave is called period. Its unit is second.

(iii) Frequency (n) : The number of oscillations completed in one second is called frequency n. Its unit is hertz (Hz).

$$1 \text{ Hz} = 1 \text{ osc/sec.}$$

$$\boxed{n = \frac{1}{T}}$$

(iv) Phase : The phase of a particle at any instant is the state (i.e. displacement, direction, position) of motion of particle at that instant.

The particle having same displacement, direction and position are said to be in the same phase.

In Fig. 7.1, a and m are in same phase, b and n are in same phase. But notes a and e are not in same phase even though they seem to be, because they are not identical by all means i.e. their positions are different.

(v) Wavelength (λ) : The distance between two consecutive points in the wave, which are in same phase, is called wavelength and denoted by symbol λ (lambda).

In short, it is the length of one full wave. Its unit is meter or Angstrom unit, denoted by A° or A.U.

$$1 \text{ A}° = 1 \times 10^{-10} \text{ m}$$

7.1.2 Relation between Velocity, Frequency and Wavelength

We have, $$\text{Velocity} = \frac{\text{Distance covered}}{\text{Time taken}}$$

When disturbance travels through one full wave, then,

$$\text{Distance covered} = \text{Wavelength}$$

$$\text{Time taken} = \text{Period}$$

$$\therefore \quad \text{Velocity} = \frac{\text{Wavelength}}{\text{Period}}$$

$$\boxed{v = \frac{\lambda}{T}} \qquad \text{But } \frac{1}{T} = n \quad \therefore \quad \boxed{v = n\lambda}$$

This is the relation between velocity, frequency and wavelength.

7.1.3 Transverse Wave and its Characteristics

In this wave, the particles of material medium are vibrating up and down but (disturbance) wave travels in horizontal (right) direction.

1. **The wave, in which the direction of vibration of particles of material medium are perpendicular to the direction of propagation of wave, is called transverse wave.**
2. Wave travels in the form of alternate crest and trough.
3. Material medium needs elasticity of shape.
4. Density and pressure of medium remains same.
5. Wave travels through solids only.
6. e.g. Light wave.
7. Every particle of material medium performs SHM of same amplitude and period.

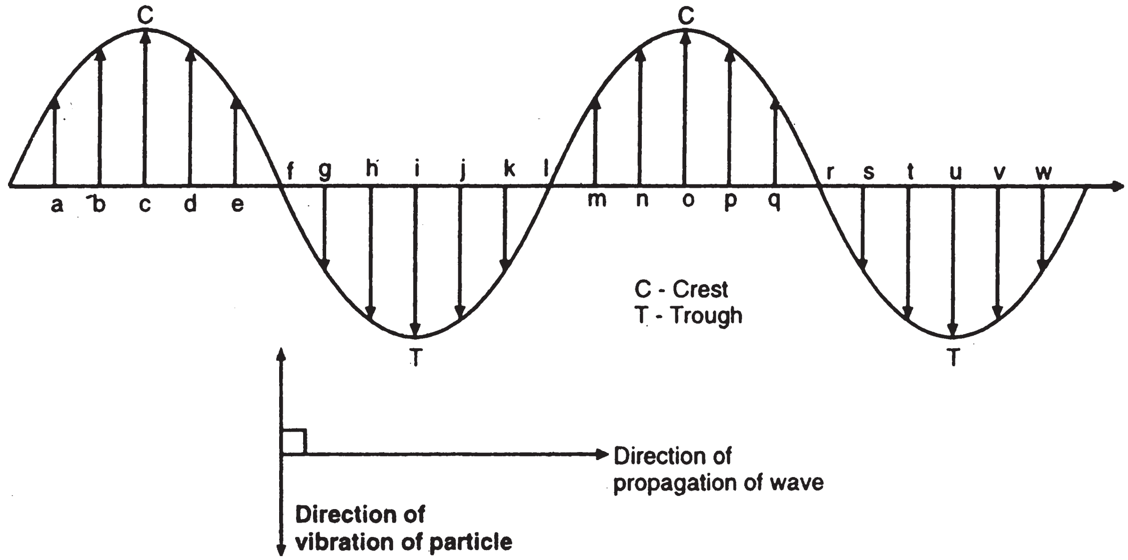


Fig. 7.3

7.1.4 Longitudinal Wave and its Characteristics

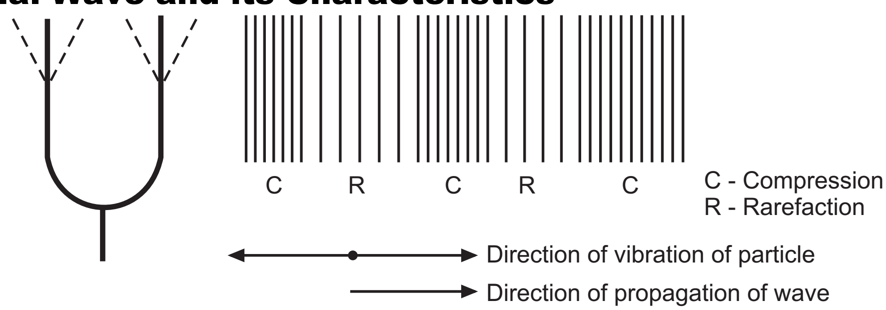


Fig. 7.4

In this wave, the particles of material medium are vibrating to and fro and the wave is travelling in the horizontal direction (right).

1. **The wave in which the direction of vibration of particle is parallel to the direction of propagation of wave is called a longitudinal wave.**
2. Wave travels in the form of alternate compression and rarefaction.
3. Material medium needs elasticity of volume.
4. Pressure and density of medium changes. It is more at compression and less at rarefaction.
5. Wave travels through liquids and gases.
6. e.g. Sound wave.
7. Every particle of material medium performs SHM of same amplitude and period.

Difference between transverse and longitudinal waves :

Transverse wave	Longitudinal wave
1. **Definition :** The wave in which the direction of vibration of particles of material medium is perpendicular to the direction of propagation of wave is called transverse wave.	1. **Definition :** The wave in which the direction of vibration of particles of material medium is parallel to the direction of propagation of wave is called longitudinal wave.
2. Wave travels in the form of alternate crests and troughs.	2. Wave travels in the form of alternate compressions and rarefactions.
3. Material medium needs elasticity of shape.	3. Material medium needs elasticity of volume.
4. Density and pressure of medium remain same.	4. Density and pressure of medium change. It is maximum at compression and minimum at rarefaction.
5. Wave travels through solid only.	5. Wave travels through liquids and gases.
6. e.g. Light wave.	6. e.g. Sound wave.

7.1.5 Stationary Waves or Standing Waves

Principle of Superposition of Waves :

It states that, when two waves travelling through the medium arrive at a point simultaneously, each wave produces its own displacement independent of the other.

The resultant displacement at that point changes.

The resultant displacement at that point is equal to the vector sum of the displacements due to the two waves.

Definition of stationary or standing wave : A stationary or standing wave is the resultant wave produced due to the superposition of two identical progressive waves with the same amplitude, frequency, wavelength and velocity and travelling along the same straight line but in opposite direction.

Thus, in short, two identical waves but travelling in opposite directions form stationary waves.

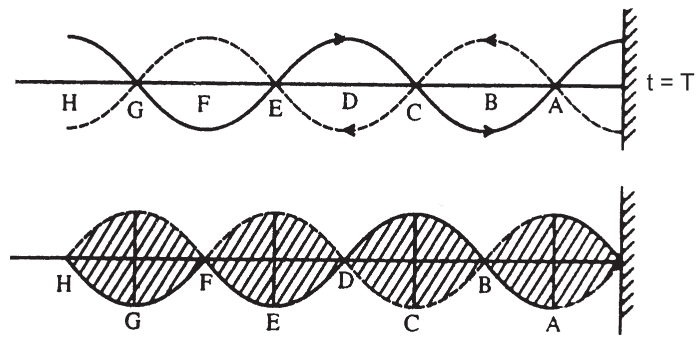


Fig. 7.5 : Stationary waves

Nodes, Antinodes, Distance between two successive nodes and antinodes :

Nodes : The points in the stationary waves which have resultant displacement zero are called nodes.

Antinodes : The points in the stationary waves which have resultant displacement maximum are called antinodes.

Distance between two successive nodes and antinodes (Nodes and antinodes are equally spaced) :

The distance between two successive nodes is $\frac{\lambda}{2}$ i.e. nodes are equally spaced.

The distance between two successive antinodes is $\frac{\lambda}{2}$ i.e. antinodes are equally spaced.

The distance between two successive nodes and antinodes is $\frac{\lambda}{4}$.

7.1.6 Free and Forced Oscillations (Vibrations)

Free oscillations (vibrations) :

Consider a simple pendulum or cricketers hanging ball : if it is displaced (disturbed) from mean position and released, it will oscillate with definite time period and frequency. This time period or frequency will not change even if displacement is changed. In such a case, we can say that pendulum (or ball) is making free vibrations.

A body, if disturbed from its position of rest and allowed to vibrate freely on its own, it vibrates with a frequency called the "natural frequency" and the oscillations are called free or natural oscillations.

OR

When an object is made to oscillate freely on its own, then such oscillations are called free oscillations.

(If the amplitude of oscillations continuously goes on decreasing with time, then the oscillations are called "damped oscillations".)

Every body, small or large, has its own natural frequency of vibration. A machine, a building, a dam or a bridge has its own natural frequency of vibration.

Free vibrations or Natural vibrations : The vibrations produced in a body, on being slightly disturbed from its mean position are called **free vibrations** or **natural vibrations (oscillations)**.

Natural time period (T) : The periodic time of a body executing natural vibrations is called **natural time period**.

OR

The time required for one oscillation for a body executing natural vibrations is called **natural time period (or periodic time)**.

Natural frequency (n) : The number of oscillations (vibrations) performed in one second by a freely vibrating body is called **natural frequency**.

Examples of free vibrations :

1. A freely suspended cricketer's hanging ball if displaced and released.
2. A freely suspended pendulum oscillating about its mean position.
3. A tuning fork on being stuck on a rubber pad.
4. A metal blade clamped at one end and disturbed.
5. String of musical instrument (guitar) being plucked.

Forced oscillations (vibrations) :

Consider a cricketer's hanging ball : if it is displaced from its mean position and released, then it performs natural oscillations. Now if cricketer (hit) apply a stroke of bat with certain rhythm then ball may oscillate with some new frequency and period. In this case if natural frequency of oscillations of a ball is equal to forced frequency of cricketer's hitting and rhythm then ball will perform oscillations with larger amplitude.

Consider a simple pendulum performing natural oscillations, now if support of a pendulum itself vibrates with a certain frequency, then the pendulum start performing oscillations with some new frequency and period, these oscillations are forced oscillations (vibrations).

Forced vibrations : The vibrations which take place under the influence of external periodic force are called **forced vibrations**.

OR

When a body oscillates with a some new frequency other than natural frequency because of external periodic force, then the oscillations are called **forced oscillations**.

Forced frequency (n) : The number of oscillations (vibrations) performed in one second by a body performing forced oscillations.

Periodic time (T) of forced oscillations : The time required to complete one oscillation for a body performing forced oscillations.

Properties of forced vibrations :

- Body performing forced oscillations vibrate with new frequency and period.
- If natural frequency and forced frequency are different then the body oscillate with small amplitude.
- If natural frequency of vibration and forced frequency are equal then body oscillates with larger amplitude (i.e. resonance).

Examples of forced vibrations (oscillations) :

1. Cricketer's hanging ball, hit by a cricketer with certain periodic rhythm.
2. Oscillating simple pendulums support itself vibrates with certain frequency.
3. Vibrating tuning fork held at open end of the tube make the air column to vibrate with new frequency.
4. Needle of a gramophone player moves with forced vibrations.

7.1.7 Resonance

Definition : If a frequency is forced upon a body and if the forced frequency matches (is equal to) the natural frequency of the body, the body vibrates with a large amplitude. The phenomenon is called "Resonance".

Hence, "resonance" may be defined as a phenomenon in which a body vibrates with a large amplitude due to the effect of a forced frequency which is equal to natural frequency of the body.

Examples of Resonance :

1. If two exactly tuned 'Tanpuras' are kept side by side and if a wire of one is plucked, then the corresponding wire of the other Tanpura also starts vibrating even though it is not actually plucked.
2. When we tune a radio receiver set, the frequency of radio waves received in the set is adjusted equal to the natural frequency of the set. or Frequency of radio receiver becomes equal to frequency of transmitted waves of transmitting station, then we get clear and loud sound.
3. The phenomenon of resonance is of importance in 'Acoustics' where it is useful to obtain a strong response to a weak sound.

4. The soldiers are not allowed to march in regular steps while crossing a bridge as there is a possibility that due to resonance between the forced frequency of their regular (left-right) steps and the natural frequency of vibration of the bridge, large amplitude vibrations may be developed in the bridge and it may subsequently collapse.
5. If natural frequency of oscillations hanging ball becomes equal to forced frequency of bat hitting rhythem of cricketer, then ball oscillates with larger amplitude.

7.1.8 Formula for Velocity of Sound with End Correction

A metal tube of known diameter 'D' is immersed in water and kept at downmost position. A tuning fork of known frequency 'n' is made to vibrate and held at the mouth of the tube as shown in Fig. 7.6. The metal tube is adjusted (by moving up) such that loud sound is heard. Fix the position of the tube where loud sound is heard and record the length 'l' of air column.

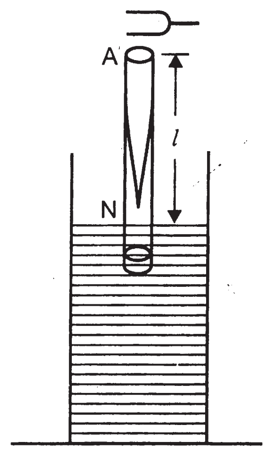


Fig. 7.6

Derivation of formula of velocity of sound :

Loud sound at certain position is heard due to the resonance of air column in the tube. Here sound waves travel from vibrating tuning fork towards the water surface in the column. This wave gets reflected from water surface and stationary wave is formed such that node at water level and antinode at open end of the tube as shown in Fig. 7.6.

Thus, resonating length of air column

$$l = \frac{\lambda}{4}$$

$\therefore$ $\lambda = 4l$

Since resonance occurs, natural frequency of vibrating air column is equal to forced frequency of tuning fork 'n'.

We have $v = n(\lambda)$ but $\lambda = 4l$ (from above)

$\therefore$ $v = n\,(4l)$

$$\boxed{v = 4nl}$$

But practically it is observed that antinode does not situated at open end exactly but it lies slightly above the open end, this difference is called as end correction.

End correction, $e = 0.3\text{ D}$ where D = diameter

New corrected resonating length 'L'

$L = l + e$ where l = resonating length

$= l + 0.3D$ L = corrected resonating length

Now $v = 4nl$ e = end correction

becomes $\boxed{v = 4nL}$ θ = internal diameter of the tube

i.e. $\boxed{v = 4n\,(l + 0.3D)}$

Important Points

- Wave motion is a form of disturbance which travels through the medium due to repeated periodic motion of the medium particle about its mean position.
- **Amplitude :** The maximum displacement of the oscillation in the wave is called amplitude.
- **Periodic time or Period (T) :** The time required to complete one oscillation in the wave is called as period.
- **Frequency (n) :** The number of oscillations completed in one second is called as frequency.
- **Phase :** The phase of a particle at any instant is the state i.e. displacement, direction, position of motion of particle at that instant.
- **Wavelength (λ) :** The distance between two consecutive points in the wave, which are in same phase, is called as wavelength.
- Relation between v, n and λ is **v = nλ.**

- **Transverse wave :** The wave in which direction of vibration of particles of material medium is perpendicular to the direction of propagation of wave.
- **Longitudinal wave :** The wave in which direction of vibration of particle is parallel to the direction of propagation of wave is called as longitudinal wave.
- **Stationary or standing waves :** It is the resultant wave produced due to superposition of two identical progressive waves with same amplitude, frequency, wavelength and velocity and travelling along same straight line but in opposite direction.
- **Free oscillations :** A body if disturbed from its mean position of rest and allowed to vibrate freely on its own, vibrates with natural frequency, then these vibrations are called free vibrations (oscillations).
- **Forced oscillations :** The oscillations in which another frequency is forced upon the body are called forced oscillations.
- **Resonance :** It is defined as a phenomenon in which a body vibrates with a large amplitude due to effect of forced frequency matches with natural frequency of the body.

SOLVED EXAMPLES

Examples on relation v = nλ and resonance only (As per curriculum)

Example 1 : *A tuning fork of frequency 384 Hz produces sound wave of wavelength 86 cm. Find the velocity of sound.*

Solution : Given : n = 384 Hz, λ = 86 cm = 0.86 m, v = ?

We have, $v = n\lambda = (384)(0.86) \quad \therefore \quad \boxed{v = 330.24 \text{ m/s}}$

Example 2 : *The velocity of wave is 300 m/s. If the frequency of vibration of wave is 300 Hz, calculate the wavelength.*

Solution : Given : v = 300 m/s, n = 300 Hz, λ = ?

We have, $v = n\lambda$

$\therefore \quad \lambda = \frac{v}{n} = \frac{300}{300} \quad \therefore \quad \boxed{\lambda = 1 \text{ m}}$

Example 3 : *A body produces waves of wavelength 33 cm. What is the frequency of vibration if velocity of propagation is 330 m/s ?*

Solution : Given : λ = 33 cm = 0.33 m, n = ?, v = 330 m/s

We have, $v = n\lambda$

$\therefore \quad n = \frac{v}{\lambda} = \frac{330}{0.33} \quad \therefore \quad \boxed{n = 1000 \text{ Hz}}$

Example 4 : *An air column of length 17 cm in a resonance tube resonates when fork of 500 Hz is used. Calculate the velocity of sound.*

Solution : Given : l = 17 cm = 0.17 m, n = 500 Hz, v = ?

$v = 4nl = 4 \times (500) \times (0.17) \quad \therefore \quad \boxed{v = 340 \text{ m/s}}$

Example 5 : *Find the corrected length of air column in a resonance tube of diameter 4 cm, if the length of the resonating air column, for the first resonance is 18.8 cm.*

Solution : l = 18.8 cm, diameter = D = 4 cm, corrected resonating length = ?

Corrected resonating length, $L = (l + 0.3\,D) = [18.8 + (0.3 \times 4)] \quad \boxed{L = 20 \text{ cm}}$

Example 6 : *A tuning fork of frequency 512 Hz resonates with an air column of length 14.4 cm. The end correction is 6 mm. Calculate the velocity of sound in air.*

Solution : Given : n = 512 Hz, l = 14.4 cm, End correction, e = 6 mm = 0.6 cm

$v = 4n(l + e) = 4(512)(14.4 + 0.6) = 30720 \text{ cm/s} \quad \therefore \quad \boxed{v = 307.2 \text{ m/s}}$

Example 7 : *A vibrating body sends waves of 110 cm wavelength in air and 480 cm wavelength in water. If velocity of sound in air is 330 m/s, find velocity of sound in water.*

Solution : Given : $\lambda_{air} = 110 \text{ cm} = 1.1 \text{ m}, \; \lambda_{water} = 480 \text{ cm} = 4.8 \text{ m}$

$$v = n\lambda$$

$\therefore$ $v_{air} \propto \lambda_{air}$ ... (1)

and $v_{water} \propto \lambda_{water}$ ... (2)

Dividing equation (2) by (1), $\frac{v_{water}}{v_{air}} = \frac{\lambda_{water}}{\lambda_{air}}$

$\therefore$ $v_{water} = \frac{\lambda_{water}}{\lambda_{air}} \times v_{air} = \frac{4.8}{1.1} \times 330 \quad \therefore \quad \boxed{v_{water} = 1440 \text{ m/s}}$

Example 8 : *In a resonating tube experiment, resonating length for a tuning fork of 512 Hz frequency is 16 cm. Calculate the resonating length for tuning fork of frequency 384 Hz. Also find velocity of sound at room temperature neglecting end correction.*

Solution : Given : $n_1 = 512$ Hz, $l_1 = 16$ cm | $n_2 = 384$ Hz, $l_2 = ?$

We have, $v = 4nl$

i.e. $v = 4\, n_1 l_1 = 4 \times (512) \times (16) = 32768 \text{ cm/s}$ or $\boxed{v = 327.68 \text{ m/s}}$

Now, $n \propto \frac{1}{l}$

$\therefore$ $n_1 \propto \frac{1}{l_1}$ ... (1)

and $n_2 \propto \frac{1}{l_2}$ ... (2)

Dividing equation (1) by (2), $\frac{n_1}{n_2} = \frac{l_2}{l_1}$

$\therefore$ $l_2 = \frac{n_1}{n_2} \times l_1 = \frac{512}{384} \times 16 \quad \therefore \quad \boxed{l_2 = 21.33 \text{ cm}}$

Questions

1. Define wave motion.
2. Define amplitude, period, frequency, phase and wavelength.
3. State relation between velocity, frequency and wavelength.
4. Define transverse wave and longitudinal wave.
5. Define transverse wave and state its characteristics.
6. Define longitudinal wave and state its characteristics.
7. Distinguish between transverse and longitudinal waves.
8. State equation of progressive wave with symbol meanings.
9. Define stationary (standing) wave.
10. Define nodes and antinodes.
11. Define free oscillations, forced oscillations and resonance.
12. State formula for velocity of sound with end correction with symbol meanings.

Problems

1. Find the corrected length of air column in a resonance tube of diameter 2.5 cm, if the length of the resonating air column, for the first resonance is 16 cm.

   Ans. Corrected length L = 16.75 cm.

2. A tuning fork of frequency 512 Hz produces sound wave of wavelength 64 cm. Find the velocity of sound.

   Ans. v = 327.68 m/s.

3. The velocity of wave is 320 m/s. If the frequency of vibration of wave is 480 Hz, calculate the wavelength.
   Ans. λ = 0.667 m.
4. A body produces waves of wavelength 50 cm. What is the frequency of vibration if velocity of propagation is 320 m/s ?
   Ans. n = 640 Hz.
5. Calculate the velocity of sound if resonating length 14 cm is observed for a tuning fork of frequency 512 Hz.
   Ans. v = 286.72 m/s.
6. A tuning fork of frequency 480 Hz resonates with air column of length 16 cm. The end correction is 0.5 cm. Calculate the velocity of sound in air.
   Ans. v = 316.8 m/s.

7.2 ACOUSTICS OF BUILDINGS

7.2.1 Acoustic Concept and Definition

- Sound waves undergo reflection, refraction, diffraction, transmission etc.

Acoustics :

- Acoustics is a branch of physics which deals with design, generation, transmission (propagation) and reception (detection) of sound waves.
- Building or Architectural acoustics is a branch of science which deals with the planning of various buildings such that sound produced inside the building (hall) is clearly heard at all places in the hall without any confusion.

Building or Architectural acoustics (Definition) :

- It is defined as the branch of science which deals with the planning of a building or a hall in order to provide the best audible (clear) sound to the audience.
- Earlier many civil engineers and architects had poor knowledge about the considerations of acoustical properties and hence very few buildings used to be satisfactory in this respect in those times. Prof. W. C. Sabine was the first scientist to study the problem in detail in the year 1910-1911. To understand the subject of acoustics, it is necessary to have knowledge about various quantities and the terms used in it.

7.2.2 Limit of Intensity and Loudness

We sometimes, say that some sounds are more intense than others. The energy carried by the sound wave is the measure of intensity of sound.

7.2.2.1 Intensity of Sound

- **The intensity of sound at a point is defined as the sound energy passing normally at a point through the unit area of the medium in unit time.**
- If sound energy spreads uniformly in all directions, then the intensity of sound at a point is inversely proportional to the square of the distance from the source.

$$I \propto \frac{1}{d^2}$$

- S.I. unit of intensity is watt/meter^2 (W/m^2). The weakest sound to which human ear can respond is roughly 10^{-12} W/m^2 and when the sound intensity is as high as 1 W/m^2, it starts giving pain.

$$I_O = 10^{-12} \text{ W/m}^2 \rightarrow 0 \text{ dB}$$

Why ? Because, we have $\text{dB} = 10 \log_{10}\left(\frac{I_1}{I_2}\right) = 10 \log_{10}\left(\frac{I_1}{10^{-12}}\right)$

Put $I_1 = 10^{-12} \text{ W/m}^2 = 10 \log_{10}\left(\frac{10^{-12}}{10^{-12}}\right) = \mathbf{0\ dB}$

$$I = 1 \text{ W/m}^2 \rightarrow 120 \text{ dB}$$

Why ? Because, we have $\text{dB} = 10 \log_{10}\left(\frac{I_1}{I_2}\right) = 10 \log_{10}\left(\frac{I_1}{10^{-12}}\right)$

Put $I_1 = 1 \text{ W/m}^2 = 10 \log_{10}\left(\frac{1}{10^{-12}}\right) = 10 \log_{10}(10^{12}) = \mathbf{120\ dB}$

Hence, the intensity level corresponding to $I_O = 10^{-12}$ W/m^2 is reckoned as zero (decibel) and the intensity level corresponding to I = 1 W/m^2 is 120 dB.

7.2.2.2 Loudness of Sound

- Loudness is a subjective measurement of sound power. Loudness and intensity though related to each other are not the same. Intensity does not depend on the response of the human ear. **Loudness on the other hand, depends on both intensity and response of the ear**.
- **Loudness can also be defined as an observer's auditory impression of the strength of sound.**
- For example, in case of a young person and old person sitting on the same bench away from a speaker, the intensity is same but loudness for the younger person is more than that of the older person.

7.2.2.3 Weber and Fechner's Law

- As we have seen, loudness of sound varies from one listener to another.
- Weber and Fechner's law states that the magnitude of loudness of sound is directly proportional to the logarithm of the sound intensity.

i.e. $L \propto \log I$

- Greater the intensity of sound wave, greater is the loudness.

The following are the factors upon which intensity (loudness) depends.

1. Amplitude of vibration of the source.
2. Distance of the observer from the source.
3. Density of the medium.
4. Presence of other bodies (if resonant body is placed near the source, intensity of sound increases).
5. Frequency (Intensity $\propto$ Frequency^2).

- The unit of loudness is phons and sones.

7.2.2.4 Graph of Loudness versus Frequency

- Loudness is expressed in decibels most of the times. In audio-frequency amplifiers, we convert an a.c. signal into the corresponding sound signal. There are two advantages in expressing loudness in dB for amplifiers.

(i) While calculating net gain of amplification of different stages, we get addition of gains as the net power gain is log to the base 10 of (output/input).

(ii) Human ear responds to sound i.e. dB directly and not current and voltages. So the direct response of the amplifier can be tested.

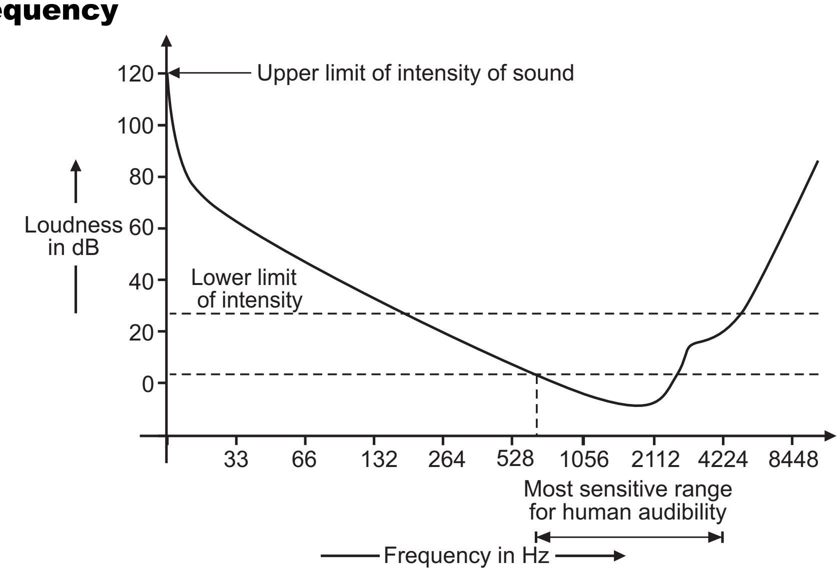


Fig. 7.7 : Graph of loudness versus frequency

7.2.3 Requirements of Satisfactory Acoustics (Conditions for Good Acoustics)

It has been observed that the following general conditions should be satisfied to obtain good acoustics for a hall :

1. The sound produced should be uniformly distributed throughout the hall and should be clearly heard in all parts of the hall.
2. The sound should be heard at all points in the hall sufficiently loudly. (The loudness of sound depends upon the sound energy put in.)
3. The consecutive musical notes or spoken words should be distinctly heard and there should be no overlapping of them. Thus, mixing up between two successive sounds should be absent.
4. The reverberation time of a hall should have a proper value depending upon the type of sound produced; about 1.25 to 2.5 seconds for music and about 1 to 2 seconds for speech.
5. The echelon effect should be absent. In this effect, the successive reflections from various steps of a staircase or some similarly designed structures produce overall effect of confusion.

6. There should not be any concentration of sound at particular points in the hall as in case of 'whispering galleries'; because in such circumstances, the sound is clearly heard at some points only, but not heard at all other points in the hall which is most undesirable.
7. The external sound should not be allowed to enter the hall as these sounds from the outside will produce undesirable confusion in the sound which is produced in the hall and is to be heard by the audience (listeners) present in the hall.

7.2.4 Factors Affecting Acoustical Planning of Auditorium

- As we know in auditorium or theatre, the quality of sound produced and picture quality plays a very important role.
- The quality of sound produced is said to be good if the sound is clearly heard at all places in a hall without any confusion.
- The factors that affect quality of sound are explained below.

7.2.4.1 Echo

- The reflection of sound is a common experience. If the time interval between the instants of hearing the original sound produced and the reflected sound is less than $1/10^{th}$ of a second, the two sounds cannot be separately heard as the effect of sound remains in the human ear for about $1/10^{th}$ of a second.
- However, if this time interval is greater than $1/10^{th}$ of a second, the original sound and the reflected sound can be separately heard.
- Such a reflection of sound takes place if the distance between the source of sound and the reflecting surface is greater than 16.5 metre.
- **The echo is defined as the same sound heard again after an interval of $1/10^{th}$ of second due to the reflection of the original sound from a surface which is at a distance greater than 16.5 metre from the source of sound**.
- An echo is also called 'resounding'. If the reflection of sound takes place from a concave surface, the reflected sound is focused at a definite point as happens in case of 'whispering galleries'.
- In a hall of sufficiently large dimensions, the sound is reflected from different surfaces of the hall again and again and this phenomenon is called 'multiple echoes'.
- A doctor's stethoscope is a good example of multiple reflections of sound.

7.2.4.2 Reverberation

- When a sound is produced in a hall, it gets reflected from the ceiling, walls, doors, floor, etc. of the hall number of times forming 'multiple reflections'. The sound is thus heard a number of times, once directly from the source and again many times due to reflections.
- Thus, if we cut-off the sound producing source, we continue to hear the sound afterwards also or the sound persists for some more time in the hall.
- **The reverberation of sound is defined as the persistance of sound due to multiple reflections in a hall even after the source of sound is cut-off.**
- **However, the sound goes on becoming weaker and weaker after every reflection as part of the sound energy is absorbed by the reflecting surface.**

7.2.4.3 Reverberation Time

- **The time for which the sound persists in a hall even after the source is cut-off is called as 'reverberation time'.** According to Prof. Sabine, reverberation time is the time required to decrease the intensity of sound at cut-off to one millionth (10^{-6} times) value.
- Reverberation time also depends upon the type of sound produced, e.g. human speech, musical sound, noise, etc. The reverberation time (t) is given by **Sabine's formula**,

$$t = \frac{0.164\ V}{A}$$

where, V = Total volume of the hall in m^3

A = Total absorption of sound in the hall = $\sum a.S$

(a is the coefficient of absorption and S is the surface area in m^2).

i.e. $$t = \frac{0.164\ V}{\sum a \cdot S}$$

Standard reverberation time : This is the value of reverberation time which is suitable for certain volume of a hall.

It is defined as the time required for the sound pressure level to reduce by 60 dB, measured after the generated test signal abruptly ended.

7.2.4.4 Creep

- Creep occurs because of reflection of sound along a curved surface. Most of the time from architecture point of view, the ceilings of auditorium are in **dome shape**.
- If the source of sound is close to the dome then energy of sound flows without much absorption and can be heard distincity at the other side (point) of the surface.

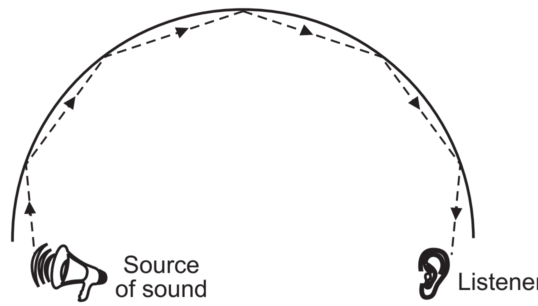


Fig. 7.8 : Creep

7.2.4.5 External Noise

- The outside noise can mix up with the sound of music or speech in the hall and can thus create confusion for the audience. This includes noise travelling through air, noise due to vibrations of structures, noise of machinery or typewriters, etc.
- The external noises can be decreased by making the hall sound-proof and constructing small sound-proof cabins for machinery and typewriters.

7.2.4.6 Audience and Upholstered Seats

- The human body and clothes act as good absorbing materials.
- The sound can be better heard in a hall full of audience than in an empty hall.
- The presence of upholstery (foam, cushions etc.) also affects the acoustics of the hall as it can absorb sound energy.

Important Points

- Acoustics is a branch of science which deals with planning of various buildings such that sound produced inside it is clearly heard at all places in the hall without any confusion.
- Intensity of sound at a point is defined as the sound energy passing normally at the point through unit area in one second. Its unit is W/m^2.
- Loudness depends on the intensity of sound and response of the ear. Its unit is decibel.
- Weber's and Fechner's law states that the magnitude of loudness of sound is directly proportional to the logarithm of the sound intensity.
- Echo, reverberation, creep, focussing of sound, absorption of sound, noise are the factors affecting acoustics of sound.
- **Echo** : Echo is defined as same sound is heard again after an interval of $(1/10)^{th}$ sec due to reflection of original sound from the surface which is at a distance greater than 16.5 m from the source of sound.
- **Reverberation time :** The time for which sound persists in a hall even after the source of sound is cut-off. $t = \frac{0.164\ V}{\sum aS}$.

Important Formulae

1. $$t = \frac{0.164\ V}{A} = \frac{0.164\ V}{\sum aS} = \frac{0.164\ V}{a_1S_1 + a_2S_2 + \ldots}$$

where
t = Reverberation time of hall
V = Volume of hall
A = Total absorption in hall
a = Absorption coefficient
S = Surface area.

(Note : The value of 'a' always lies between 0 to 1.)

2. $$\text{Velocity} = \frac{\text{Total distance covered}}{\text{Time taken}}$$

$$v = \frac{2h}{t}$$ where, v = Speed of sound

h = Depth of sea water

t = Time.

SOLVED EXAMPLES

Example 1 : *The volume of an auditorium is 6000 m^3. The reverberation time is to be controlled to 2 seconds by fixing a sound absorbing material of coefficient of absorption 0.12 O.W.U. Find the surface area of the absorbing material.*

Solution : Given : $V = 6{,}000\ m^3$, $t = 2$ sec., $a = 0.12$ O.W.U., $S = ?$

$$t = \frac{0.164\ V}{aS}$$

$$\therefore \quad S = \frac{0.164\ V}{at} = \frac{0.164 \times (6000)}{(0.12) \times (2)} \qquad \boxed{S = 4100\ m^2}$$

Example 2 : *An auditorium of volume 6525 m^3 has reverberation time of 2.1 seconds. If the total absorbing surface area in the hall is 4980 m^2, find the coefficient of absorption.*

Solution : Given : $V = 6525\ m^3$, $t = 2.1$ sec., $S = 4980\ m^2$, $a = ?$

$$t = \frac{0.164\ V}{aS}$$

$$\therefore \quad a = \frac{0.164\ V}{t\ S} = \frac{0.164 \times (6525)}{(2.1) \times (4980)} \qquad \boxed{a = 0.102 \text{ O.W.U.}}$$

Example 3 : *An interval of 1.6 second was observed between sending of a wave to the bottom of sea and return of the reflected wave. If the velocity of sound in the sea water is 2400 m/s, calculate the depth of sea.*

Solution : Given : $t = 1.6$ sec., $v = 2400$ m/s, $h = ?$

$$v = \frac{2h}{t} \qquad \left(\because \text{Velocity} = \frac{\text{Total distance covered}}{\text{Time}}\right)$$

$$\therefore \quad h = \frac{vt}{2} = \frac{(2400) \times (1.6)}{2} \quad \therefore \boxed{\text{Depth } h = 1920 \text{ m}}$$

Example 4 : *A lecture hall has a total surface absorption of 180 O.W.U. The reverberation time so calculated is 3.55 sec. Find the volume of the hall.*

Solution : Given : $A = \sum aS = 180$ O.W.U., $t = 3.55$ sec, $V = ?$

$$t = \frac{0.164\ V}{\sum aS}$$

$$\therefore \quad V = \frac{t \times \sum aS}{0.164} = \frac{(3.55) \times (180)}{0.164} \qquad \boxed{V = 3896.3\ m^3}$$

Example 5 : *A concrete hall of volume 2500 m^3 has a total surface absorption of 205. Find the reverberation time.*

Solution : Given : $V = 2500\ m^3$, $\sum aS = 205$, $t = ?$

$$t = \frac{0.164\ V}{\sum aS} = \frac{0.164 \times (2500)}{205} \quad \therefore \boxed{t = 2 \text{ sec.}}$$

Example 6 : *A hall of volume 5000 m^3 has reverberation time of 2 seconds. If the absorbing surface in the hall amounts to 3320 m^2, determine the coefficient of absorption.*

Solution : Given : $V = 5000\ m^3$, $t = 2$ sec., $\sum S = 3320\ m^2$, $a = ?$

$$t = \frac{0.164\ V}{\sum aS}$$

$$\therefore \quad a = \frac{0.164\ V}{t \sum S} = \frac{0.164 \times (5000)}{(2) \times (3320)} \quad \therefore \boxed{a = 0.123 \text{ O.W.U.}}$$

Example 7 : *Intensity of sound produced by thunder is 0.2 Wm^{-2}. Calculate the level in decibel.*

Solution : $I_1 = 0.2\ Wm^{-2}$, $I_2 = 10^{-12}\ Wm^{-2}$

$$\text{Intensity level} = 10 \log \frac{I_1}{I_2} = 10 \log_{10} \left(\frac{0.2}{10^{-12}}\right)$$

$$\boxed{\text{Intensity level} = 113\ dB}$$

Example 8 : *Calculate the intensity level of a jet plane just bearing the runway having a sound intensity of about 100 W/m^2.*

Solution : Given : $I_1 = 100\ W/m^2$, $I_2 = 10^{-12}\ W/m^2$ (zero decibel)

The relative intensity, $\beta = 10 \log_{10} \left(\frac{I_1}{I_2}\right) = 10 \log_{10} \left(\frac{100}{10^{-12}}\right) \quad \therefore \quad \boxed{\beta = 140\ dB}$

Example 9 : *The volume of a hall is 2000 m^3 and its total absorption equals 100 sabine-m^2 of an open window. Entry of people in the hall raises the total absorption by 2000 sabine-m^2. Calculate the change in reverberation time.*

Solution : Given : $V = 2000\ m^3$, Initial $A_1 = 100$ sabine-m^2, $t_1 = ?$

$$t_1 = \frac{0.166\ V}{A_1} = \frac{0.166 \times 2000}{100} = 3.32\ sec$$

Final (when people entered in the hall)

$$A_2 = 100 + 2000\ \text{sabine-}m^2 = 2100\ \text{sabine-}m^2$$

$$t_2 = ?$$

$$t_2 = \frac{0.166 \times 2000}{2100} = 0.158\ sec$$

$\therefore$ Change in reverberation time $t = t_1 - t_2 = 3.32 - 0.158$

$$\boxed{\text{Change in reverberation time, } t = 3.162\ sec}$$

Questions

1. Define echo and reverberation.
2. Distinguish between echo and reverberation.
3. Define reverberation time and how it is reduced in a good auditorium ?
4. What are the factors affecting reverberation time ?
5. State the conditions for good acoustics.
6. Why is acoustical planning of an auditorium necessary ?
7. Draw a graph of loudness versus frequency and show the different limits of intensity.
8. Write a note on noise and its insulation.

MSBTE Questions

1. Define echo and reverberation.
2. Intensity of sound produced by thunder is 0.2 Wm^{-2}. Calculate the level in decibel.
3. Define echo and reverberation.
4. A concrete hall of volume 2500 m^3 has a total surface absorption of 20 S. Find the reverberation time.
5. External noise creates confusion for the audience in a hall, suggest remedies to reduce effect of external noise.
6. Define reverberation of sound. Give Sabine's equation to find the reverberation time.
7. State the requirements of good acoustics of a building.
8. Define loudness and intensity of sound.
9. Why is acoustical planning of an auditorium necessary ? Which factors affect the acoustical planning of auditorium ?
10. State any four factors affecting acoustical planning of a building. State how they are to be adjusted for good acoustics.
11. Explain any four factors to be considered in acoustical planning of an auditorium.

Unsolved Problems

1. A drama hall of volume 2500 m^3 has an absorption coefficient of 0.125 sabine/m^2. If the surface area of absorption is 1660 m^2, find out the reverberation period.
   (**Ans.** 1.975 sec.)
2. The volume of the hall is 475 cubic meter. The area of the wall is 200 sq. meter; area of the floor and ceiling is 100 sq. meter each. If the absorption coefficients of the wall, ceiling and floor are 0.025, 0.02 and 0.055 respectively, calculate the reverberation time of the hall.
   (**Ans.** 6.23 sec.)
3. For a hall, the total absorbing surface is 1660 m^2 and absorption coefficient is 0.125 MKS unit. Find the volume of the hall if reverberation time is 2 sec.
   (Ans. 2530.5 m^3)
4. Reverberation time of an auditorium is 1.64 sec. Volume is 12×10^4 m^3 and sound absorbing surface is 25000 m^2. Calculate the coefficient of absorption.
   (**Ans.** 0.48 O.W.U.)
5. The volume of a hall is 8000 m^3 and reverberation time is 2 sec. If the absorbing surface of the hall is 6400 m^2, determine the coefficient of absorption. Mention its units.
   (**Ans.** 0.1025 O.W.U.)

MCQs on Acoustics and Sound

1. *The maximum displacement of the oscillating particle in the wave is called ……*
   (a) frequency *(b) period* *(c) amplitude* *(d) wavelength*
2. *The time taken to complete one oscillation in the wave is called ……*
   (a) frequency *(b) period* *(c) amplitude* *(d) wavelength*
3. *The number of oscillations completed in one second is called ……*
   (a) frequency *(b) period* *(c) amplitude* *(d) wavelength*
4. *The distance between two consecutive points in the wave, which are in the same phase, is called ……*
   (a) frequency *(b) period* *(c) amplitude* *(d) wavelength*
5. *The distance between two successive compressions or rarefactions is called as ……*
   (a) frequency *(b) period* *(c) amplitude* *(d) wavelength*
6. *The unit of amplitude is ……*
   (a) meter *(b) hertz* *(c) second* *(d) m/s*
7. *The unit of wavelength is ……*
   (a) meter *(b) hertz* *(c) second* *(d) m/s*
8. *One angstron unit (1 A° or 1 AV) is equal to ……*
   (a) 1×10^{-6} *m* *(b)* 1×10^{-9} *m* *(c)* 1×10^{-10} *m* *(d)* 1×10^{-12} *m*
9. *Out of the following the relation between velocity, frequency and wavelength is ……*
   (a) $v = n + \lambda$ *(b)* $v = n\lambda$ *(c)* $n = v\lambda$ *(d)* $n = v + \lambda$
10. *Out of the following the relation between velocity, period and wavelength is ……*
    (a) $v = \lambda + T$ *(b)* $\lambda = v + T$ *(c)* $\lambda = v/T$ *(d)* $v = \lambda/T$
11. *In transverse wave the direction of vibrations of particles of material medium and direction of propagation of wave are ……*
    (a) perpendicular to each other *(b) parallel to each other*
    (c) opposite to each other *(d) angle of 45° making*
12. *In longitudinal wave the direction of vibration of particles of material medium and direction of propogation of wave are ……*
    (a) perpendicular to each other *(b) parallel to each other*
    (c) making angle of 45° *(d) making angle of 60°*
13. *The material medium through which transverse wave travels needs ……*
    (a) plasticity of shape *(b) elasticity of shape*
    (c) plasticity of volume *(d) elasticity of volume*
14. *The material medium through which longitudinal waves travels needs ……*
    (a) plasticity of shape *(b) elasticity of shape*
    (c) plasticity of volume *(d) elasticity of volume*

15. *When transverse wave travels through material medium ……*
    (a) *density and pressure of medium change*
    (b) *density and pressure of medium change alternatively*
    (c) *density and pressure of medium remain same*
    (d) *pressure is high but density of medium is less*
16. *When longitudinal wave travels through material medium ……*
    (a) *density and pressure of medium change*
    (b) *density and pressure of medium change alternatively*
    (c) *density and pressure of medium remain same*
    (d) *pressure is high but density of medium is less*
17. *In transverse as well as longitudinal waves, every particle of material medium performs S.H.M. of ……*
    (a) *same amplitude, same period, same frequency*
    (b) *same amplitude but different period and frequency*
    (c) *different amplitude but same period and frequency*
    (d) *different amplitude different period, different frequency*
18. *Principle of superposition of waves states, that when two waves travelling through the medium arrive at a point simultaneously ……*
    (a) *each wave produces its own displacement independent of the other*
    (b) *the resultant displacement at that point changes*
    (c) *the resultant displacement at that point is equal to vector sum of displacements due to two waves*
    (d) *all of the above*
19. *A stationary wave or standing wave is the resultant wave produced due to the superposition of two waves with ……*
    (a) *same amplitude, frequency, wavelength, velocity but travelling in opposite direction*
    (b) *same amplitude, frequency, direction but of different wavelength*
    (c) *same frequency, direction, wavelength but different amplitude*
    (d) *same direction, wavelength, amplitude but different frequency*
20. *The points in the stationary waves which have resultant displacement zero, are called ……*
    (a) *nodes* (b) *antinodes* (c) *amplitudes* (d) *beats*
21. *The points in the stationary waves which have resultant displacement maximum are called ……*
    (a) *nodes* (b) *antinodes* (c) *amplitudes* (d) *beats*
22. *The nodes are the points in the stationary waves where resultant displacement is ……*
    (a) *zero* (b) *maximum* (c) *positive* (d) *negative*
23. *Antinodes are the points in the stationary waves where resultant displacement is ……*
    (a) *zero* (b) *maximum* (c) *positive* (d) *negative*
24. *The distance between two successive nodes is ……*
    (a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ
25. *The distance between two successive antinodes is ……*
    (a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ
26. *The distance between two successive nodes and antinodes is ……*
    (a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ
27. *If the amplitude of oscillations continuously goes on decreasing with time, then the oscillations are called ……*
    (a) *intense oscillations* (b) *resonance* (c) *damped oscillations* (d) *stationary waves*
28. *The vibrations of a body, on being slightly disturbed from mean position are called ……*
    (a) *resonating vibrations* (b) *beats* (c) *forced vibrations* (d) *free vibrations*
29. *Out of the following examples, which is not of free vibrations (oscillations) is ……*
    (a) *a freely suspended pendulum oscillating about its mean position*
    (b) *needle of a gramophone player*
    (c) *a metal blade clamped at one end and disturbed*
    (d) *string of musical instrument like guitar*

30. *Out of the following examples, which is not of forced vibrations (oscillations) is ……*
    (a) *oscillating simple pendulum's support itself vibrates*
    (b) *needle of gramophone player*
    (c) *a tuning fork on being stuck on a rubber pad*
    (d) *vibrations of air column made to vibrate using tuning fork*
31. *The phenomenon in which a body vibrates with a large amplitude due to the effect of forced frequency which is equal to natural frequency of the body is called ……*
    (a) *beats* (b) *resonance* (c) *damped oscillations* (d) *acoustics*
32. *Out of the following examples which is not of resonance is ……*
    (a) *tuning of radio receiver* (b) *marching in regular steps of soldiers*
    (c) *exactly tuned two 'tanpuras'* (d) *amplitude of oscillations goes on decreasing*
33. *In the resonance tube experiment, resonance occurs when natural frequency of air column ……*
    (a) *is greater than forced frequency of tuning fork* (b) *is less than forced frequency of tuning fork*
    (c) *becomes zero* (d) *becomes equal to forced frequency of tuning fork*
34. *The velocity of sound using resonance tube experiment can be calculated using formula ……*
    (a) $v = 4n\,(l + 0.3\,D)$ (b) $v = n\,(l + 0.3\,D)$ (c) $v = \frac{4n}{(l + 0.3D)}$ (d) $v = 4n + (l + 0.3\,D)$
35. *A tuning fork of frequency 384 Hz produces sound wave of wavelength 85 cm. The velocity of sound will be ……*
    (a) *393 m/s* (b) *365.5 m/s* (c) *326.4 m/s* (d) *305.5 m/s*
36. *The velocity of wave is 350 m/s. If the frequency of vibration of wave is 175 Hz, the wavelength will be ……*
    (a) *0.5 m* (b) *1 m* (c) *1.5 m* (d) *2 m*
37. *A body produces waves of wavelength 55 cm. If the velocity of propagation is 330 m/s, then the frequency of vibration will be ……*
    (a) *181.5 Hz* (b) *282.5 Hz* (c) *600 Hz* (d) *900 Hz*
38. *The resonating length of air column observed is 19.2 cm and the diameter of the tube is 3 cm. Then the corrected resonating length will be ……*
    (a) *20.1 cm* (b) *25 cm* (c) *28.5 cm* (d) *30.2 cm*
39. *A tuning fork of frequency 512 Hz resonates with an air column of length 15 cm. The end correction is 0.8 cm, the velocity of sound will be ……*
    (a) *28.04 m/s* (b) *302.6 m/s* (c) *323.6 m/s* (d) *346.5 m/s*
40. *A tuning fork of frequency 384 Hz resonates with air column of length 21.5 cm. The diameter of the tube is 2 cm. The velocity of sound in air is ……*
    (a) *280.57 m/s* (b) *339.46 m/s* (c) *375.2 m/s* (d) *382.6 m/s*
41. *A vibrating body sends waves of 100 cm wavelength in air and 500 cm wavelength in water. If velocity of sound in air is 330 m/s, then the velocity of sound in water will be ……*
    (a) *1500 m/s* (b) *600 m/s* (c) *1650 m/s* (d) *2000 m/s*
42. *The resonating length for a tuning fork of frequency 384 Hz is 21.34 cm. The resonating length for frequency 512 Hz will be ……*
    (a) *16 cm* (b) *14 cm* (c) *11.8 cm* (d) *10.6 cm*
43. *The resonating length obtained is 17 cm for frequency 480 Hz. If resonating length obtained for other tuning fork is 22 cm then the frequency of that tuning fork will be ……*
    (a) *312.8 Hz* (b) *320.6 Hz* (c) *340.5 Hz* (d) *370.9 Hz*
44. *The science which deals with the planning of a building or a hall in order to provide the best audible sound to the audience is known as ……*
    (a) *thermodynamics* (b) *kinematics* (c) *acoustics* (d) *kinetics*
45. *Sound energy passing normally at a point through the unit area of the medium in unit time is called ……*
    (a) *loudness of sound* (b) *intensity of sound* (c) *audibility of sound* (d) *range of sound*
46. *Observer's auditory impression of the strength of sound is called as ……*
    (a) *loudness of sound* (b) *intensity of sound* (c) *audibility of sound* (d) *range of sound*

47. *As per Weber and Fechner's law the relation between loudness and sound intensity is given by ……*
    (a) $L \propto \frac{1}{\log I}$ (b) $L \propto I$ (c) $L \propto \log I$ (d) $L \propto I^2$
48. *Out of the following the parameter on which intensity (loudness) of sound does not depend is ……*
    (a) amplitude of vibration (b) distance of observer from source
    (c) density of medium (d) reverberation
49. *Out of the following, the condition (requirement) which is not required for satisfactory acoustics is ……*
    (a) sound should be uniformly distributed throughout the hall
    (b) there should be concentration of sound
    (c) reverberation time should have proper value
    (d) echolon effect should be absent
50. *Out of the following the factor which is not affecting acoustical planning of auditorium is ……*
    (a) echo (b) reverberation (c) external noise (d) cost of the building
51. *Echo is defined as the same sound is heard again after an interval of ……*
    (a) 10 seconds (b) $\frac{1}{10^{th}}$ seconds (c) 20 seconds (d) 30 seconds
52. *Reverberation is ……*
    (a) persistence of sound (b) decreasing sound (c) increasing sound (d) creeping of sound
53. *Sabine's formula (with usual symbol meaning) for reverberation of time is ……*
    (a) $t = 0.164\ V + \sum aS$ (b) $t = \frac{\sum aS}{0.164\ V}$ (c) $t = \frac{0.164\ V}{\sum a \cdot S}$ (d) $t = 0.164\ V - \sum a \cdot S$
54. *An auditorium of volume 6000 m^3 has reverberation time of 2 sec. If the total absorbing surface area in the hall has 5000 m^2, then the coefficient of absorption is ……*
    (a) 0.25 O.W.U. (b) 0.2 O.W.U. (c) 0.098 O.W.U. (d) 0.05 O.W.U.
55. *An interval of 2 seconds was observed between sending a wave to the bottom of sea and return of the reflected wave. If the velocity of sound in the sea water is 2400 m/s, then the depth of the sea will be ……*
    (a) 800 m (b) 1000 m (c) 1500 m (d) 2400 m
56. *Intensity of sound produced by thunder is 0.3 W/m^2. The level of sound loudness in decibel will be ………*
    (a) 85.55 dB (b) 114.77 dB (c) 127.65 dB (d) 132.23 dB
57. *The volume of a hall is 3000 m^3 and its total absorption equals 100 sabine-m^2 of an open window. Entry of the people in the hall raises the total absorption by 2000 sabine-m^2. The change in reverberation time will be ……*
    (a) 4.69 sec (b) 3.27 sec (c) 3.10 sec (d) 2.90 sec
58. *A sound wave is produced when an object ……*
    (a) accelerates (b) rotates (c) vibrates (d) remains stationary
59. *The wavelength of a wave is measured in ……*
    (a) meters (b) hertz (c) seconds (d) decibels
60. *An echo occurs when a sound wave is ……*
    (a) defracted (b) transmitted (c) refracted (d) reflected
61. *The frequency of wave is also measured in ……*
    (a) cycles per second (b) seconds per cycle (c) meters per second (d) seconds/meter
62. *Which of the following does not describe a sound wave ?*
    (a) transverse wave (b) longitudinal wave (c) compression wave (d) push-pull wave
63. *High pressure region of tightly packed molecules is known as ……*
    (a) amplitude (b) compression (c) rarefaction (d) frequency
64. *Low pressure region of loosely packed molecules is known as ……*
    (a) amplitude (b) compression (c) rarefaction (d) frequency
65. *The unit used to measure the frequency of sound is ……*
    (a) decibel (b) ultrasound (c) rarefaction (d) hertz
66. *When a wound can be heard, it is called ?*
    (a) decibel (b) ultrasound (c) audible (d) rarefaction
67. *The speed of sound in air is about ……*
    (a) 100 m/s (b) 300 m/s (c) 1000 m/s (d) 3×10^8 m/s

68. *If the level of sound in the room is said to be 70 dB this means that the sound power intensity is ……*
(a) 7.0×10^{-12} W/m^2 (b) 1.0×10^{-7} W/m^2 (c) 1.0×10^{-5} W/m^2 (d) 70×10^{-12} W/m^2

69. *If the level of sound in the room increases to 76 dB (from 70 dB) the sound power intensity has gone up ……*
(a) only a fraction (b) double (c) quadruple (d) six times

Choose the alternative that best completes the statement or answers the question.

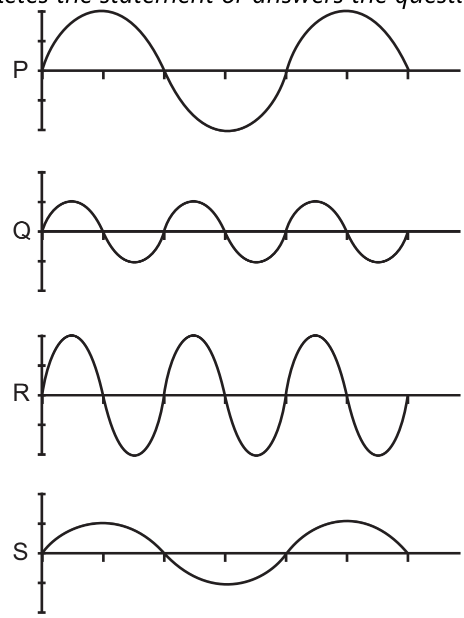


Fig. 7.9

70. *Refer above figure, compared to wave P, which wave has the same wavelength but a smaller amplitude ?*
(a) wave P (b) wave Q (c) wave R (d) wave S

71. *Refer above figure, compared to wave P, which wave has a shorter wavelength but the same amplitude ?*
(a) wave P (b) wave Q (c) wave R (d) wave S

72. *Refer above figure, which wave has the same frequency as wave P ?*
(a) wave P (b) wave Q (c) wave R (d) wave S

73. *Sound is an example of ……*
(a) a longitudinal wave (b) a wave that can travel through a vacuum
(c) a transverse wave (d) a light wave

74. *Sound waves cannot travel in ……*
(a) air (b) water (c) aluminium (d) vacuum

75. *Any vibrating body produces ……*
(a) light (b) sound (c) laser (d) electromagnetic waves

76. *The loudness of a musical sound is a measure of sound wave's ……*
(a) wavelength (b) frequency (c) speed (d) amplitude

77. *The vibrations of particles in a material medium in transverse wave are ……*
(a) in the same direction as the wave travels (b) at right angles to the direction of wave travel
(c) parallel to the moving wave (d) circular direction of the wave travel

78. *Compressions and rarefactions are characteristics of ……*
(a) longitudinal waves (b) transverse waves (c) both of these (d) stationary waves

79. *In a longitudinal wave, compressions and rarefactions travel in ……*
(a) the same direction that the wave travels (b) a direction opposite to the wave travel
(c) perpendicular direction that wave travels (d) up and down that wave travels

80. *The amplitude of a particular wave is 0.5 m. The top to bottom (crest to trough) distance of the wave is ……*
(a) 0.5 m (b) 1 m (c) 2 m (d) 3 m

81. *The distance between two consecutive crests of a wave is called the ……*
   (a) *amplitude* (b) *frequency* (c) *wave speed* (d) *wavelength*
82. *As the period of a wave decreases ……*
   (a) *the amplitude of the wave decreases* (b) *the amplitude of the wave increases*
   (c) *the frequency of the wave decreases* (d) *the frequency of the wave increases*
83. *In damped harmonic motion, the …… decreases.*
   (a) *amplitude* (b) *frequency* (c) *period* (d) *energy*
84. *The wave in which the displacement of vibrating particles is perpendicular to the direction of propagation of the wave, is called ……*
   (a) *stationary wave* (b) *transverse wave* (c) *longitudinal wave* (d) *Doppler wave*

Answers and Hints : Acoustics, Sound

1. (c)	2. (b)	3. (a)	4. (d)	5. (d)	6. (a)	7. (a)	8. (c)	9. (b)	10. (d)
11. (a)	12. (b)	13. (b)	14. (d)	15. (c)	16. (a)	17. (a)	18. (d)	19. (a)	20. (a)
21. (b)	22. (a)	23. (b)	24. (b)	25. (b)	26. (a)	27. (c)	28. (d)	29. (b)	30. (c)
31. (b)	32. (d)	33. (d)	34. (a)	35. (c)	36. (d)	37. (c)	38. (a)	39. (c)	40. (b)
41. (c)	42. (a)	43. (d)	44. (c)	45. (b)	46. (a)	47. (c)	48. (d)	49. (b)	50. (d)
51. (b)	52. (a)	53. (c)	54. (c)	55. (d)	56. (b)	57. (a)	58. (c)	59. (a)	60. (d)
61. (a)	62. (a)	63. (b)	64. (c)	65. (d)	66. (c)	67. (b)	68. (c)	69. (a)	70. (d)
71. (c)	72. (d)	73. (a)	74. (d)	75. (b)	76. (d)	77. (b)	78. (a)	79. (a)	80. (b)
81. (d)	82. (d)	83. (a)	84. (b)						

35. **Hint :** $v = n\lambda = 384 \times 0.85 = 326.4$
36. **Hint :** $v = n\lambda \quad \therefore \quad \lambda = \frac{v}{n} = \frac{350}{175} = 2$ m
37. **Hint :** $v = n\lambda \quad \therefore \quad n = \frac{v}{\lambda} = \frac{330}{0.55} = 600$ Hz
38. **Hint :** $L = l + 0.3\ D = 19.2 + 0.3 \times 3 = 20.1$ cm
39. **Hint :** $v = 4n\ (l + e) = 4 \times 512 \times (15 + 0.8) = 32358.4$ cm/s = 323.6 m/s
40. **Hint :** $v = 4n\ (l + 0.3\ D) = 4 \times 384 \times (21.5 + 0.3 \times 2) = 33946$ cm/s = 339.46 cm/s
41. **Hint :** $\frac{V_w}{V_a} = \frac{\lambda_w}{\lambda_a} \quad \therefore \quad V_w = \frac{\lambda_w}{\lambda_a} \times V_a = \frac{500}{100} \times 330 = 1650$ m/s
42. **Hint :** $n \propto \frac{1}{l} \quad \therefore \quad n_1 l_1 = n_2 l_2 \quad \therefore \quad 384 \times 21.34 = 512 \times l_2. \quad l_2 =$ **16 cm**
43. **Hint :** $n_1 l_1 = n_2 l_2 \quad \therefore \quad 480 \times 17 = n_2 \times 22$
54. **Hint :** $t = \frac{0.164\ V}{aS} \quad \therefore \quad a = \frac{0.164\ V}{tS} = \frac{0.164 \times 6000}{2 \times 5000} \quad \therefore \quad a =$ **0.098**
55. **Hint :** $v = \frac{\text{Total distance}}{\text{Time}} = \frac{2h}{t} \quad \therefore \quad h = \frac{vt}{2} = \frac{2400 \times 2}{2} =$ **2400 m**
56. **Hint :** $I_1 = 0.3\ W/m^2, \quad I_2 = 10^{-2}\ W/m^2,$ Intensity level $= 10 \log \frac{I_1}{I_2} = 10 \log_{10} \left(\frac{0.3}{10^{-12}}\right) =$ **114.77 dB**
57. **Hint :** $t_1 = \frac{0.164}{A_1} = \frac{0.164 \times 3000}{100} =$ **4.92 sec**
   After entry of people, $A_2 = A_1 + 2000 = 100 + 2000 = 2100$
   $t_2 = \frac{0.164 \times V}{A_2} = \frac{0.164 \times 3000}{2100} = 0.234$ sec , $t = t_1 - t_2 = 4.92 - 0.234 =$ **4.69 sec**
68. **Hint :** $70 = 10 \log_{10} \left(\frac{I}{10^{-12}}\right) \quad \therefore \quad 7 = \log \left(\frac{I}{10^{-12}}\right) \quad \therefore$ Antilog (7) $= \frac{I}{10^{-12}}$, $10^7 = \frac{I}{10^{-12}} \quad \therefore$ $\mathbf{10^{-5} = 1}$

❐❐❐

EXPERIMENTS

1. TO KNOW YOUR PHYSICS LABORATORY

Aim : *To know about various instruments, least count of the instruments, range of instruments, interpretation of graph.*

1. How to read the reading using given instrument.

This is very important to know how to read readings.

There are two types of instruments :

(A) Instruments with only one scale. (e.g. simple scale, voltmeter, ammeter, thermometer).

(B) Instruments with two scales, one as a main scale and other supporting (e.g. vernier calliper, micrometer screw, spherometer, travelling microscope etc.).

(A) Simple instruments with single scale :

Example 1 :

Meter scale or scale in our compass box.

Purpose : To measure length, breadth.

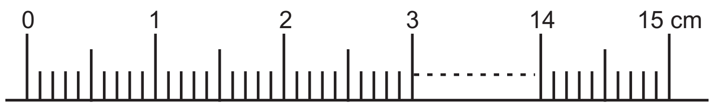


Fig. 1.1

Least count of the instrument : It is the smallest possible reading that can be measured accurately using given instrument.

In the above diagram in between 0 cm and 1 cm there are 10 divisions. It simply means that 1 cm is equally divided into 10 parts.

$\therefore \quad \frac{1 \text{ cm}}{10} = 0.1 \text{ cm} \rightarrow$ L.C. of the scale.

Range of the instrument : Maximum reading that can be measured (one time) accurately.

Referring above example.

Range of compass scale $\rightarrow$ 0 – 15 cm.

Range of foot scale $\rightarrow$ 0 – 30 cm.

Range of meter scale $\rightarrow$ 0 – 100 cm.

Example 2 : Thermometer :

Purpose : To measure temperature of liquid, body.

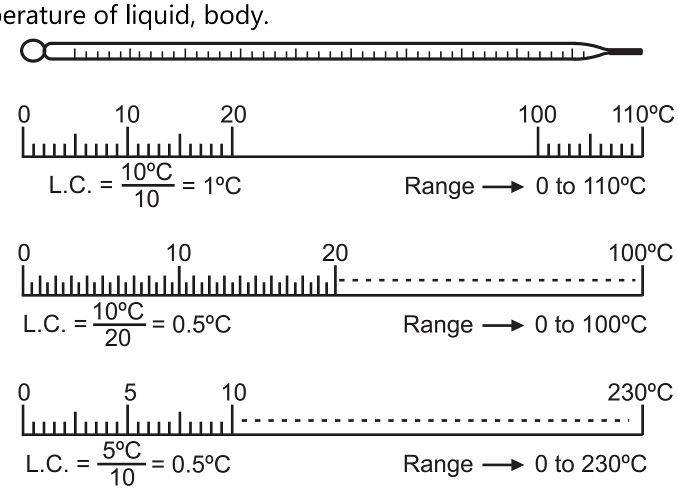


Fig. 1.2

Example 3 : Ammeter :

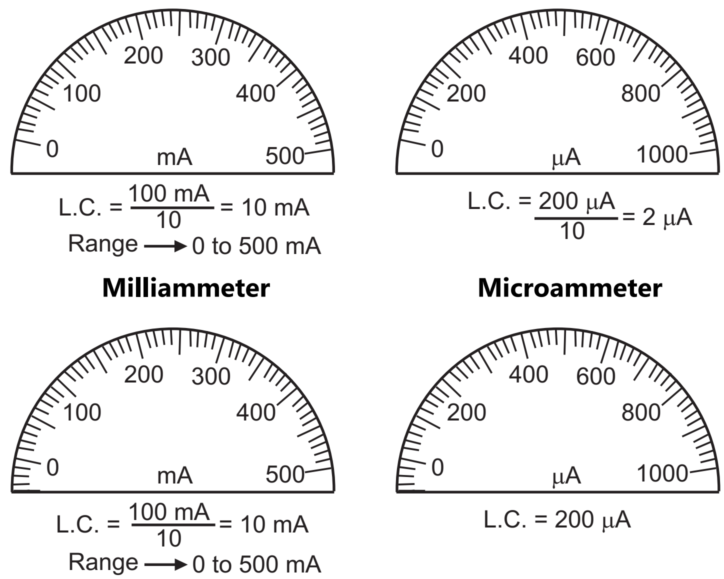


Fig. 1.4

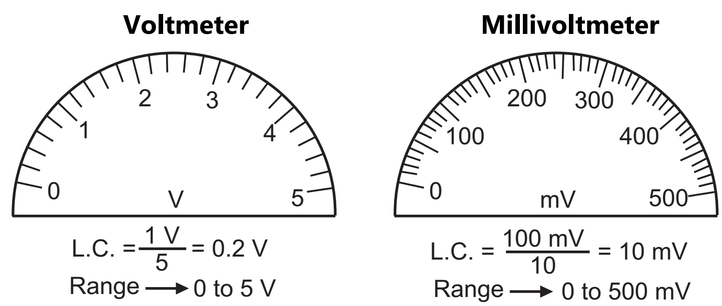


Fig. 1.5

Stop clock :

Purpose : To measure time slab.

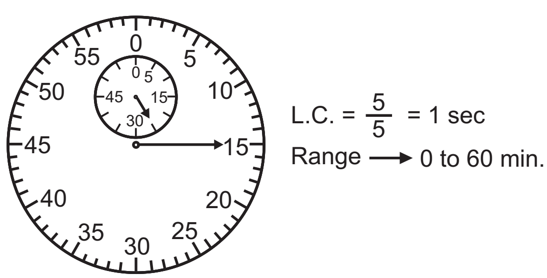


Fig. 1.6

(B) Instruments with two scales (one as a main scale and other as a supporting scale) :

For example : Vernier calliper, micrometer screw gauge, travelling microscope, spherometer.

In such scales, we have to use in general simple formula as below :

$$\text{L.C.} = \frac{\text{Smallest divisions on main scale}}{\text{Total number of divisions on the supporting scale}}$$

$$\text{Total reading} = \text{Completed main scale reading} + \left(\frac{\text{Coinciding division}}{\text{number of supporting scale}} \times \text{L.C.}\right)$$

Example 1 : Vernier calliper :

Purpose : To measure dimensions of given object accurately.

Important feature of vernier calliper is in addition to outer diameter we can measure inner diameter as well as depth of tube.

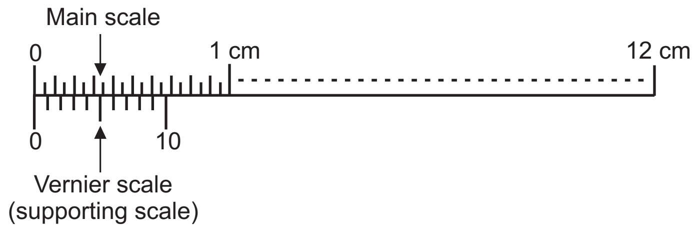


Fig. 1.7

$$\text{Now here L.C.} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on vernier scale}} = \frac{1\text{ cm}/20}{10} = \frac{0.05}{10}$$

$$\boxed{\text{L.C.} = 0.005\text{ cm}}$$

Range $\rightarrow$ **0 – 12 cm**

Note : There are different types of vernier callipers (i.e. having different L.C. and range) available in the market.

Example 2 : Micrometer screw gauge :

Purpose : To measure dimensions of small objects with more accuracy.

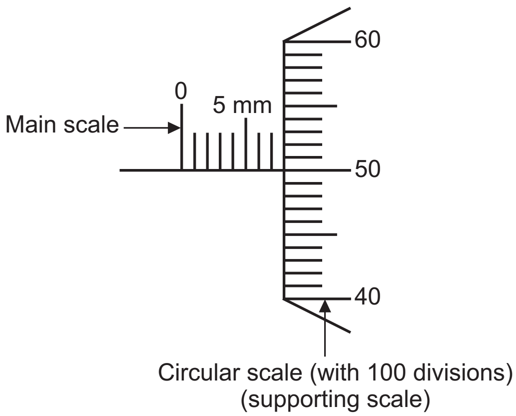


Fig. 1.8

$$\text{L.C.} = \frac{\text{Pitch of screw}}{\text{Total number of divisions on circular scale}} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}}$$

$$= \frac{0.1\text{ cm}}{100} \quad \boxed{\therefore\ \text{L.C.} = 0.001\text{ cm}}$$

Range $\rightarrow$ 0 – 2.5 cm (In general)

Example 3 : Spherometer :

Purpose : To measure radius of curvature using part of curved sphere.

It works on the principle of micrometer screw gauge.

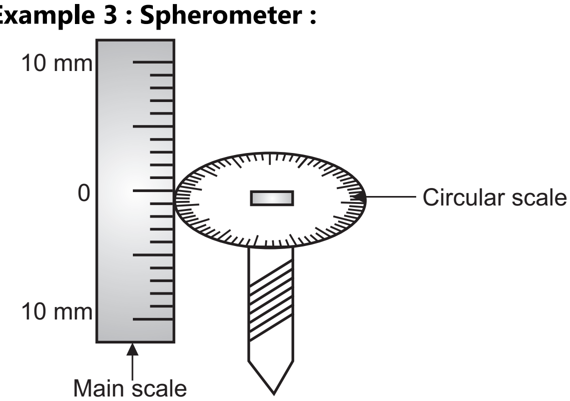


Fig. 1.9

$$\text{L.C.} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}}$$

$$= \frac{0.1\text{ cm}}{100} = 0.001 \quad \boxed{\text{L.C.} = 0.001\text{ cm}}$$

Example 4 : Travelling microscope : It consists of microscope with cross wires inside it, this microscope can travels up-down, left-right. The extend by which it travel can be measured using either horizontal scale or vertical scale.

Purpose : • To measure diameter (inner as well as outer) of small capillary.

- To measure rise of liquid in capillary.
- To measure dimensions of minute objects.

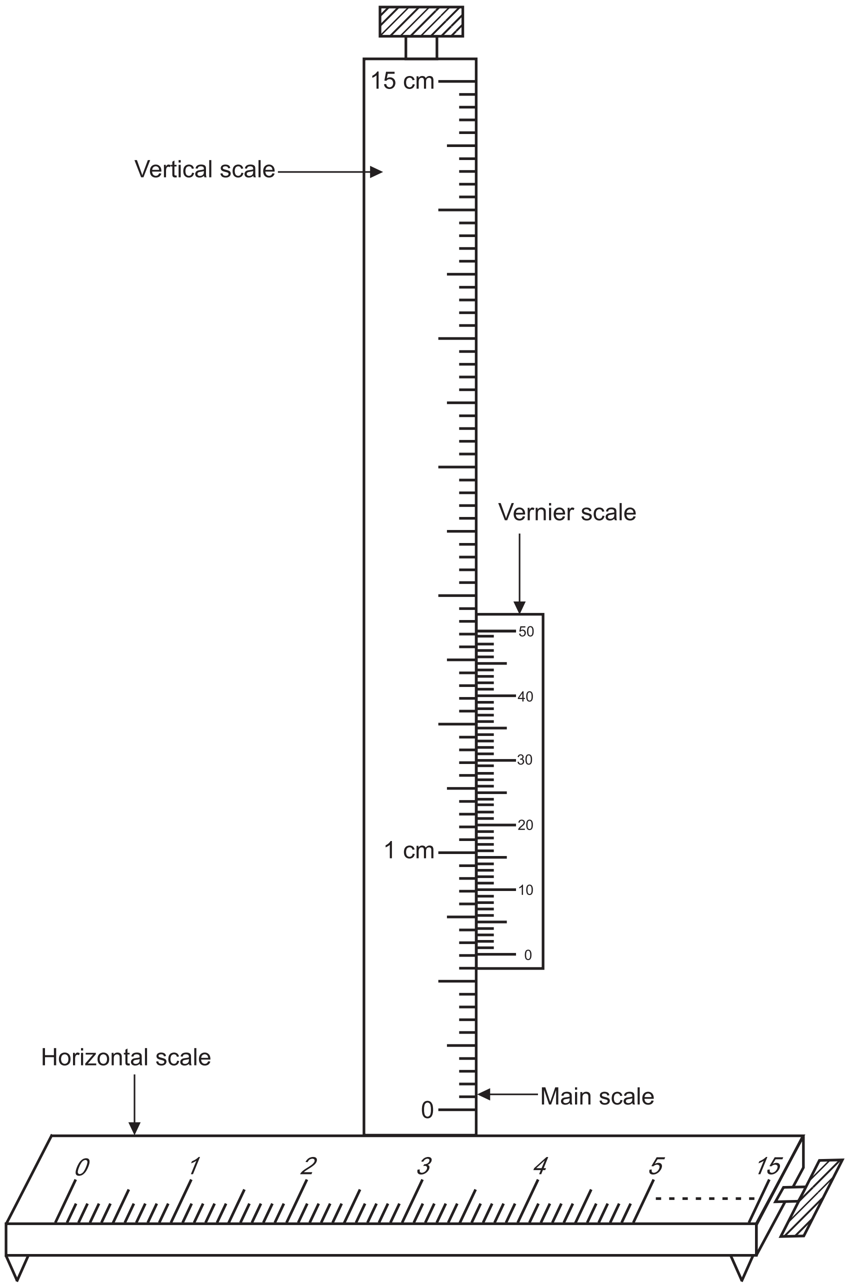


Fig. 1.10

$$\text{L.C.} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on vernier scale}} = \frac{1\text{ cm}/20}{50} = \frac{0.05}{50} = \mathbf{0.001\text{ cm}}$$

Range $\rightarrow$ 0 – 15 cm

Observation Table :

Student should read different instruments available in the laboratory and complete the following table.

Sr. No.	Name of the instrument	L.C.	Range	Purpose (Use)
1.	Large scale (meter scale)	–	–	–
2.	Vernier	–	–	–
3.	Micrometer	–	–	–
4.	Spherometer	–	–	–
5.	Travelling microscope	–	–	–
6.	Thermometer 1	–	–	–
7.	Thermometer 2	–	–	–
8.	Stop clock	–	–	–
9.	Ammeter	–	–	–
10.	Milliammeter	–	–	–
11.	Microammeter	–	–	–
12.	Voltmeter	–	–	–
13.	Millivoltmeter		–	–
14.	D.C. power supply	–	–	–

2. CALCULATOR FUNCTIONS

Practice simple calculations, the answers of which are known to us like,

(1) $2 + 3$	(2) $2 \times 3 =$	(3) $2^3 =$	(4) $2^2 =$
(5) $\frac{(2+2)}{(2+2)} =$	(6) $\frac{2}{3} =$	(7) $10^{-2} =$	(8) $10^{-3} =$
(9) $2 \times 10^3 =$	(10) $5^3 =$	(11) $\sqrt{4} =$	(12) $\sqrt[3]{27} =$
(13) $\sin 30 =$	(14) $\sin 90 =$	(15) $\cos 90 =$	(16) $\cos 0 =$
(17) $\tan 45 =$	(18) $\sin^{-1} 1$	(19) $\sin^{-1} 0 =$	(20) $\cos^{-1} 1 =$
(21) $\cos^{-1} 0 =$	(22) $\tan^{-1} 1$		

Very Important :

While calculating : To avoid mistake put all the values which are at numerator in one bracket and the values which are at denominator into another bracket (i.e. use parenthesis wherever required).

For example :

$$Y = \frac{Mg}{\pi r^2} \times \frac{L}{l} \quad \begin{matrix} \rightarrow \text{Numerator} \\ \rightarrow \text{Denominator} \end{matrix}$$

$$= \frac{(2 \times 9.81 \times 1.5)}{(3.142 \times (0.002)^2 \times 0.3)}$$

OR In the beginning (learner), calculate numerator separately and denominator separately and then divide numerator by denominator.

Simple example : How MISTAQUE occurs when bracket (parenthesis) is not used ?

Examples :

1. $\frac{2+2}{2}$ and if we do not use bracket then answer will get 3 which is wrong. Now do it like $\frac{(2+2)}{2}$.

2. $\frac{2+2}{2+2}$ and if we do not use bracket then we get wrong answer i.e. 5 instead of 1. Now perform it like $\frac{(2+2)}{(2+2)} = 1$.

3. Some important prefixes required for conversion (commonly used)

Prefix : Which comes earlier (before) e.g. km → kilometer → her kilo comes earlier (before) meter then **kilo** is called **prefix**.

Prefix	Symbol	Power of 10 i.e. 10^n
1. tera	T	10^{12}
2. giga	G	10^{9}
3. mega	M	10^{6}
4. kilo	k	10^{3}
5. hecto	h	10^{2}
6. deca	da	10^{1}
7. deci	d	10^{-1}
8. centi	c	10^{-2}
9. milli	m	10^{-3}
10. micro	μ	10^{-6}
11. nano	n	10^{-9}
12. pico	p	10^{-12}
13. 1 AU = 1 A° = 1 Angstron unit = 1×10^{-10} m		
14. 1 eV = $1 \times 1.6 \times 10^{-19}$ J		
15. 1 min = 60 sec		
16. 1 hr = 60 min = 60×60 sec		

How to convert :

(1) 1 kW = …… W, here kilo is the prefix → kilo means 10^3

$$\therefore 1 \text{ kW} = 1 \times 10^3 \text{ W}$$

(2) 1 mm = …… m → here milli is the prefix → milli means → 10^{-3}

$$\therefore 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$$

(3) Now 1 m = …… mm. This is reverse procedure of example (2).

We have 1 mm = 1×10^{-3} m

$\therefore$? = 1 m

$$? = \frac{1 \times 1}{10^{-3}} = 10^3$$

Thus $\boxed{1 \text{ m} = 1 \times 10^3 \text{ m}}$

(4) 1 micrometer = $\boxed{1\ \mu\text{m} = 1 \times 10^{-6} \text{ m}}$ $\boxed{\therefore 1 \text{ m} = 1 \times 10^{6}\ \mu\text{m}}$

Observation table : Convert the following :

Sr. No.	Convert	Sr. No.	Convert
1.	1 MW = …… W	13.	1 AU = …… m
2.	1 W = …… MW	14.	1 m = …… AU
3.	1 kg = …… gm	15.	1 eV = …… J
4.	1 gm = …… kg	16.	1 J = …… eV
5.	1 mm = …… m	17.	1 cm^2 = …… m^2
6.	1 m = …… mm	18.	1 m^2 = …… cm^2
7.	1 cm = …… m	19.	1 cm^3 = …… m^3
8.	1 m = …… cm	20.	1 m = …… cm^3
9.	1 nm = …… m	21.	1 gm/cm^3 = …… kg/m^3
10.	1 m = …… nm	22.	1 kg/m^3 = …… gm/cm^3
11.	1 μA = …… A	23.	1 km/hr = …… m/s
12.	1 A = …… μA		

4. How to plot a graph

1. Draw vertical line on left most side as a Y-axis.
2. Draw horizontal line at bottom most side as a X-axis.
3. As far as possible take dependent variable on Y-axis and independent variable on X-axis.
4. Observe lowest and highest readings to be plotted.
5. Now choose a suitable scale so that the plotted graph will spread on the entire paper.
6. Whenever law is to be verified take (0, 0) as a origin.
7. X-axis readings should increase from left to right with constant difference.
8. Y-axis readings should increase from bottom to top with constant difference.

Nature of graph and interpretation :

1. If graph is a straight line inclined to X-axis then, Y-axis quantity is directly proportional to X-axis quantity.

   OR

   If Y-axis quantity is directly proportional to X-axis quantity, then graph is a straight line.
2. For straight line graph, slope gives Y/X.

For example : If graph is I versus V thus slope gives I/V. Interpretation using nature of different graphs :

1. Ohm's law :

$$I \propto V$$

Graph is a straight line

$$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{I}{V}$$

But $$\frac{V}{I} = R$$

$$\therefore \quad \frac{1}{\text{Slope}} = R$$

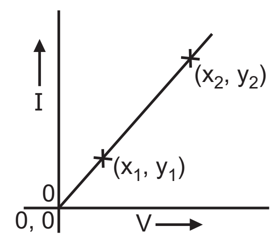


Fig. 1.11

2. Angle of deviation versus angle of incidence :

As angle of incidence increases, angle of deviation decreases, reaches certain minimum value and again increases.

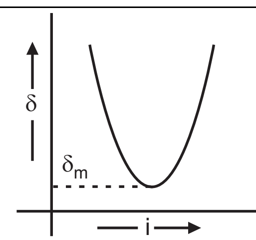


Fig. 1.12

3. Resonance :

To determine velocity of sound by resonance tube,

$$v = 4nL$$

$$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{n}{1/L} = nL$$

We have, $$v = 4 \times nL$$

$$\therefore \quad v = 4 \times \text{Slope}$$

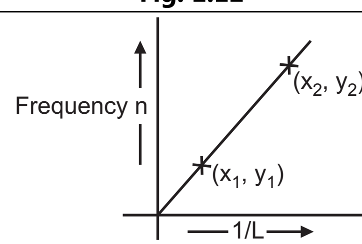


Fig. 1.13

4. Graph of stress versus strain :

Young's modulus of elasticity,

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

Slope of graph,

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \text{Slope}$$

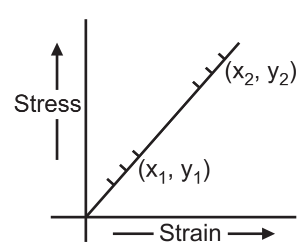


Fig. 1.14

5. Newton's cooling curve temperature versus time :

This interprets that initially temperature fall is speedy and then it is very slow.

i.e. Rate of cooling goes on decreasing as time increases.

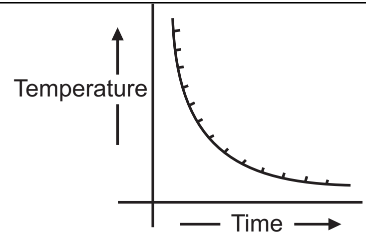


Fig. 1.15

Observation Table :

To plot graph of following readings, different values of current at different potential difference are given as follows :

Obs. No.	V volts	I amp
1.	2	0.3
2.	4	0.6
3.	6	0.9
4.	8	1.2
5.	10	1.3
6.	12	1.5

2. TO USE VERNIER CALLIPERS FOR MEASUREMENT OF DIMENSIONS OF GIVEN OBJECT

Aim : *To measure inner diameter, outer diameter and height of hollow cylinder (pipe).*

Apparatus : (1) Vernier calliper, (2) Object – hollow cylinder

Some required concepts :

Least count (L.C.) : L.C. of a given instrument is a smallest possible reading that can be measured accurately using given instrument. e.g. L.C. of normal compass scale is 0.1 cm i.e. 1 mm.

L.C. of regular wrist watch is 1 sec.

"Less the least count of instrument, more is the accuracy".

Least counter (L.C.) of vernier calliper :

$$\text{L.C. of vernier} = \frac{\text{Smallest division on main scale (m)}}{\text{Total number of divisions on vernier scale (n)}}$$

Diagram : Vernier calliper

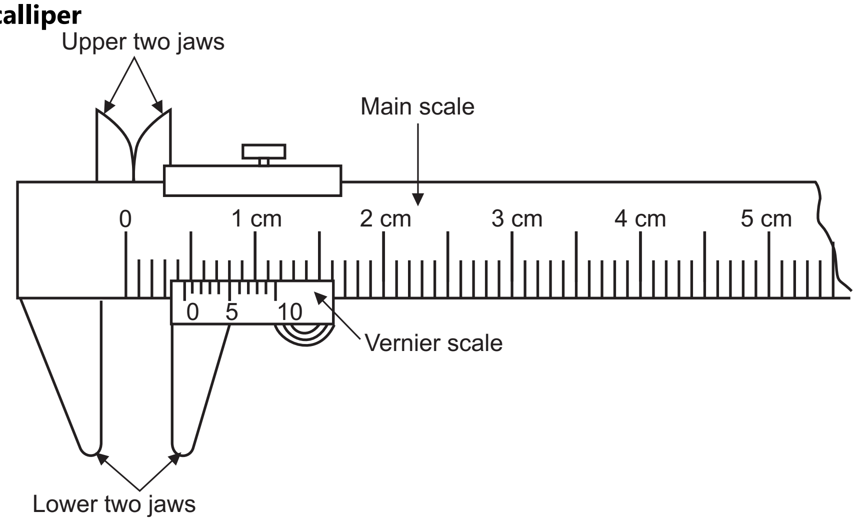


Fig. 2.1

[I] Determination of L.C. :

$$\text{L.C. of vernier shown in above figure} = \frac{\text{Smallest division on main scale (m)}}{\text{Total number of divisions on vernier scale (n)}} = \frac{0.1 \text{ cm}}{10}$$

L.C. of vernier = 0.01 cm

Concept of zero error : Concept understanding.

Note : Dear students zero error is the name of instrumental error, it does not mean that error is zero.

Now consider example of wrist watch.

Possibility 1 : Wrist watch showing exact time i.e. no zero error ∴ no correction required.

Possibility 2 : e.g. Wrist watch lagging behind by 2 minutes i.e. negative error ∴ correction is positive. e.g. add 2 minutes in the time shown by watch.

Possibility 3 : e.g. wrist watch leading by 3 minutes. i.e. positive zero error ∴ correction is negative i.e. 3 minutes are subtracted from time shown by watch.

For example :

1. If no zero error → no correction. 2. If negative zero error → correction is positive. 3. If positive zero error → correction is negative.

[II] Determination of zero error of instrument :

Dear students as we have seen, zero error is the name of the instrumental error but it does not mean that error is zero.

Hold and adjust the vernier calliper so that two jaws are in contact (touch) with each other and note down the zero error if any as shown below.

(a) Possibility 1 : When the two jaws are in contact with each other and if zero of vernier scale coincide with zero of main scale, then instrument is said to have no zero error (accurate)

correction → no correction required.

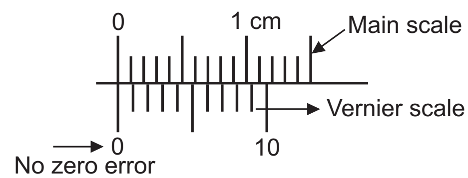


Fig. 2.2

(b) Possibility 2 : When the two jaws are in contact with each other and if zero of vernier scale lies left hand side (earlier) of main scale zero, then error is said to be negative.

If error is negative → correction is positive.

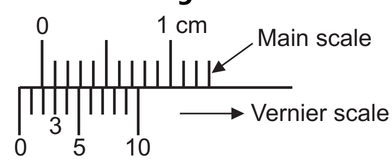


Fig. 2.3

In the above diagram third division of vernier is matching with one of the division from main scale.

∴ Error = $3 \times$ L.C. = 3×0.01 = **0.03 cm**

Here error is negative. ∴ Correction is positive. i.e. add 0.03 cm in every (total) reading.

(c) Possibility 3 : When the two jaws are in contact and if zero of vernier scale lies right hand side (after) zero of main scale then error is said to be positive.

If error is positive → correction is negative.

In the above diagram sixth division of vernier is matching with one of the division from the main scale.

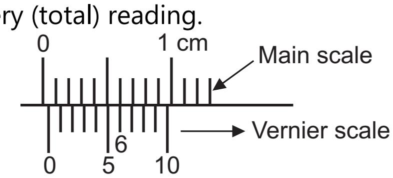


Fig. 2.4

∴ Error = 6×0.01 = 0**.06 cm**

Here error is positive. Therefore correction is negative i.e. subtract 0.06 cm from every (total) reading.

[III] Process of recording readings using vernier calliper :

Hold the job between two jaws as shown in the figure.

Sample Reading :

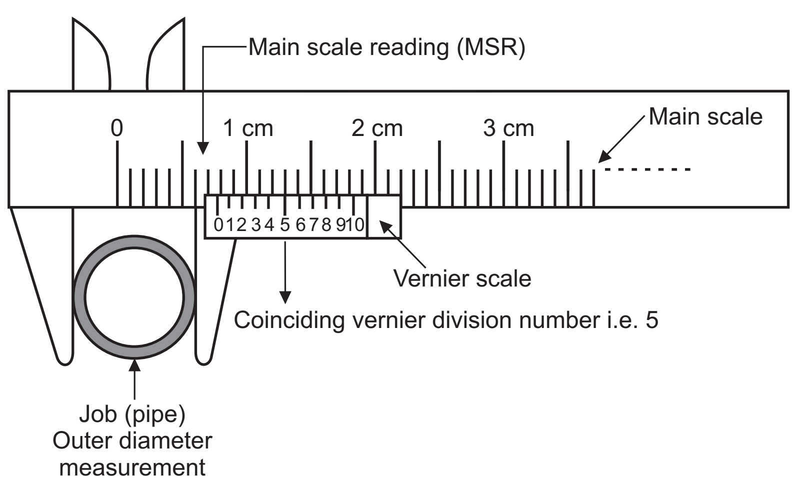


Fig. 2.5

Major steps for recording total reading : Watch above diagram carefully.

1. Record main scale reading (MSR). 2. Record coinciding vernier division number (VSD).
3. Calculate and record total reading.

(I) How to record main scale reading (M.S.R.) : Focus on zero of vernier scale and see the corresponding completed reading by it on main scale. In the above diagram zero of vernier is lying between 1.2 cm and 1.3 cm of main scale. i.e. completed reading is $\boxed{X = 1.2 \text{ cm}}$.

(II) How to record coinciding vernier division number : Go on watching vernier divisions from 0 to 10 carefully.

You will observe that lower venier scale divisions and upper main scale divisions they are mismatching. But out of 10 divisions of vernier any one division will coincide with one of the divisions from the main scale.

In the above diagram '5' of vernier is coinciding (matching) with one of the main scale division.

i.e. coinciding vernier division number $\boxed{n = 5}$.

(III) How to write total (complete) reading : Multiply coinciding vernier division number by L.C. and add it to M.S.R. (main scale reading).

Thus, sample reading will be as below :

Object	Dimension	Main scale reading (MSR)	Coinciding vernier scale division (VSD) number	VSR = VSD × LC Y = n × LC	Total reading T.R. MSR + VSR Z = X + Y
		X	n	Y	Z
Pipe	Outer diameter	1.2 cm	5	0.05	1.25 cm

Procedure :

1. Calculate least counter (L.C.) of the given vernier calliper as explained above.
2. Calculate zero error of the instrument as explained above.
3. Hold the object in the jaws of vernier caliper. (Note : To measure inner diameter, upper jaws should be used).
4. Measure completed main scale reading MSR as shown above and record it.
5. Observe and record coinciding vernier division number 'n' and VSR = n × LC.
6. Calculate total reading using formula M.S.R. + V.S.R.
7. Calculate corrected reading by adding zero error if error is negative and by subtracting zero error if error is positive.
8. Take average of three readings.

Observations :

1. Smallest division on main scale = m = …… cm (e.g. 0.1 cm).
2. Total number of divisions on vernier scale = n = …… (e.g. 10).
3. Least count of vernier = L.C. = $\frac{m}{n}$ = …… cm (e.g. 0.01 cm).
4. Zero error = (L.C. × coinciding vernier division number when two jaws are in contact) = Z = …… cm.
5. Correction → Add 'Z' if error is negative or subtract 'Z' if error is positive.

Observation table :

Dimension	Reading No.	M.S.R. 'X' cm	Coinciding VSD 'n'	VSR = 'Y' VSD × LC = n × LC 'Y' cm	Total reading TR = MSR + VSR = X + Y	Corrected reading TR + Z	Average cm
Outer diameter	1.						
	2.						
	3.						
Inner diameter	1.						
	2.						
	3.						
Height h	1.						
	2.						
	3.						

Results :

1. Average outer diameter of hollow pipe = …… cm.
2. Average inner diameter of hollow pipe = ……… cm.
3. Average height of hollow cylinder = ……… cm.

3. TO USE MICROMETER SCREW GAUGE FOR THE MEASUREMENT OF DIMENSIONS (DIAMETER, THICKNESS)

Aim : *To measure diameter of metal sphere, diameter of metal wire and thickness of metal plate.*

Apparatus : 1. Micrometer screw gauge. 2. Metal sphere, metal wire, metal plate.

Some required concepts :

Least count of a given instrument. As we have seen earlier L.C. of given instrument is a smallest possible reading that can be measured accurately using given instrument.

e.g. L.C. of normal compass scale is 0.1 cm i.e. 1 mm. L.C. of regular wrist watch is 1 sec.

"Less the least count of the instrument more is the accuracy".

Least count of micrometer screw gauge :

$$\text{L.C. of micrometer} = \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}} = \frac{\text{Smallest division on main scale}}{\text{Total number of divisons on circular scale}}$$

Pitch : Pitch of the screw is the distance between two consecutive threads of the screw.

In other words, it is the distance covered by main scale when one complete rotation is given.

Normally pitch is equal to smallest possible division on the main scale.

Diagram : Micrometer screw gauge :

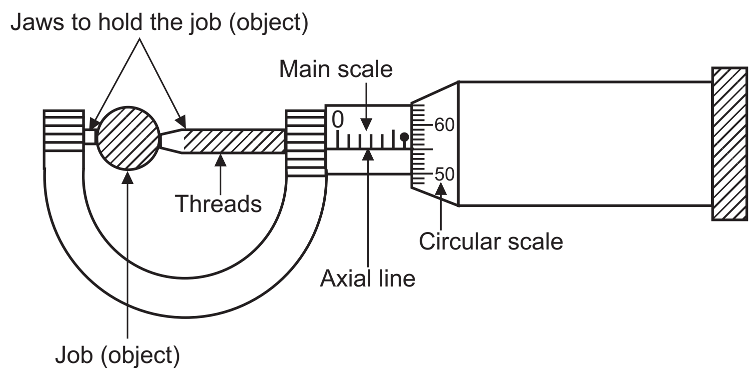


Fig. 3.1

(I) Determination of least count (L.C.) :

$$\text{L.C. of micrometer shown in above figure} = \frac{\text{Pitch of the screw}}{\text{Total number of divisions on circular scale}}$$

$$= \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}} = \frac{0.1}{100}$$

$$\boxed{\text{L.C.} = 0.001 \text{ cm}}$$

Concept of zero error :

As explained earlier, zero error is the name of instrumental error, it does not mean that error is zero.

(II) Determination of zero error :

Hold and adjust the micrometer screw so that the two jaws are in contact (touch) with each other and note down the zero error if any as shown below.

(a) Possibility 1 : No zero error : When the two jaws are in contact with each other and if zero of circular scale coincide with axial line, then instrument is said to have no zero error (i.e. instrument is accurate).

Correction → no correction required.

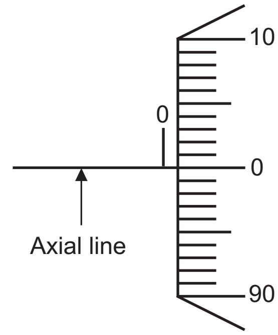


Fig. 3.2

(b) Possibility 2 : Negative error : When the two jaws are in contact with each other and if zero of circular scale lies above axial line, then error is negative.

Correction when error is positive correction is negative.

In the adjacent diagram, zero of circular scale is shifted by 6 divisions above axial.

∴ Error = 6 × L.C. × 6 × 0.001 = 0.006 cm

Here error is negative therefore correction is positive i.e. add 0.006 cm in every total reading.

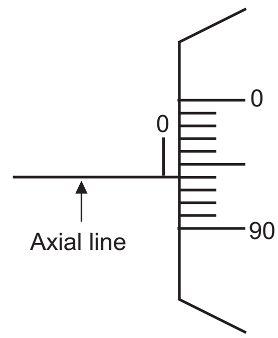


Fig. 3.3

(c) Possibility 3 : Positive error : When the two jaws are in contact and if zero of circular scale lies below axial line then error is said to be negative.

If error is positive correction is negative. In the above figure, zero of circular sale is shifted by 3 divisions below axial line.

∴ Error = 3 × L.C. = 3 × 0.001 = 0.003 cm

Here error is positive therefore correction is negative. i.e. subtract 0.003 cm from every total reading.

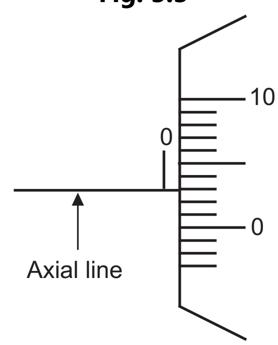


Fig. 3.4

(III) Process of recording readings using micrometer screw gauge :

Hold the job between two jaws as shown in the figure.

Simple Reading :

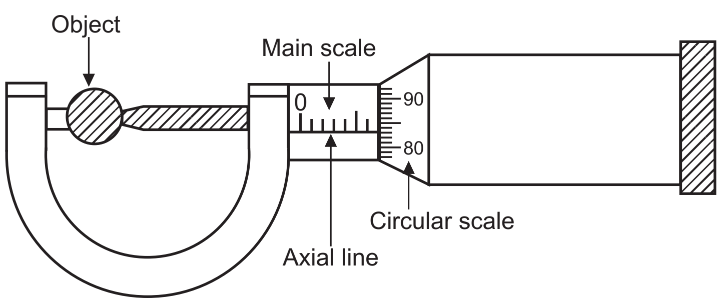


Fig. 3.5

Major steps for recording total readings :

1. Record main scale reading (M.S.R.).
2. Record coinciding circular division number (C.S.D.).
3. Calculate and record total reading (T.R.). Watch above diagram carefully.

(A) How to record main scale reading (M.S.R.) ('X') :

Focus on main scale, see the completed main scale reading which we can see. In the above diagram, we can see 0.6 cm (not more than that) i.e. completed main scale reading is 0.6 cm. X = 0.6 cm

(B) How to record coinciding circular division number (C.S.D.) ('n') :

Go on watching circular scale divisions, one division will coincide with axial line.

In the above diagram 83rd division is coinciding with axial line. i.e. n = 83.

(C) How to write total reading (T.R.) :

Multiply coinciding circular division number by L.C. and add it to M.S.R. (main scale reading).

Thus sample reading will be as below :

Object	Dimension	Main scale reading M.S.R.	Coinciding circular scale division C.S.D.	CSR = CSD × LC	TR = MSR + CSR	Corrected reading	Average
		'X' cm	n	'Y' cm	TR = X + Y cm	TR ± Z	
Metal sphere	Diameter	0.6	83	0.083	0.683	…	…

Procedure :

1. Calculate the least count (L.C.) of micrometer screw gauge as explained above.
2. Calculate zero error if any (as explained).
3. Hold the object in the jaws of micrometer.
4. Measure completed main scale reading (MSR) ('X').
5. Observe and record coinciding circular scale division number (CSD) ('n') and record CSR = n × L.C.
6. Calculate total reading using formula MSR + CSR.
7. Calculate corrected reading by adding zero error. If error is negative and by subtracting zero error if zero is positive.
8. Take average of three readings.

Observations :

1. Pitch of the screw = distance covered on main scale when one rotation is completed = smallest division on main scale (normally) = P = …… cm (e.g. 0.1 cm).
2. Total number of divisions on circular scale = n = …… (e.g. 100).
3. Least count of micrometer screw gauge = L.C. = $\frac{P}{n}$ = …… cm $\left(\text{e.g. } \frac{0.1}{100} = 0.001 \text{ cm}\right)$.
4. Zero error = (L.C. × number of divisions by which zero of circular scale is shifted up or down the axial lines) = Z = …… cm.
5. Correction → Add 'Z' if error is negative or subtract 'Z' if error is positive.

Observation Table :

Object	Dimension		MSR	CSD	CSR = CSD × LC	Total reading	Corrected reading	Average reading
		1	**X cm**	**n**	**Y cm**	**X + Y cm**		
Metal sphere	Diameter	2.						
		3.						
		1.						
Metal wire	Diameter (thickness)	2.						
		3.						
		1.						
Metal plate	Thickness	2.						
		3.						

Results :

1. Average diameter of metal sphere = …… cm.
2. Average diameter of metal wire = …… cm.
3. Average thickness of metal plate = …… cm.

4. TO USE SPHEROMETER FOR THE MEASUREMENT OF THICKNESS OF A GIVEN GLASS PLATE

Apparatus : Spherometer, concave surface, convex surface.

Spherometer : We know that convex less or concave less is a part of sphere of very large radius. This radius of curvature of spherical lense can be measured using spherometer. Working and style of recording reading of spherometer is similar to micrometer screw gauge.

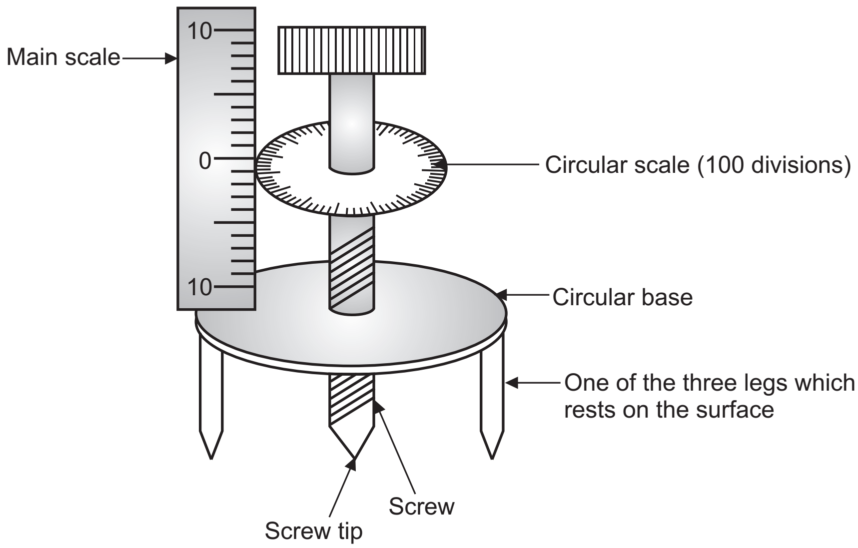


Fig. 4.1

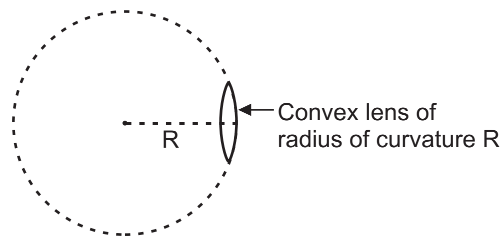


Fig. 4.2

$$\text{Least count of spherometer} = \frac{\text{Pitch}}{\text{Total number of divisions on circular scale}}$$

$$= \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on circular scale}} = \frac{0.1}{100} = 0.001 \text{ cm.}$$

Sample Reading :

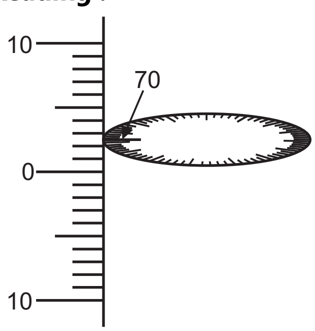


Fig. 4.3

MSR = 0.2 cm

Circular coinciding division number = n = 70

TR = $0.2 + (70 \times 0.001)$

TR = 0.270 cm

Procedure :

1. Measure least count of instrument (L.C.).
2. Mount spherometer on plane table resting its three legs on the table surface. Rotate the screw till its tip joust touches the surface of table. Record spherometer reading of plane surface 'X'.
3. Now mount spherometer on convex surface resting its legs on its surface. Rotate the screw till its tip touches convex surface and record spherometer reading 'Y'.

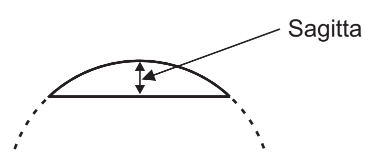


Fig. 4.4

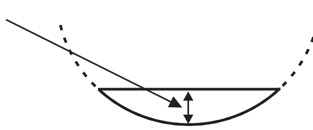


Fig. 4.5

4. Now mount spherometer on concave surface resting its legs on its surface. Rotate the screw till its tip touches concave surface and record spheroemter reading 'Z'.
5. Calculate sagitta of surface i.e. Y – X and X – Z.
6. Measure distance between legs.
7. Calculate radius of curvature R using given formula.

Observations :

1. Pitch of the screw = P = …… cm.
2. Total number of divisions on circular scale = n = …….
3. Least count of spherometer L.C. = $\frac{P}{n}$ = …… cm.

Observation Table :

Surface	MSR cm	CSD n	CSR = CSD × LC = n × LC	Total reading TR = MSR + CSR	Average reading in cm
	1.				
Plane surface	2.				X =
	3.				
	1.				
Convex surface	2.				Y =
	3.				
	1.				
Concave surface	2.				Z =
	3.				

Calculations :

1. Sagitta of convex surface, $h_1 = Y - X$ = …… cm.
2. Sagitta of concave surface, $h_2 = X - Z$ = …… cm.
3. Average distance between three legs 'd' = $\frac{(PQ + QR + PR)}{3}$.
4. Radius of curvature 'R_1' of convex surface = $R_1 = \left(\frac{d^2}{6h_1} + \frac{h_1}{2}\right)$ = …… cm.
5. Radius of curvature 'R_2' of concave surface = $R_2 = \left(\frac{d^2}{6h_2} + \frac{h_2}{2}\right)$ = …… cm.

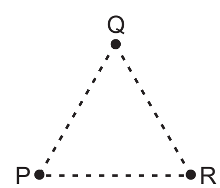


Fig. 4.6

Results :

1. Radius of curvature of convex surface = R_1 = …… cm.
2. Radius of curvature of concave surface = R_2 = …… cm.

5. TO CALCULATE YOUNG'S MODULUS OF ELASTICITY OF STEEL WIRE BY VERNIER METHOD

Apparatus : Two steel wires, vernier scale, main scale, two hangers, load.

Diagram : Set up for determination of Young's modulus 'Y' of material of wire by vernier method.

$$\text{L.C. of instrument} = \frac{\text{Smallest division on main scale}}{\text{Number of divisions on vernier scale}} = \frac{0.1 \text{ cm}}{10} \quad \boxed{\text{LC} = 0.01 \text{ cm}}$$

Formula : $$Y = \frac{Mg\,L}{\pi r^2 l}$$

where,
- M - Mass (load) attached to experimental wire
- L - Original length of experimental wire
- r - Radius of experimental wire
- l - Elongation (extension) produced in experimental wire

Experimental setup : Two steel wires A and B of same length and same diameter are taken. Wire A is called as reference wire and wire B is called as experimental wire. Top end of both the wires are fixed to rigid support. Main scale is attached at the bottom of reference wire and vernier scale is attached at the bottom of experimental wire. Fixed load is attached to reference wire and variable load with hanger is attached to experimental wire.

Why two wires : (One reference wire and other experimental wire).

Some error may include in experiment readings due to

1. Change in temperature – there may be expansion or contraction due to change in temperature because of which one error may include.
2. Bending of rigid support after certain use may be the cause of error.

Because of two wires the same change in length in both the wires (due to change in temperature or bending of support) takes place and, error can be eliminated.

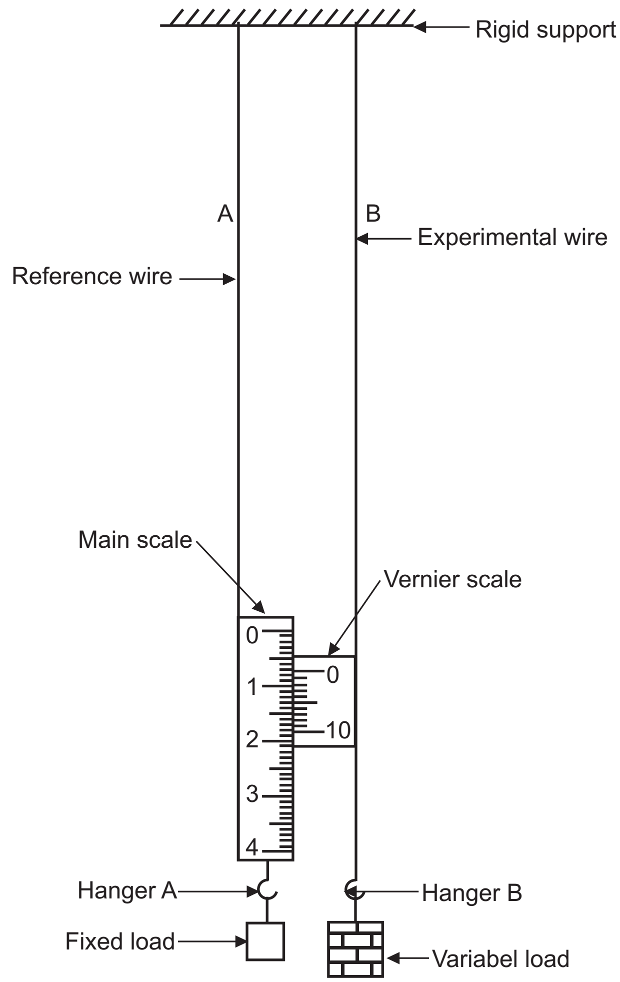


Fig. 5.1

Sample reading :

In the above diagram, zero of vernier is lying between 0.7 cm and 0.8 cm.

$\therefore$ Completed MSR = 0.7 cm

Coinciding vernier division number = 5.

$\therefore$ VSR = $5 \times LC = 5 \times 0.01 = 0.05$ cm

Total reading TR = MSR + VSR

= 0.7 + 0.05 = 0.75 cm

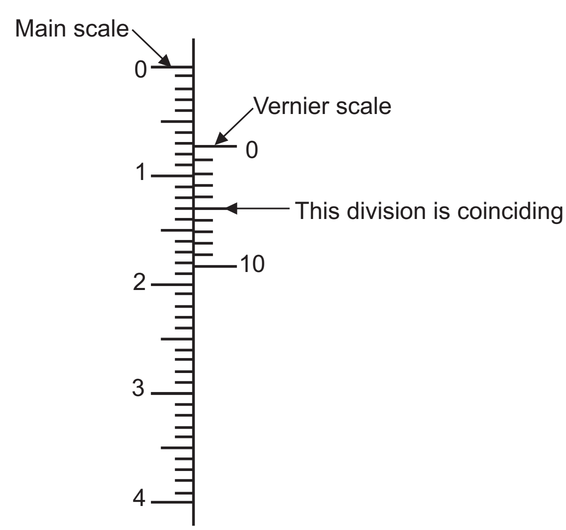


Fig. 5.2

Procedure :

1. Measure the length of the wire 'B' as a 'L'.
2. Measure diameter and hence radius of wire 'B' or a 'r'.
3. Add some load as initial load in hanger 'B' to remove some kinks and bendings in the wire.
4. Record main scale reading as well as vernier coinciding reading and then total reading.

   Total reading = Main scale reading + (Coinciding vernier division number × L.C.)

   This is initial reading TR.

5. Now add load of 500 gm in hanger 'B' and wait a little.
6. Now record main scale, vernier scale coinciding and total reading TR_1.

$$TR_1 - TR = l_1 = \text{elongation in the wire '}l_1\text{' due to load } M_1 \text{ (500 gm)}$$

7. Follow the same procedure, go on adding load of 500 gm each time and go on recording T.R. (loading).
8. In short, experimentally we have to measure elongation 'l' for different values of 'M'.
9. Calculate Y using formula : $Y = \frac{Mg\,L}{\pi r^2 l}$

Obs. No.	Load M gm	Main scale reading X cm	VSR = coinciding vernier division number VSD × LC Y cm	Total reading TR = X + Y cm	Elongation *l* cm	Young's modulus of elasticity of material of wire Y dyne/cm²
1.	Initial load (hanger)			TR =	–	–
2.	M_1 = 500			TR_1 =	$l_1 = TR_1 - TR = \ldots$	$Y_1 = \frac{M_1 g\,L}{\pi r^2\,l_1} = \ldots$
3.	M_2 = 1000			TR_2 =	$l_2 = TR_2 - TR = \ldots$	$Y_2 = \frac{M_2 gL}{\pi r^2 l_2} = \ldots$
4.	M_3 = 1500			TR_3 =	$l_3 = TR_3 - TR = \ldots$	$Y_3 = \frac{M_3 gL}{\pi r^2 l_3} = \ldots$
5.	M_4 = 2000			TR_4 =	$l_4 = TR_4 - TR = \ldots$	$Y_4 = \frac{M_4 g l}{\pi r^2 l_4} = \ldots$
6.	M_5 = 2500			TR_5 =	$l_5 = TR_5 - TR = \ldots$	$Y_5 = \frac{M_5 gL}{\pi r^2 l_5} = \ldots$
						Average Y = …

Result : Young's modulus of material of wire Y = …… dyne/cm^2 = …… N/m^2.

6. TO STUDY CAPILLARY PHENOMENON AND TO VERIFY THAT HEIGHT OF LIQUID IN A CAPILLARY IS INVERSELY PROPORTIONAL TO THE RADIUS OF CAPILLARY

Apparatus : Travelling microscope (T.M.), capillary tubes of different diameters, pointer, beaker, water.

Some required concepts :

Capillary : It is a hollow glass tube of very small inner diameter.

Capillarity : When glass capillary is immersed in liquid, then liquid either rises up or depresses down in a capillary.

For example :

1. When capillary is immersed in water, then water rises up inside the capillary.
2. When capillary is immersed in mercury, mercury depresses down inside the capillary.

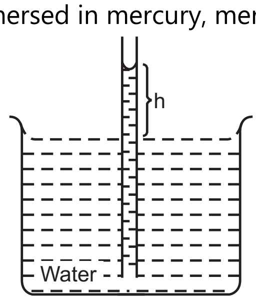


Fig. 6.1

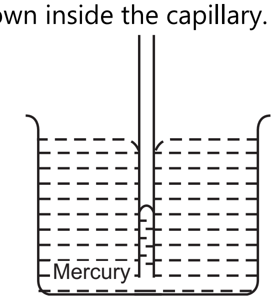


Fig. 6.2

This rise or fall of a liquid inside the capillary is called capillarity.

How to measure travelling microscope reading :

1. Determine L.C. of T.M. $\quad L.C. = \frac{\text{Smallest division on main scale}}{\text{Total number of divisions on vernier scale}} = \frac{0.05}{50} = \mathbf{0.001\ cm}$

Diagram showing sample reading :

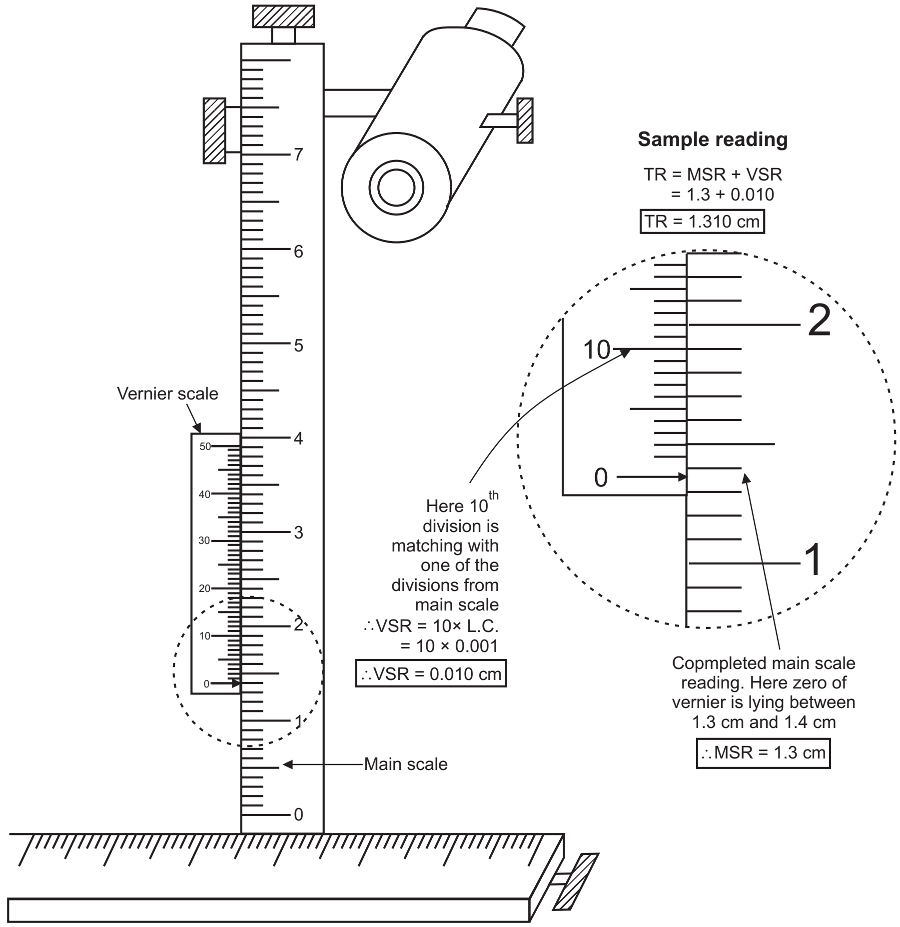


Fig. 6.3

Procedure :

(I) To measure inner (diameter) radius of capillary.

1. Focus travelling microscope (T.M.) on diameter of capillary.
2. Adjust vertical cross wire of T.M. on L.H.S. of inner bore of capillary record horizontal reading 'x_1' of T.M.

   Now go on moving the screw of T.M. so that vertical cross wire will touch on R.H.S. of inner bore of capillary and record horizontal reading 'x_2' of T.M.

   Now diameter 1 = $x_2 - x_1$.

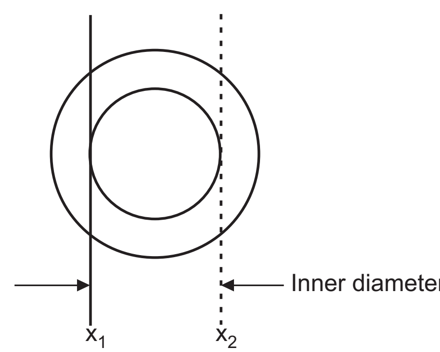


Fig. 6.4

3. Now using same procedure adjust horizontal cross wire on downside and then topside of inner bore of capillary and record vertical readings of T.V. 'y_1' and 'y_2'.

   Diameter 2 = $y_2 - y_1$

   Now diameter = $\frac{\text{(Diameter 1 + Diameter 2)}}{2}$

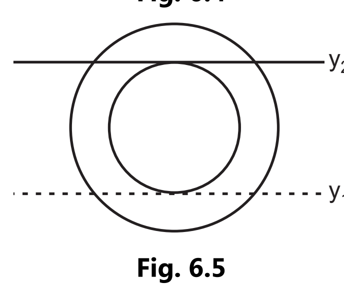


Fig. 6.5

Observation Table :

Capillary	Horizontal reading of T.M. when vertical cross wire of T.M. is adjusted at							Average diameter
	L.H.S. of inner bore			R.H.S. of inner bore			Diameter 1	
	T.R. 'x_1' cm			T.R. 'x_2' cm				
	MSR	VSR	TR_1	MSR	VSR	TR_2	$TR_2 - TR_1$	
1.								
2.								
3.								
4.								
Capillary	Vertical reading of T.M. when horizontal cross wire of T.M. is adjusted at							
	Bottom side of inner bore			Topside of inner bore			Diameter 2	
	T.R. 'Y_1' cm			F.R. 'Y_2' cm			$(Y_2 - Y_1)$	
	MSR	VSR	TR_1	MSR	VSR	TR_2	$TR_2 - TR_1$	
1.								
2.								
3.								
4.								

$$\text{Now, Average diameter of capillary} = \frac{(\text{Diameter 1 of capillary 1 + Diameter 2 of capillary 1})}{2}$$

Using this method diameter of every capillary is measured and hence radius r_1, r_2, r_3, r_4 are recorded.

To verify that height of liquid in a capillary is inversely proportional to radius of capillary :

1. All the 4 capillary tubes whose radius is known to us are to be cleaned using water and dry it.
2. Now clamp these capillaries vertically such that they get immersed in water in a beaker.
3. Now adjust the pointer such that its tip just touches the water level in a beaker.

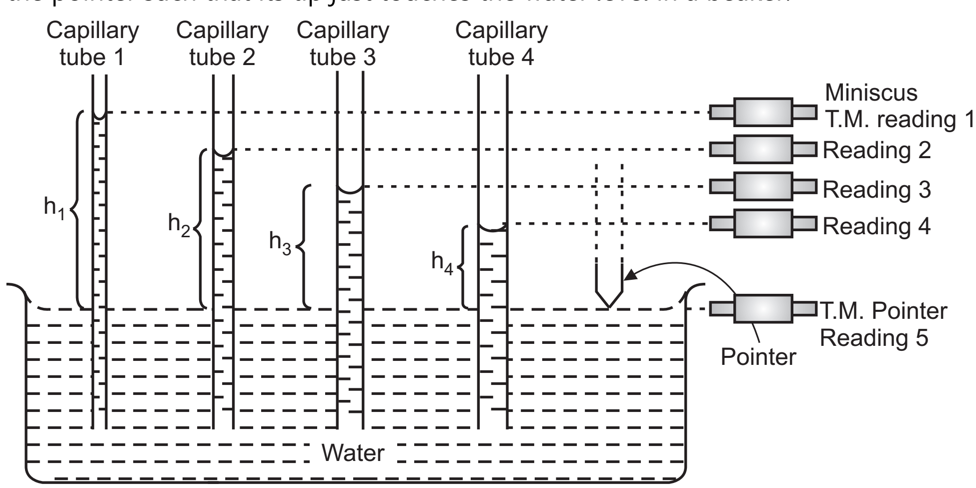


Fig. 6.6

4. Focus T.M. on miniscus of risen water in capillary 1 and record reading 1. [Note : Image through T.M. get inverted].
5. Similarly follow the same procedure for other three tubes and record reading 2, reading 3, reading 4.
6. Now remove the beaker from downside so that tip of the pointer suspend in air. Now focus T.M. on tip of the pointer and record **reading 5**, this reading will be common for all the capillaries.

Observation table to measure rise of liquid 'h'.

Observation table :

Observation No.		**Readings of T.M. of liquid miniscus (m)**			
		MSR cm	**VSR cm**	**TR = MSR + VSR**	**Height of liquid in capillary**
	Capillary 1			$m_1 = \ldots$	$(m_1 - p) = h_1 = \ldots$
	Capillary 2			$m_2 = \ldots$	$(m_2 - p) = h_2 = \ldots$
	Capillary 3			$m_3 = \ldots$	$(m_3 - p) = h_3 = \ldots$
	Capillary 4			$m_4 = \ldots$	$(m_4 - p) = h_4 = \ldots$
	Reading of T.M. when it is focused on tip of the pointer				
		MSR	**VSR**	**TR = MSR + VSR**	
	tip of pointer			P = (this reading is common) = …	

Now we have radius of capillaries as r_1, r_2, r_3, r_4 and also we have rise of liquid in capillaries (i.e. height of liquid in capillaries) h_1, h_2, h_3, h_4.

Practically verify that less the radius of capillary, more is the rise of liquid in it and vice a versa.

i.e. $$r_1h_1 = r_2h_2 = r_3h_3 = r_4h_4 = \text{constant} \quad \boxed{\text{i.e. } h \propto \frac{1}{r}}$$

Result : Height of liquid in a capillary is inversely proportional to radius of capillary (i.e. less the radius of the capillary, more is the rise of liquid in it.)

7. TO DETERMINE COEFFICIENT OF VISCOSITY OF GIVEN LIQUID USING STOKE'S METHOD

Aim : *To determine coefficient of viscosity of glycerine.*

Apparatus : Glycerine (liquid of which coefficient of viscosity is to be determined), tall glass jar (tube), micrometer screw gauge, metal sphere (ball bearing), stop watch.

Formula : $\eta = \frac{2}{9} \frac{r^2 (d - \rho) g}{v}$

Diagram :

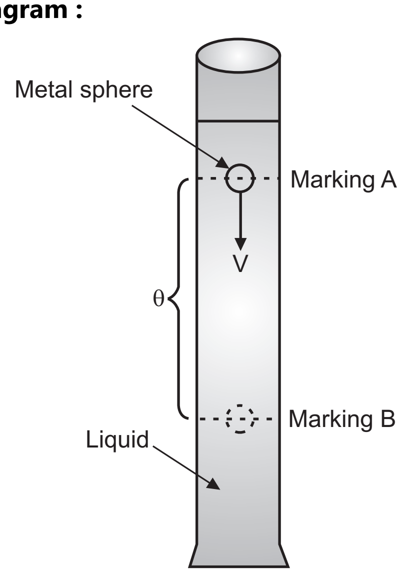


Fig. 7.1

where,
- η = coefficient of viscosity of liquid (which is to be determined)
- r = radius of metal sphere
- d = density of metal sphere
- ρ = density of liquid
- v = terminal velocity
- g = gravitational acceleration

Concept of terminal velocity : When metal sphere is allowed to fall freely through liquid, it is practically observed that, after covering certain distance this metal sphere attains constant velocity called as terminal velocity.

Procedure :

To understand the procedure and to make it simple, go through the above formula.

In this d, ρ, g are constant (given) and only r, v are unknown in order to calculate η.

i.e. In simple words we have to measure r and v practically.

1. Measure diameter of metal sphere using micrometer screw gauge. Therefore calculate radius of metal sphere 'r'.
2. Take liquid in a tall glass tube. Mark line A on tube at some lower side of liquid level. Now mark line B on tube at 50 cm apart from mark A. i.e. distance D = AB = 50 cm.
3. Practice STAR-STOP-RESET function of stop watch.
4. Release ball bearing of radius 'r' from the centre of tube through the liquid.
5. The moment ball bearing crosses marking 'A' start the stop watch and the moment it crosses marking 'B' stop the stop watch and record time as a 't' sec. Now reset the stop watch. This gives us time required for ball bearing to cover distance AB = D.
6. Now calculate terminal velocity $v = \frac{D}{t}$.
7. Now calculate 'η' using formula : $\eta = \frac{2}{9}\frac{r^2 (d - \rho) g}{v}$ where d, ρ, g are given.

Observations :

1. Liquid used = …… (given) (e.g. glycerine).
2. Density of metal sphere d = …… kg/m^3 (given).
3. Density of liquid ρ = …… kg/m^3 (given).
4. Distance between two markings A and B. i.e. D = …… cm = …… m (e.g. 50 cm).
5. Gravitational acceleration 'g' = 9.81 m/s^2 (given).

Observation table :

1. To determine diameter and hence radius of metal sphere (ball bearings) using micrometer screw gauge :

Obs. No.	Main scale reading MSR	Coinciding circular scale division number CSD	CSR = CSD × LC	Total readings TR = MSR + CSR	Corrected reading TR + CR	Average
	X cm	n	Y cm	TR = X + Y		
1.						
2.						
3.						

2. To determine terminal velocity 'v' :

Obs. No.	Time required to cover distance AB i.e. D	Terminal velocity $v = \frac{D}{t} = \frac{AB}{t}$
1.	t sec	v m/s

Calculate coefficient of viscosity of given liquid using formula :

$$\eta = \frac{2}{g}\frac{r^2 (d - \rho) g}{v} = \text{………} \ Ns/m^2$$

Result : Coefficient of viscosity, η = ……… Ns/m^2

8. TO CALCULATE LINEAR THERMAL COEFFICIENT OF EXPANSION FOR COPPER BY USING PULLINGER'S APPARATUS

Aim : *To determine coefficient of linear expansion (α) of copper.*

Apparatus : Pullinger's apparatus, steam (vessel) chamber, gas burner, copper rod.

Linear expansion concept : If metal rod is taken and it is heated, then its length increases i.e. linear expansion takes place.

$$\text{Change in length, } l = L_{t_2} - L_{t_1}$$

More the original length, more is the change in length. More the change in temperature, more is the change in length.

Change in length also depends on the materials of the rod.

Thus linear expansion i.e. 'l' depends on :

1. Original length of the rod. 2. Change in temperature. 3. Material of the rod.

Practically it is observed that, change in length (linear expansion) is directly proportional to the original length and change in temperature.

$$l = (L_{t_2} - L_{t_1}) \propto L \text{ and } (t_2 - t_1)$$

$$\therefore \quad l = (L_{t_2} - L_{t_1}) \propto L \times (t_2 - t_1)$$

$$(L_{t_2} - L_{t_1}) = \alpha L \times (t_2 - t_1)$$

$$\therefore \quad \alpha = \frac{(L_{t_2} - L_{t_1})}{[L \times (t_2 - t_1)]}$$

where,
α = coefficient of linear expansion of material of rod
L = original length at room temperature
L_{t_1} = length at temperature t_1
L_{t_2} = length at temperature t_2
t_1 = initial temperature
t_2 = final temperature

$\alpha \rightarrow$ coefficient of linear expansion of material of a rod is defined as, change in length per unit original length per unit change in temperature. Its unit is /°C. It is always constant for a given material of a rod. It changes from material to material.

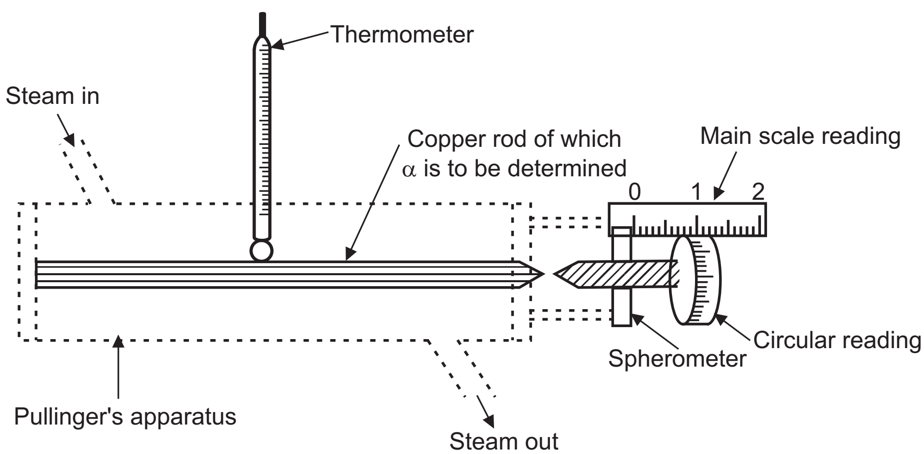


Fig. 8.1

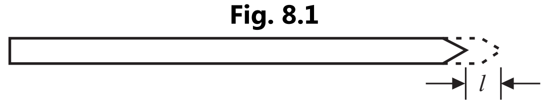


Fig. 8.2

Procedure :

1. Determine L.C. of spherometer.
2. Take a copper rod of length L = 50 cm at room temperature and fix it as shown in figure.
3. Insert thermometer so that its bulb just touches the rod.
4. Now adjust spherometer screw so that it just touches tip of the copper rod. Now note down room temperature as 't_1' and spherometer reading say S_1.
5. Now rotate screw of spherometer and maintain little gap between tip of screw and tip of copper rod (i.e. scope for expansion).
6. Now pass steam through inlet pipe.
7. After sufficient time, adjust the screw of sphereometer such that it touches tip of copper rod and record spherometer reading as s_2 and temperature by thermometer as t_2.

8. Find increase in length $l = s_2 - s_1$.
9. Calculate coefficient of linear expansion

$$\alpha = \frac{l}{L \times (t_2 - t_1)} = \frac{(s_2 - s_1)}{L \times (t_2 - t_1)} = \ldots\ldots /°C$$

Result : Coefficient of linear expansion of copper rod α = …… /°C.

9. TO DETERMINE REFRACTIVE INDEX OF GLASS SLAB BY PIN METHOD

Aim : *To determine refractive index of glass slab by pin method.*

Apparatus : Drawing board, white sheet paper, rectangular glass (e.g. crown glass) slab, office pins, protractor, pencil, scale.

Refractive index : Concept.

When light enters from one medium to another, it changes its path.

Practically it is observed that when light ray enters from air (optically rarer medium) to glass (optically denser medium) ray bends towards normal.

When light enters from glass to air, the ray bends away from the normal.

Case 1

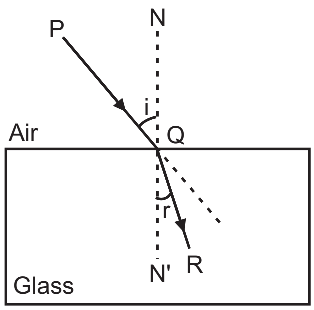


Fig. 9.1

Ray enters from air to glass

Case 2

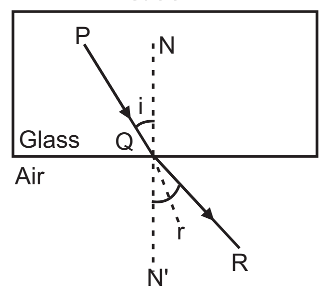


Fig. 9.2

Ray enters from glass to air

PQ - incident ray
QR - refracted ray
NN' - normal to glass slab
$\angle i$ = angle of incidence
$\angle r$ = angle of refraction

Case 1 : Here $\angle i > \angle r$

Case 2 : Here $\angle i < \angle r$

Snell's law : For a given pair of media, sin r is directly proportional to sin i.

$$\sin r \propto \sin i$$

i.e. $$\frac{\sin i}{\sin r} = \text{Constant}$$

$$\frac{\sin i}{\sin r} = \text{Refractive index of second medium w.r.t. first medium}$$

Case I

Here $\frac{\sin i}{\sin r}$ = Refractive index of glass w.r.t. air

$= {}_a\mu_g$

OR

$= {}_an_g$

Case 2

Here $\frac{\sin i}{\sin r}$ = Refractive index of air w.r.t. glass

$= {}_g\mu_a$

OR

$= {}_gn_a$

Thus in case 1 $\rightarrow {}_a\mu_g > 1$ i.e. ${}_an_g > 1$

case 2 $\rightarrow {}_g\mu_a < 1$ i.e. ${}_gn_a < 1$

Thus in

Case 1 : Refractive index of glass w.r.t. air i.e. ${}_a\mu_g > 1$ **OR** ${}_an_g > 1$.

Case 2 : Refractive index of air w.r.t. gass. i.e. ${}_g\mu_a < 1$ **OR** ${}_gn_a < 1$.

Procedure :

1. Fix a white sheet paper on drawing board.
2. Draw a line AB using pencil on it.
3. Draw perpendicular NN' to it. NN' crosses line AB at point Q.
4. Draw a line PQ making an angle of 35° with the normal. i.e. angle of incidence $\angle i_1 = 35°$.
5. Now place a rectangular glass slab so that its length will coincide with line AB drawn on paper and draw outline of glass slab on paper by pencil.
6. Now fix 2 pins in errect position a, b on line PQ at sufficient large distance.
7. Now keep your one eye open and bring it on the opposite side of slab and in the same level of glass slab.

   Now fix another 2 pins c, d so that they will coincide with image of earlier two pins (i.e. a, b). In short, now all the 4 pins will appear to be on one line.
8. Now remove glass slab. Now remove pins and draw circle around impression of pin points i.e. a, b, c, d.

   Now join points a and c and d and extend them.
9. Note crossing point as a R where extended line cd will cross out line of glass slab.
10. Now draw lines PQ, QR and RS.
11. Draw perpendiculars N" N"' to glass slab at R.
12. Angle PQN = $\angle i$

    $\angle$ N'QR = $\angle$ N"RQ = $\angle r_1$ i.e. angle of refraction.

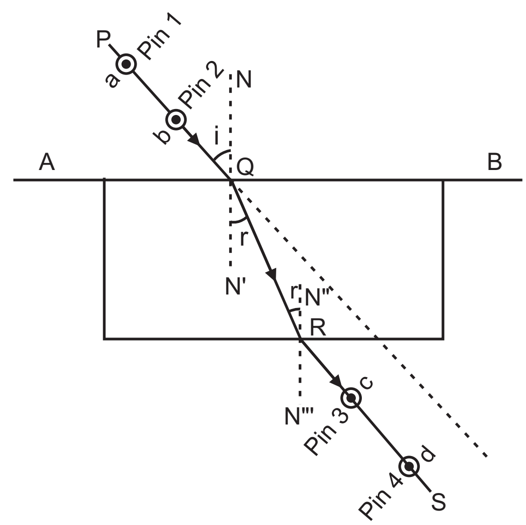


Fig. 9.3

13. Follow same procedure for different angles of incidence $i_2 = 45$, $i_3 = 55°$ and measure angle of refraction r_2 and r_3.

Observation table :

Obs. No.	Angle of incidence	Angle of refraction	Refractive index of glass slab w.r.t. air	Average R.I. of glass slab w.r.t. air
	i	r	$_a\mu_g$	$_a\mu_g$
(1)	$i_1 = 35°$	$r_1 = ...$	$= \frac{\sin i_1}{\sin r_1} = ...$	
(2)	$i_2 = 45°$	$r_2 = ...$	$= \frac{\sin i_2}{\sin r_2} = ...$	......
(3)	$i_3 = 55°$	$r_3 = ...$	$= \frac{\sin i_3}{\sin r_3} = ...$	

Result : Refractive index of given glass slab $= {}_a\mu_g = ...$ (no unit).

10. TO DETERMINE THE VELOCITY OF SOUND BY USING RESONANCE TUBE

Aim : *To determine the velocity of sound by using resonance tube.*

Apparatus : Resonance tube (pipe of certain diameter), meter scale, vernier calliper, set of tuning forks, vessel, water, stand with clamp.

Resonance : When natural frequency air column becomes equal to forced frequency of tuning fork then loud sound is produced i.e. resonance takes place.

Diagram :

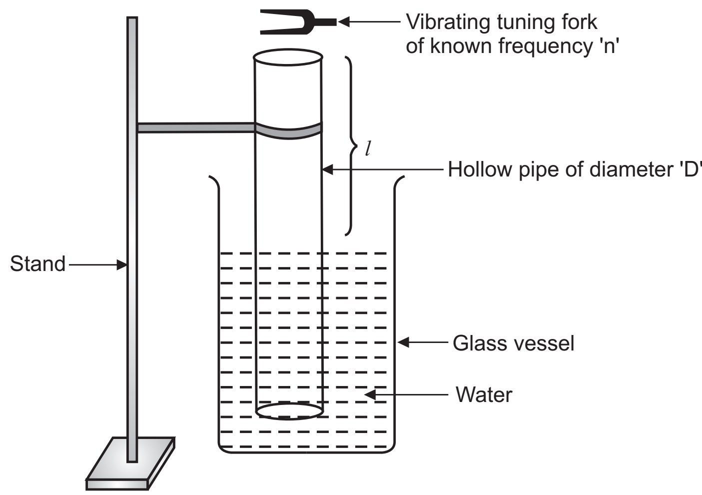


Fig. 10.1

Procedure :

1. Find internal diameter 'D' of resonance tube using vernier calliper.
2. Calculate end correction $e = 0.3 \times D$.
3. Arrange the tuning forks in decreasing order of their frequencies. Select highest frequency tuning fork. Strike the tuning fork on rubber pad to generate vibrations in it.
4. Suspend resonance tube at lowermost position (i.e. minimum length of air column in it).
5. Hold the vibrating tuning fork at open end as shown in figure and slowly go on raising pipe up (i.e. increase the length of air column). At one position of tube you will get loud sound from pipe (i.e. resonance occurs).

   Find the exact position of pipe at which you get maximum sound.

   Measure the length of air column 'l' (i.e. length of pipe from open end till the water level in vessel). Take 3 trials.
6. Now calculate corrected resonating length $L = l + e = l + 0.3D$.
7. Calculate velocity of sound using formula $v = 4nL$.
8. Now take 2nd highest frequency tuning fork and make it to vibrating and hold it on tube end. Now go on raising the tube in upward direction (because as frequency of tuning fork decreases, resonating length will go on increasing i.e. $l \propto \frac{1}{n}$) and determine the resonating length.
9. Follow the same procedure for other frequency tuning forks.
10. Calculate velocity of sound using formula $v = 4nL = 4n\,(l + e) = 4n\,(l + 0.3D)$.
11. Plot graph of frequency (Y-axis) versus $\frac{1}{L}$ (X-axis). Nature of graph will be straight line (inclined). Slope of the graph i.e. $\frac{(y_2 - y_1)}{(x_2 - x_1)}$ gives $\frac{y}{x}$ which gives $n \times L$.

    $\therefore$ $v = 4 \times$ slope gives velocity of sound by graph.

Observation table :

1. To determine internal diameter of the resonance tube :

Obs. No.	Main scale reading MSR	Coinciding vernier division no. VSD	Vernier scale reading VSR	Total reading TR	Corrected total reading CR	Mean diameter D cm
	X cm	n	$Y = n \times LC$	$TR = X + Y$	$CR = TR \pm Z$	
1.						
2.						
3.						

Average diameter D = …… cm $\therefore$ End correction (e) = $0.3 \times D$ = …… cm

2. To calculate velocity of sound :

Obs. No.	Frequency of tuning fork	Resonating length of air column (three trials)			Mean resonating length	Corrected resonating length	$\frac{1}{L}$ cm	Velocity of sound v = 4nL cm/s
	n Hz	l_1 cm	l_2 cm	l_3 cm	l cm	L = l + 0.3D		v
1.	… (e.g. 512)							
2.	… (e.g. 480)							
3.	… (e.g. 426)							
4.	… (e.g. 384)							
5.	… (e.g. 341)							
6.	… (e.g. 320)							

Velocity of sound in air (v) = average of last column = …… cm/s = …… m/s.

3. To calculate velocity of sound by graph :

Plot graph of frequency 'n' (on Y-axis) and $\frac{1}{L}$ (on X-axis).

Draw straight line such that maximum points get covered (i.e. maximum points lie on line).

Select any two points of larger distance on line (graph) i.e. point 1 and point 2. Note down their co-ordination as (x_1, y_1) and (x_2, y_2).

Find slope of graph = $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{dy}{dx} = \frac{n}{1/L}$ = nL = ……

Now find velocity of sound by graph = v = 4 × slope = …… cm/s = …… m/s.

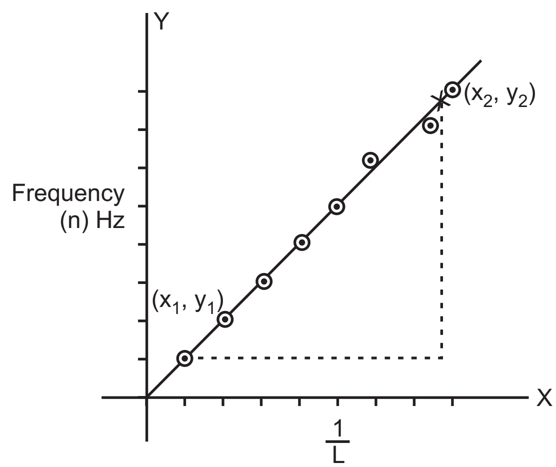


Fig. 10.2

Results :

1. Velocity of sound by calculation = v = …… cm/s = …… m/s.
2. Velocity of sound by graph = v = …… cm/s = …… m/s.

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