Chapter 3

Design of Shaft, Keys, Couplings and Gears

3.1 Introduction

1.Shaft

- A shaft is a rotating machine element which is used to transmit power from one place to another.
- The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.
- In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it.
- These members along with the forces exerted upon them causes the shaft to bending.
- In other words, we may say that a shaft is used for the transmission of torque and bending moment.
- The -various members are mounted on the shaft by means of keys or splines.
- The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

2. Axle

- An axle though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only.
- It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave**.**

3. Spindle

- A spindle a short shaft that imparts motion either to a cutting tool (e.g., drill press spindles) or work piece (e.g., lathe spindles).

3.2 Material Used for Shafts

- The material used for ordinary shafts is carbon steel of grades 40 C 8 , 45 C 8, 50 C 4, 50 C12.
- When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

3.3 Properties of material used for shafts

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

3.4 Types of Shafts

The following two types of shafts are important from the subject point of view:

1. Transmission shafts.

- These shafts transmit power between the source and the machines absorbing power.
- The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts.
- Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. Machine shafts.

- These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

3.5 Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are:

- 25 mm to 60 mm with 5 mm steps; 60mm to 110mm with 10mm steps; 110mm to 140 mm with 15 mm steps; and 140 mm to 500 mm with 20 mm steps.
- The standard length of the shafts are 5 m, 6 m and 7 m.

3.6 Design of shafts

The shafts may be designed on the basis of

1. Strength, and 2. Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Case (a) Shafts subjected to twisting moment or torque only

- When the shaft is subjected to a twisting moment (or torque) only, then the d shaft may be obtained by using the torsion equation. We know that -

$$\frac{T}{J} = \frac{\tau}{r} \qquad ...(i)$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre

$= d/2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (*i*), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad ...(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad ...(iv)$$

From the equations (*iii*) or (*iv*), the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stro material and they may be forged on a mandrel, thus making the material more hom would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2) R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Case (b) Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \text{...}(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \text{ and } y = \frac{d}{2}$$

Substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4(1 - k^4) \quad \text{...(where } k = d_i / d_o)$$

and $y = d_o / 2$

Again substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4(1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Case (c) Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.
2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of σ_b and τ we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as *equivalent twisting moment* and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \quad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \quad ...(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi d^3}\right)^2 + 4\left(\frac{16\,T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

or $$\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \quad ...(iv)$$

The expression $\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$ is known as *equivalent bending moment* and is denoted by M_e. The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment. By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \quad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

- ***It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.***

Case (d) Shafts Subjected to Fluctuating Loads

- In the previous articles we have assumed that the shaft is subjected to consta bending moment.
- But in actual practice, the shafts are subjected to fluctuating torque and bending moments.
- In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (T) and bending moment (M).
- Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and
K_t = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for K_m and K_t.

Design of Shafts on the Basis of Rigidity

- Sometimes the shafts are to be designed on the basis of rigidity.
- The torsional rigidity is important in the case of camshaft of an IC. engine where the timing of the valves would be affected.
- The permissible amount of twist should not exceed 0.25° per meter length of such shafts.
- For line shafts or transmission shafts, deflections 2.5 to 3 degree per meter length may be used as limiting value.
- The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.
- The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G\,.\,\theta}{L} \quad \text{or} \quad \theta = \frac{T\,.\,L}{J\,.\,G}$$

Where,

θ = Torsional deflection or angle of twist in radians,
T = Twisting moment or torque on the shaft,
J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \qquad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$ ...(For hollow

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

Keys

3.7 Introduction

- A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.
- It is always inserted parallel to the axis of the shaft.
- Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.
- A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

3.8 Types of Keys

- The following types of keys are important from the subject point of view

1. Sunk keys.
2. Saddle keys,
3. Tangent keys,
4. Round keys, and
5. Splines.

1. Sunk Key

- The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley.
- The sunk keys are of the following types:

A. Rectangular sunk key.

- A rectangular sunk key is shown in following Fig.
- The usual proportions of this key are

Width of key,

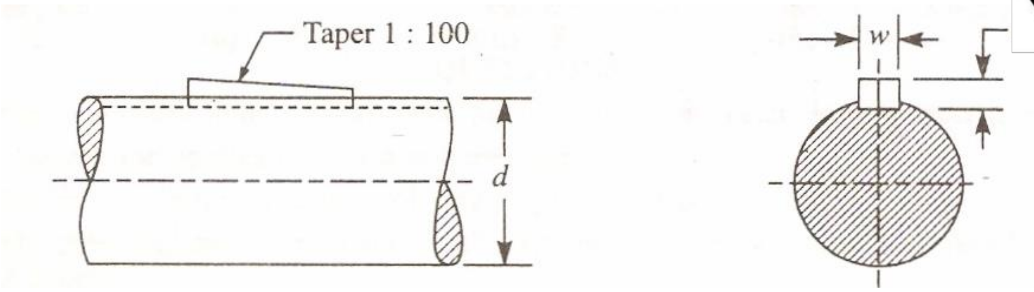
$w = d/4$, and

thickness of key,

$t = d/6$

Where ,

d = Diameter of the shaft or diameter of the hole in the hub.



- The key has taper I in 100 on the top side only.

B. Square sunk key.

- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e.

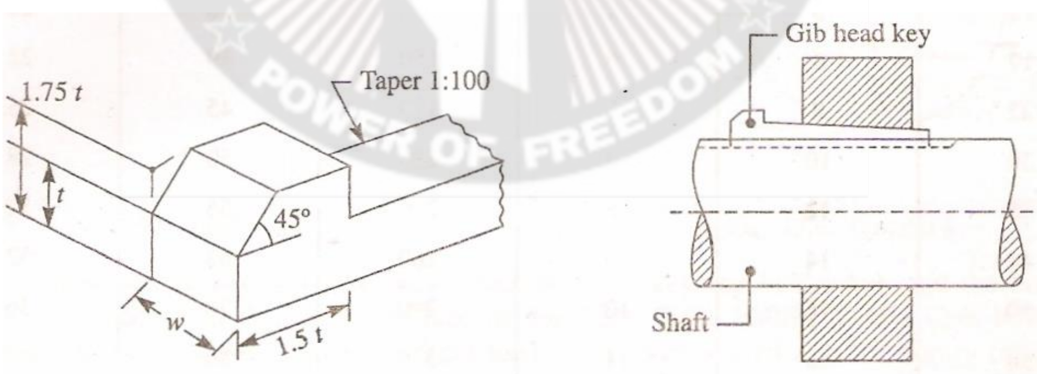
  w =t = d/4

C. Parallel sunk key.

- The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout.
- It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

D. Gib-head key.

- It is a rectangular sunk key with a head at one end known as gib head.
- It is usually provided to facilitate the removal of key.
- A gib head key is shown in following Fig. (a) and its use in shown in Fig. (b).



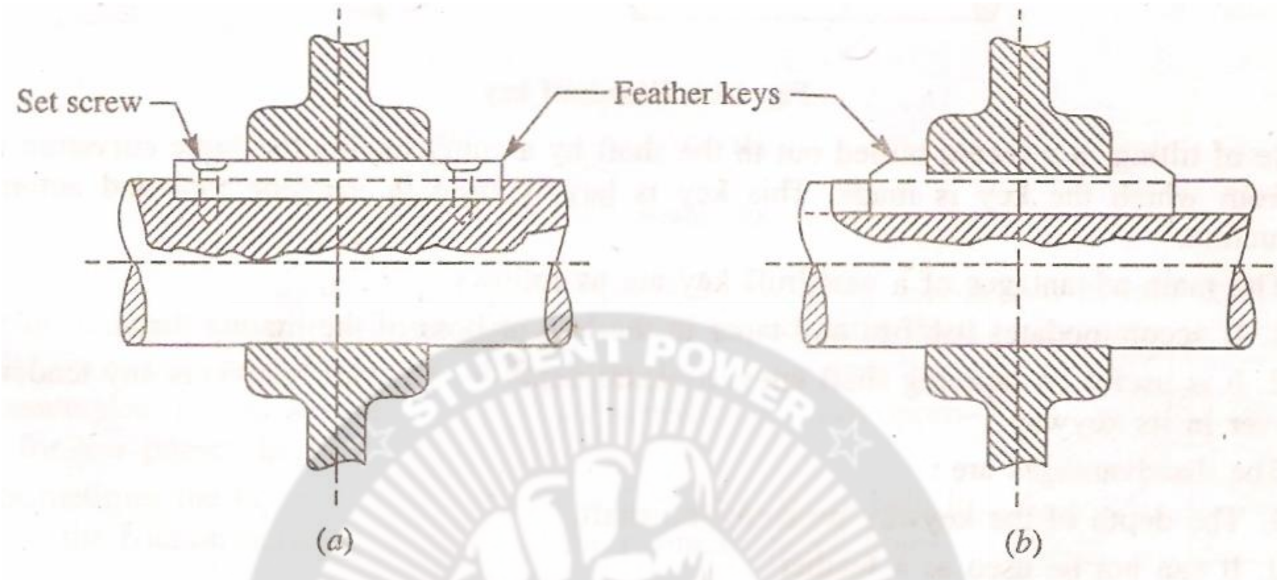
(a) (b)

- The usual proportions of the gib head key are

  Width, w = d/4 , and thickness at large end, t = d/6

E. Feather key.

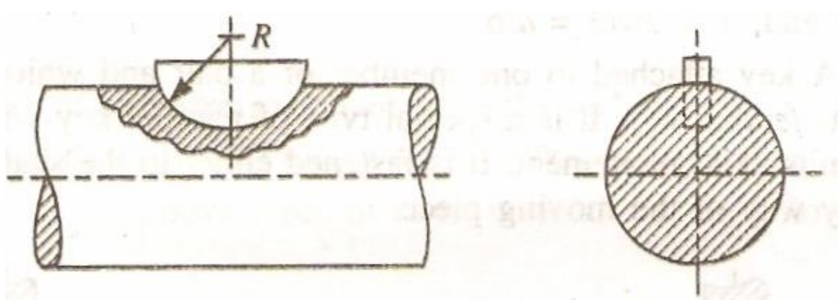
- A key attached to one member of a pair and which permits relative axial move known as feather key.
- It is a special type of parallel key which transmits a turning moment and also permits axial movement.
- It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.



- The feather key may be screwed to the shaft as shown in above Fig. (a) or it may have double gib heads as shown in above Fig. (b).
- The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

F. Woodruff key.

- The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in below Fig.
- A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



❖ The main advantages of a woodruff key are as follows

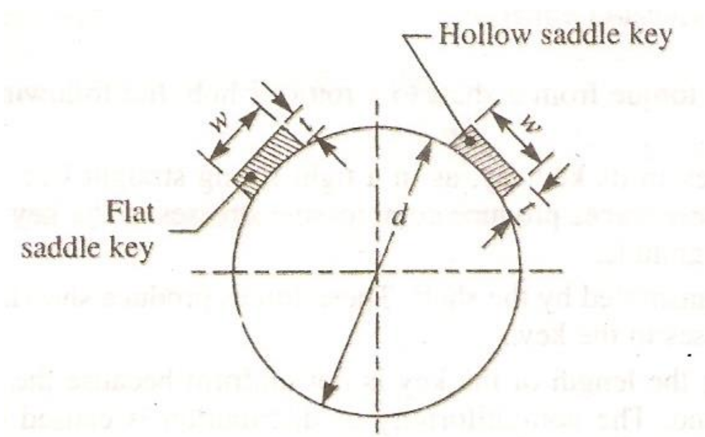
1. If accommodates itself to any taper in the hub or boss of the mating piece.
2. It is useful on tapering shaft ends, Its extra depth in the shaft “prevents any tendency to turn over in its keyway. -

❖ The disadvantages are

1. The depth of the keyway weakens the shaft.
2. It can not be used as a feather.

2.Saddle Key

- ❖ The saddle keys are of the following two types
    1. Flat saddle key. and
    2. Hollow saddle key.
- ❖ A flat saddle key is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in following Fig.
- ❖ It is likely to slip round the shaft under load.
- ❖ Therefore it is used for comparatively light loads.

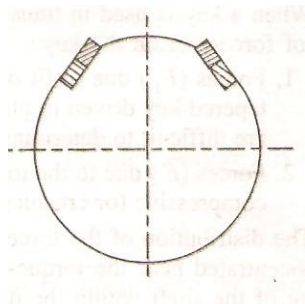


$t = w/3 = d/12$

- ❖ A hollow saddle key is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft.
- ❖ Since hollow saddle keys hold on by friction, therefore these are suitable for light loads.
- ❖ It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

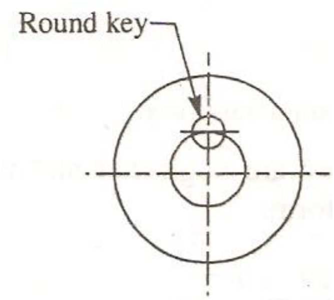
3. Tangent Keys

- ❖ The tangent keys are fitted in pair at right angles as shown in following Fig.
- ❖ Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.



4. Round Keys

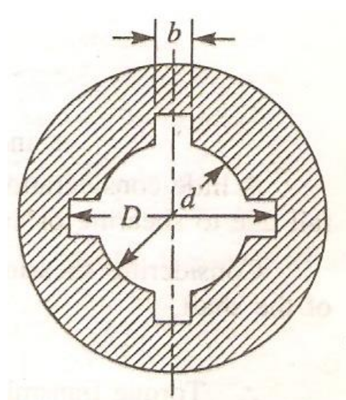
- ❖ The round keys, as shown in following Fig.



- ❖ These are circular in section partly in the shaft and partly in the hub.
- ❖ They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled.
- ❖ Round keys are usually considered to be most appropriate for low power drives.

5.Splines

- ❖ Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub.
- ❖ Such shafts are known as splined shafts as shown in following Fig.
- ❖ These shafts usually have four, six, ten or sixteen splines.
- ❖ The splined shafts are relatively stronger than shafts having a single keyway.



$D = 1.25\ d$ and $b = 0.25\ D$

- ❖ The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions.
- ❖ By using splined shafts, we obtain axial movement as well as positive drive is obtained.

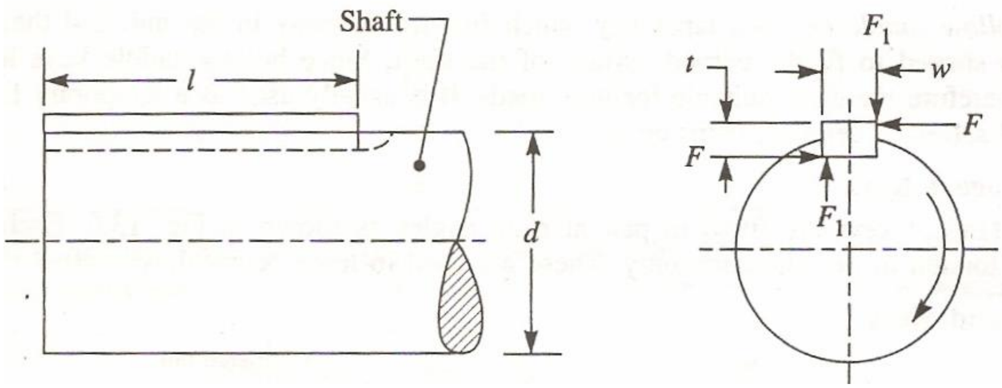
3.9 Design of Sunk Key

- When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

  1. Forces (*F*1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

  2. Forces (*F*) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

- In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.



- Let

  T = Torque transmitted by the shaft,

  F = Tangential force acting at the circumference of the shaft,

  d = Diameter of shaft,

  l = Length of key,

  w = Width of key.

  t = Thickness of key, and

  σc = Shear and crushing stresses for the material of key.

- The usual proportions of this key are

  Width of key, w = d/4 , and

  thickness of key, t = d/6

  Where ,

  d = Diameter of the shaft or diameter of the hole in the hub.

- A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Case 1 . Failure of key due to shearing

- Considering shearing of the key, the relation between tangential shearing force ,area resisting shearing and shear stress is

  shear stress , $\tau = \frac{F}{l \, x \, w}$

  Therefore , tangential force is

  $$F = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2}$$

Case 2 . Failure of key due to crushing

- Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{...}(ii)$$

Key is equally strong in shearing and crushing

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{...[Equating equations } (i) \text{ and } (ii)]$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad \text{...}(iii)$$

- The permissible crushing stress for the usual key material is at least twice the permissible shearing stress.
- Therefore from equation (iii), we have w = t.
- In other words, a square key is equally strong in shearing and crushing**.**

Length of key .

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad ...(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad ...(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \quad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \; \text{... (Taking } w = d/4) \; ...(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \quad l = 1.571\, d \quad \text{... [From equation } (vi)]$$

3.10 Effect of keyway on shaft

- A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft.
- It other words, the torsional strength of the shaft is reduced.
- The following relation for the weakening effect of the keyway is based on the experimental results by H.P. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

h = Depth of keyway = $\dfrac{\text{Thickness of key } (t)}{2}$

- It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

Couplings

3.11 Introduction

- Shafts are usually available up to 7 metres length due to inconvenience in transport.
- In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.
- Shaft couplings are used in machinery for several purposes, the most common of which are the following

Purpose of couplings

1. To provide for the connection of shafts of units that are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.
2. To provide for misalignment of the shafts or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against overloads.
5. To alter the vibration characteristics of rotating units.

Requirements of a Good Shaft Coupling

I. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. If should have no projecting parts.

3.12 Types of Shafts Couplings

- shaft couplings are divided into two main groups as follows:

1. Rigid coupling. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view

(a) Sleeve or muff coupling.
(b) Clamp or split-muff or compression coupling, and
(c) Flange coupling.

2. Flexible coupling. It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

(a) Bushed pin type coupling,
(b) Universal coupling, and

(c) Oldham coupling.

3.13 Sleeve or muff coupling.

- It is the simplest type of rigid coupling, made of cast iron.
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.
- It is fitted over the ends of the two shafts by means of a gib head key, as shown in following Fig.
- The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

Design Procedure

Step 1. Design of Shaft .

- Generally power transmitted by shaft is given , hence first of all find torque transmitted by shaft as

$$p = \frac{2\pi NT}{60}$$

T = ………N-m

T = ……x 10^3 N-mm

Now as per torsion equation ,

$$T = \frac{\pi}{16} \tau d^3$$

From above equation find (d)

Step 2. Proportions of sleeve .

- The usual proportions of a cast iron sleeve coupling are as follows

Outer diameter of the sleeve, D = 2 d + 13 mm

and length of the sleeve, L = 3.5 d

Where,

d is the diameter of the shaft.

Step 3. Design of Key

- The usual proportions for **rectangular** key are

Width of key, w = d/4 , and

thickness of key, t = d/6

Where , d = Diameter of the shaft or diameter of the hole in the hub.

- And for **square key** proportions are

  Width of key, w = d/4 , and

  thickness of key, t = d/4

  Where , d = Diameter of the shaft or diameter of the hole in the hub.

The length of the coupling key is atleast equal to the length of the sleeve (*i.e.*, 3.5 *d*). The coupling key is usually made into two parts so that the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\,d}{2}$$

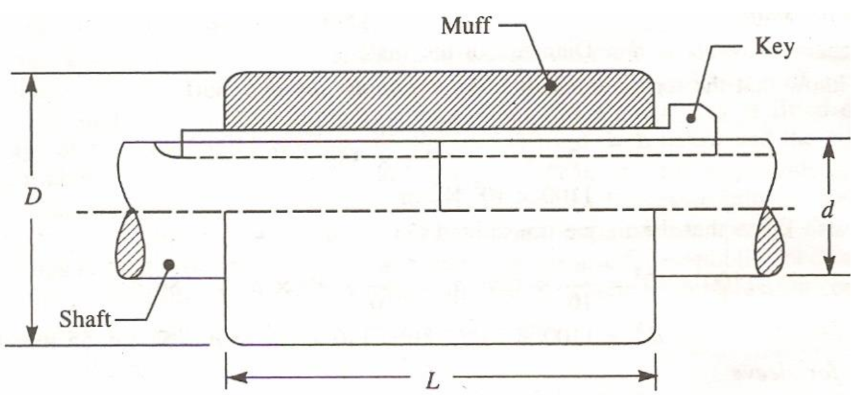
After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Step 4. Design of sleeve

- The sleeve is designed by considering it as a hollow shaft.



Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast rion. The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \times D^3\,(1 - k^4) \quad \text{... } (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

Example 13.4. *Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.*

Solution. Given : $P = 40$ kW $= 40 \times 10^3$ W ; $N = 350$ r.p.m. ; $\tau_s = 40$ MPa $= 40$ N/mm^2; $\sigma_{cs} = 80$ MPa $= 80$ N/mm^2; $\tau_c = 15$ MPa $= 15$ N/mm^2

The muff coupling is shown in Fig. 13.10. It is designed as discussed below :

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3 / 7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm } \textbf{Ans.}$$

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm } \textbf{Ans.}$$

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the muff. Let τ_c be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 10^3\, \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. *Design for key*

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, $w = 18$ mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

$\therefore$ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L/2 = 195/2 = 97.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\, \tau_s$$

$$\therefore \quad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2}$$

$$= 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

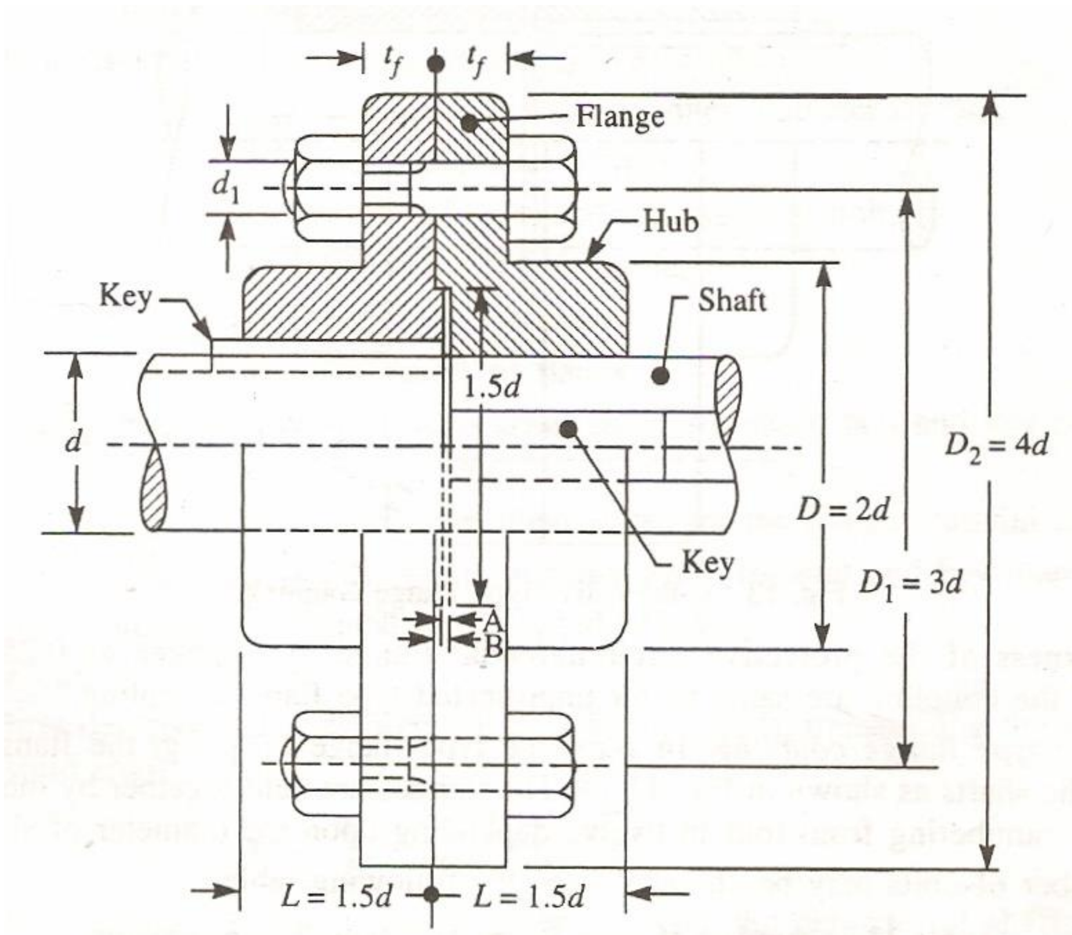
Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

3.14 Flange coupling

- A flange coupling is having two separate cast iron flanges.
- Each flange is mounted on the shaft end and keyed to it.
- The faces are turned up at right angle to the axis of the shaft.
- One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment.
- The two flanges are coupled together by means of bolts and nuts.
- The flange coupling is adopted to heavy loads and hence it is used on large shafting.
- The flange couplings are of the following **three types**

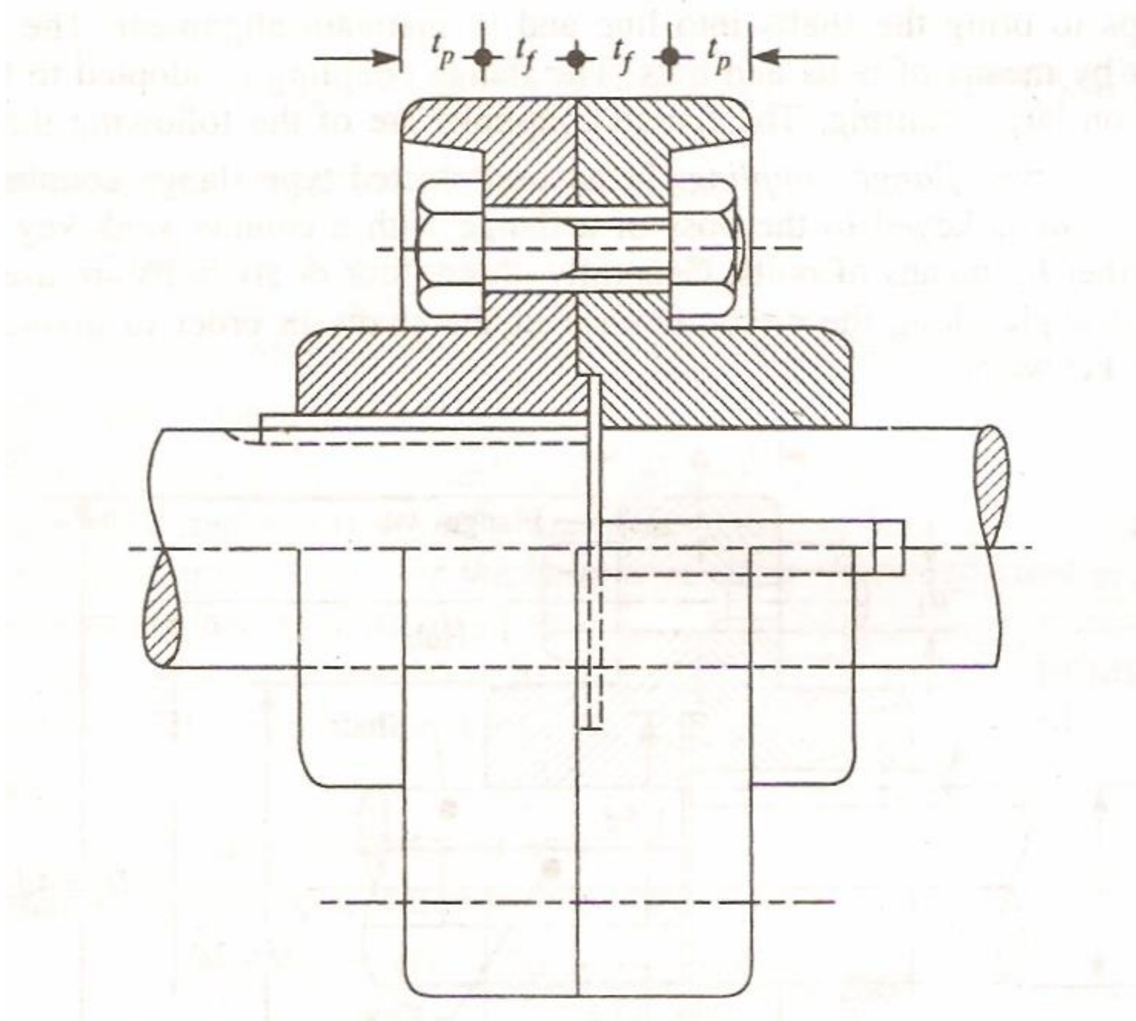
1. Unprotected type flange coupling.

- In an unprotected type flange coupling, as shown in following Fig. each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.
- Generally, three, four or six bolts are used.
- The keys are staggered at right angle along the circumference of the shafts in order to divide the weakening effect caused by keyways.



2. Protected type flange coupling.

- In a protected type flange coupling, as shown in following Fig.the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman.
- The thickness of the protective circumferential flange (t_p) is taken as 0.25 d. The other proportions of the coupling are same as for unprotected type flange coupling.



Design Procedure of Flange Coupling

- ❖ Consider a flange coupling as shown in above Fig.

Let ,

d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

d_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s , τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material *i.e.*, cast iron,

σ_{cb} , and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

Step 1. Design of Shaft

- ❖ Generally power transmitted by shaft is given , hence first of all find torque transmitted by shaft as

$$p= \frac{2\,\pi NT}{60}$$

T = ………N-m

T = ……x 10^3 N-mm

Now as per torsion equation

$$T = \frac{\pi}{16}\ \tau\ d^3$$

From above equation find (d)

Step 2 The usual proportions for both an <u>unprotected and protected type</u> cast iron flange couplings, are as follows

If **d** is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub,

D = 2d

Length of hub, L = 1.5 d

Pitch circle diameter of bolts,

D1 = 3d

Outside diameter of flange,

D2 =4d

Thickness of flange, tf = 0.5 d

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto ISO mm

Step 3. Design of Key

- The key is designed with usual proportions and then checked for shearing and crushing stresses.
- The material of key is usually the same as that of shaft.
- The length of key is taken equal to the length of hub.
- The usual proportions for **rectangular** key are

Width of key, w = d/4 , and

thickness of key, t = d/6

Where , d = Diameter of the shaft or diameter of the hole in the hub.

- And for **square key** proportions are

Width of key, w = d/4 , and

thickness of key, t = d/4

Where , d = Diameter of the shaft or diameter of the hole in the hub.

. **The length of key is taken equal to the length of hub**

l = L

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torque transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{... (Considering crushing of the key)}$$

Step 4 Hub design

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$\therefore \quad T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

Step 5 Design of Flange

3. *Design for flange*

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

Step 6 Design of Bolts

4. Design for bolts

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as $3\,d$. We know that

$$\text{Load on each bolt} = \frac{\pi}{4}(d_1)^2\,\tau_b$$

∴ Total load on all the bolts

$$= \frac{\pi}{4}(d_1)^2\,\tau_b \times n$$

and torque transmitted, $$T = \frac{\pi}{4}(d_1)^2\,\tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f)\,\sigma_{cb}$$

∴ Torque, $$T = (n \times d_1 \times t_f \times \sigma_{cb})\,\frac{D_1}{2}$$

From above equation , the induced crushing stress in the bolt may be checked .

Example: Design a rigid flange coupling to transmit a torque of 250 N-m between two co-axial shafts. The shaft is made of alloy steel, flanges out of cast iron and bolts out of steel Four bolts are used to couple the flanges. The shafts are keyed to the flange hub. The permissible stresses are given below:

Shear stress on shaft	= *100 MPa*
Bearing or crushing stress on shaft	= *250 MPa*
Shear stress on keys	= *100 MPa*
Bearing stress on keys	= *250 MPa*
Shearing stress on cast iron	= *200 MPa*
Shear stress on bolts	= *100 MPa*

After designing the various elements, make a neat sketch of the assembly indicating the important dimensions. The stresses developed in the various members may be checked if thumb rules are used for fixing the dimensions.

Soution. Given : T = 250 N-m = 250 × 10^3 N-mm ; n = 4 ; τ_s = 100 MPa = 100 N/mm^2 ; σ_{cs} = 250 MPa = 250 N/mm^2 ; τ_k = 100 MPa = 100 N/mm^2 ; σ_{ck} = 250 MPa = 250 N/mm^2 ; τ_c = 200 MPa = 200 N/mm^2 ; τ_b = 100 MPa = 100 N/mm^2

The cast iron flange coupling of the protective type is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 100 \times d^3 = 19.64\, d^3$$

$$\therefore \quad d^3 = 250 \times 10^3 / 19.64 = 12\,729 \quad \text{or} \quad d = 23.35 \text{ say } 25 \text{ mm } \textbf{Ans.}$$

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 25 = 50 \text{ mm}$$

and length of hub, $L = 1.5\,d = 1.5 \times 25 = 37.5$ mm

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(50)^4 - (25)^4}{50}\right] = 23\,013\, \tau_c$$

$$\therefore \quad \tau_c = 250 \times 10^3 / 23\,013 = 10.86 \text{ N/mm}^2 = 10.86 \text{ MPa}$$

Since the induced shear stress for the hub material (*i.e.*, cast iron) is less than 200 MPa therefore the design for hub is safe.

2. *Design for key*

From Table 13.1, we find that the proportions of key for a 25 mm diameter shaft are :

Width of key, w = 10 mm **Ans.**

and thickness of key, t = 8 mm **Ans.**

The length of key (l) is taken equal to the length of hub,

$$\therefore \quad l = L = 37.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. Considering the key in shearing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 37.5 \times 10 \times \tau_k \times \frac{25}{2} = 4688\, \tau_k$$

$$\therefore \quad \tau_k = 250 \times 10^3 / 4688 = 53.3 \text{ N/mm}^2 = 53.3 \text{ MPa}$$

Considering the key in crushing. We know that the torque transmitted (T),

$$250 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 37.5 \times \frac{8}{2} \times \sigma_{ck} \times \frac{25}{2} = 1875\, \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 250 \times 10^3 / 1875 = 133.3 \text{ N/mm}^2 = 133.3 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the given stresses, therefore the design of key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 d.

$$\therefore \quad t_f = 0.5\,d = 0.5 \times 25 = 12.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear. We know that the torque transmitted (T),

$$250 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (50)^2}{2} \times \tau_c \times 12.5 = 49\ 094\ \tau_c$$

$$\therefore \quad \tau_c = 250 \times 10^3 / 49\ 094 = 5.1\ \text{N/mm}^2 = 5.1\ \text{MPa}$$

Since the induced shear stress in the flange of cast iron is less than 200 MPa, therefore design of flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

We know that the pitch circle diameter of bolts,

$$\therefore \quad D_1 = 3\,d = 3 \times 25 = 75 \text{ mm } \textbf{Ans.}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$250 \times 10^3 = \frac{\pi}{4}(d_1)^2\,\tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\,100 \times 4 \times \frac{75}{2} = 11\ 780\ (d_1)^2$$

$$\therefore \quad (d_1)^2 = 250 \times 10^3 / 11\ 780 = 21.22 \quad \text{or} \quad d_1 = 4.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of the bolt is M 6. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 25 = 100 \text{ mm } \textbf{Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 25 = 6.25 \text{ mm } \textbf{Ans.}$$

3.15 Flexible Coupling

- We have already discussed that a flexible coupling is used to join the ends of shafts when they are not in exact alignment.
- Following are the different types of flexible couplings
  - I. Bushed pin flexible coupling,
  - 2. Oldham's coupling, and
  - 3. Universal coupling.

I. Bushed pin flexible coupling

- A bushed-pin flexible coupling, as shown in following Fig. is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction.
- A clearance of 5 mm is left between the face of the two halves of the coupling.
- There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.